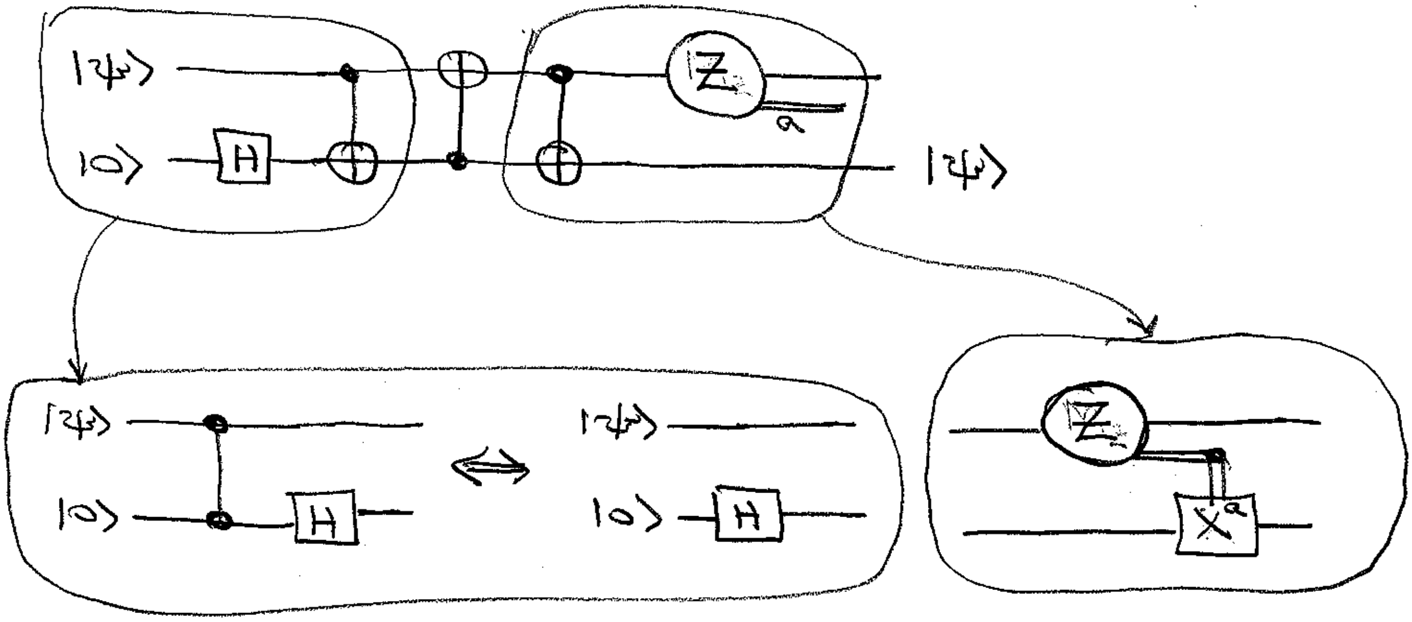
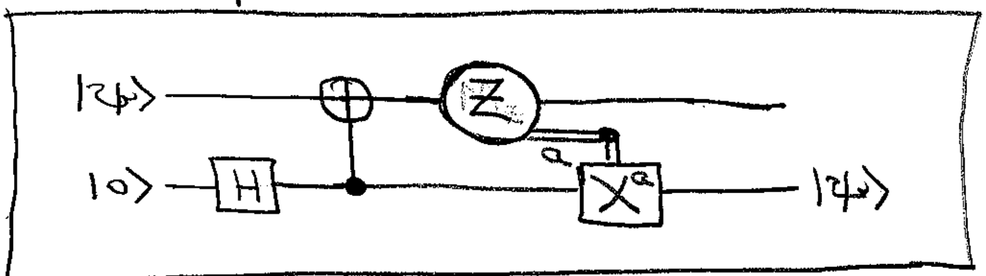
2.3 (a)



