

Homework Problem 1.5

Mutual information of three random variables? Given three random variables X , Y , and Z , one is tempted to define a “mutual information” for all three variables,

$$H(X : Y : Z) \equiv H(X : Y) - H(X : Y|Z) = H(X) - H(X|Y) - H(X|Z) + H(X|Y, Z) ,$$

meant to be the amount of information shared by all three variables.

(a) Using a three-variable version of the Venn diagram for the joint information quantities of two variables (see the lecture notes or Nielsen and Chuang Fig. 11.2), *justify* the above definition.

(b) *Show* that this $H(X : Y : Z)$ can be negative (and thus is unsuitable as a measure of shared information) by considering the example of three binary variables with probabilities

$$p_{000} = p_{111} = 0 , \quad p_{001} = p_{010} = p_{100} = \frac{1}{3} , \quad p_{110} = p_{101} = p_{011} = 0 .$$