The top circuit is equivalent to



Check: that bottom circuit works:

$$|\psi\rangle \otimes |0\rangle = \sum_b c_b |b,0\rangle \xrightarrow[I\otimes H]{} \frac{1}{\sqrt{2}} \sum_{b,c} c_b |b,c\rangle \xrightarrow[V(X)]{} \frac{1}{\sqrt{2}} \sum_{b,c} c_b |b\oplus c, c\rangle$$

$$= \frac{1}{\sqrt{2}} \sum_{b,c} c_b |b,\bar{c}\rangle$$

$$= \frac{1}{\sqrt{2}} \sum_{b,c} c_{b\oplus c} |b,c\rangle$$

$$= \frac{1}{\sqrt{2}} |0\rangle \otimes (c_0|0\rangle + c_1|1\rangle) + \frac{1}{\sqrt{2}} |1\rangle \otimes (c_1|0\rangle + c_0|1\rangle)$$

$$\Big\downarrow M_Z$$

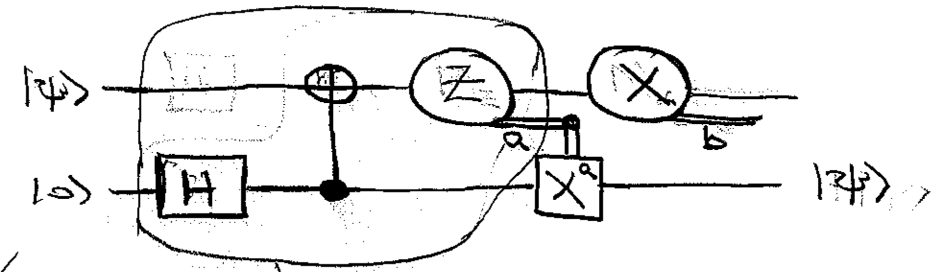
$$|a\rangle \otimes \sum_c c_{a\oplus c} |c\rangle$$

$$\Big\downarrow X^a$$

$$|a\rangle \otimes \sum_c c_{a\oplus c} |a\oplus c\rangle$$

$$= |a\rangle \otimes \sum_b c_b |b\rangle$$

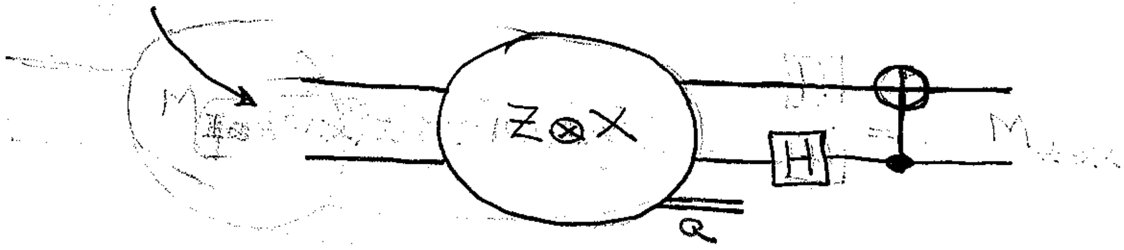
(b)

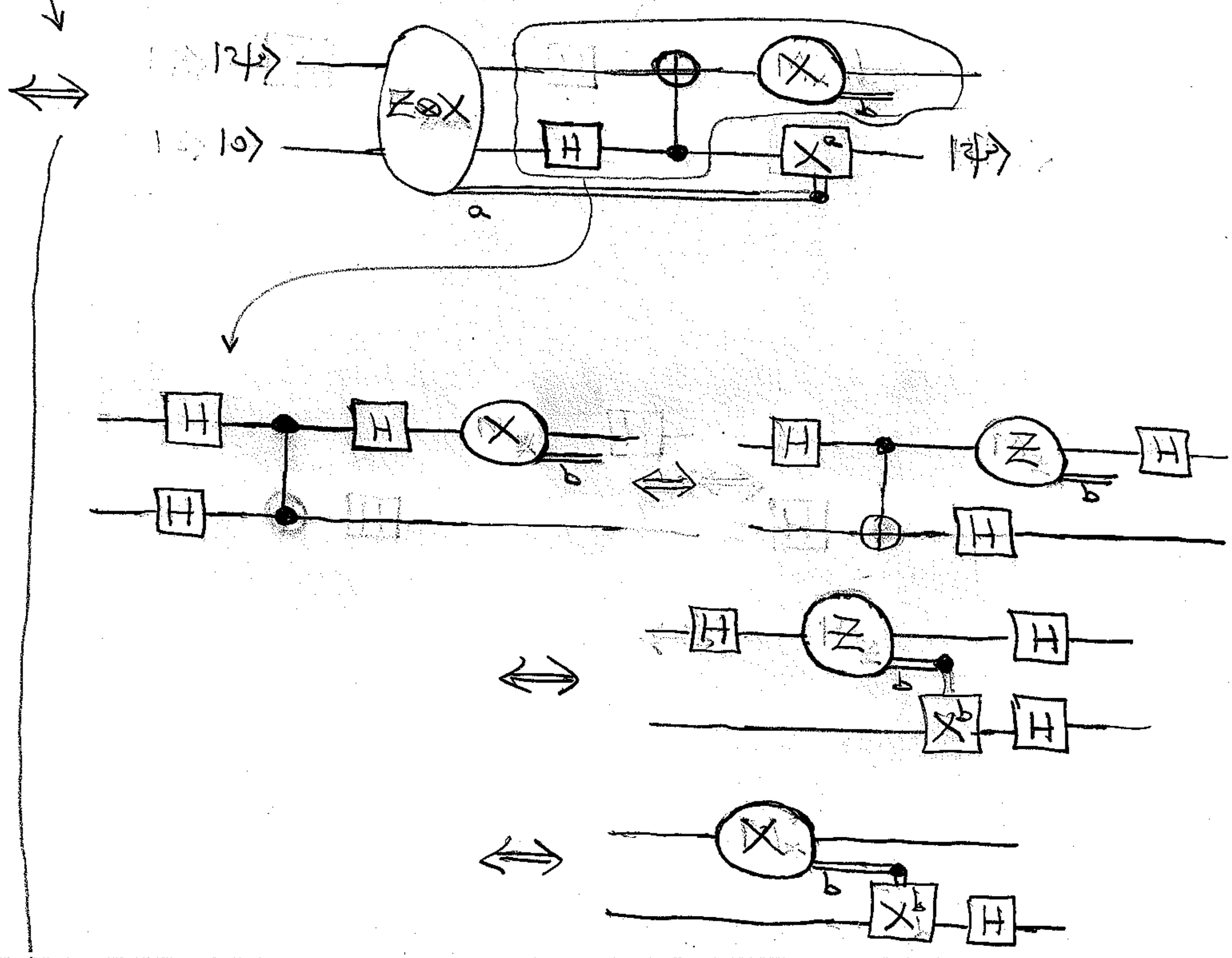


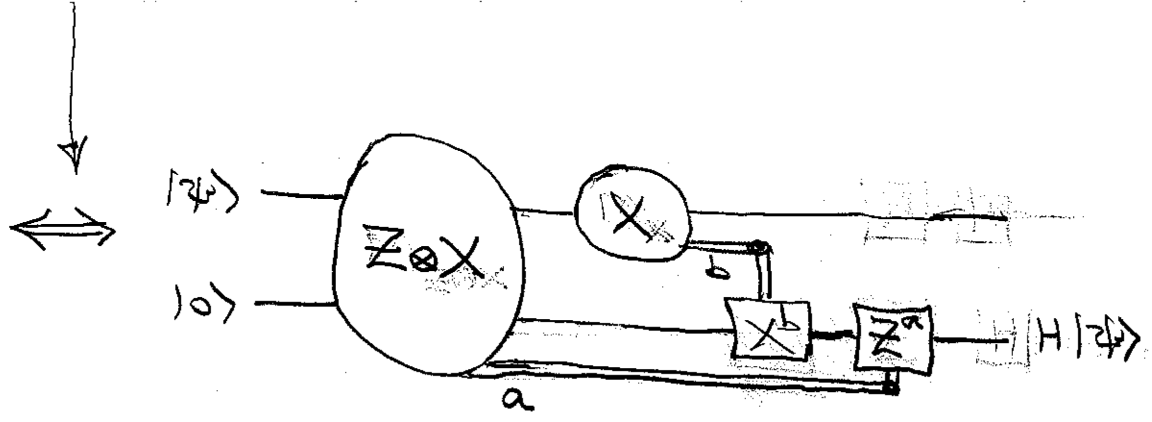
$U^\dagger = V(X)\, I\otimes H$

$I\otimes H \; V(X) \; Z\otimes I \; V(X) \; I\otimes H$

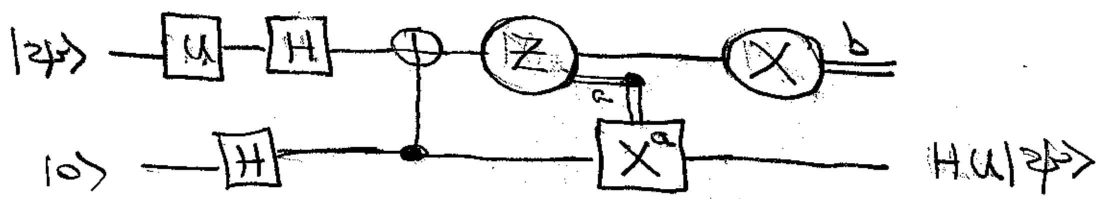
$= I\otimes H \; Z\otimes Z \; I\otimes H = Z\otimes X$

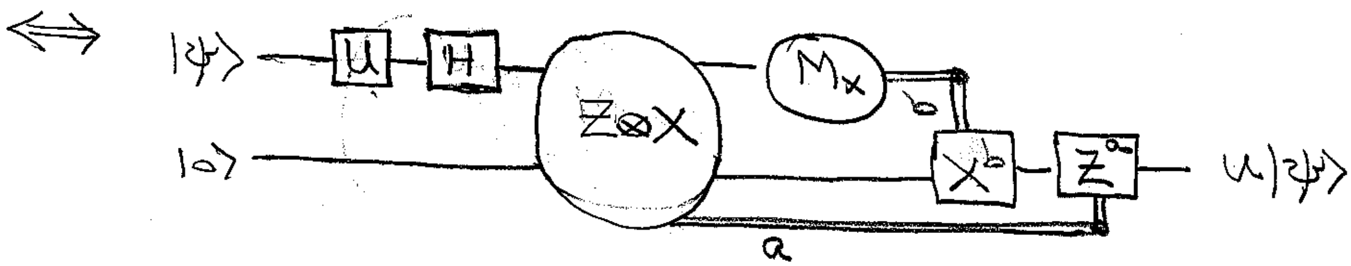


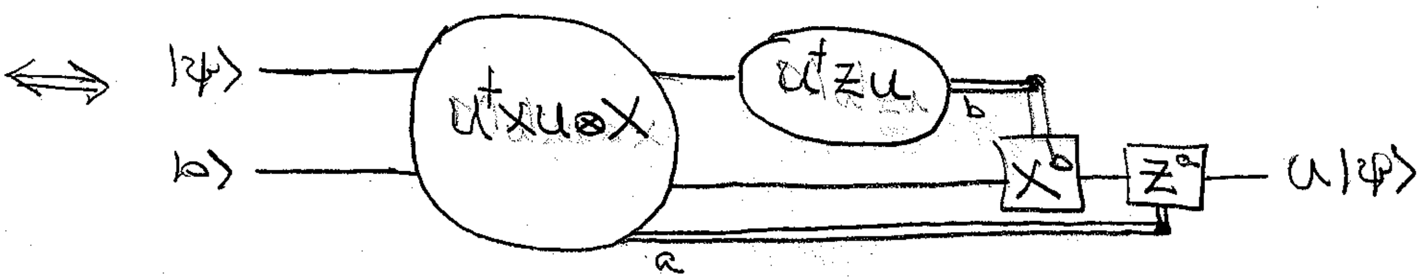




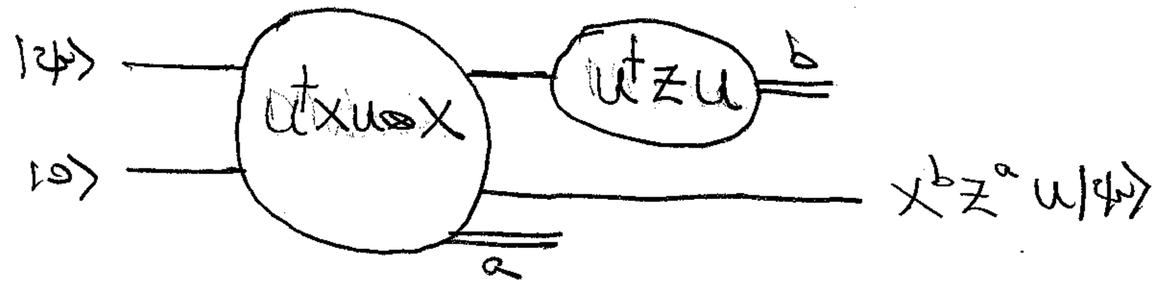
To do a unitary, we actually have to do HU at the input to get rid of the H at the output.







Leaving out the Pauli corrections, we have our single-qubit-measurement circuit up to Pauli errors.



This is the same measurement circuit we got in the lectures by a different route.