Phys 572 Quantum Information Theory Spring 2017

Homework Problem 2.6

Normal operators for qubits. Consider an arbitrary qubit operator

$$A = A_\alpha \sigma_\alpha = A_0 1 + \mathbf{A} \cdot \boldsymbol{\sigma} \ .$$

where the coefficients A_α are arbitrary complex numbers. We can write $\mathbf{A} = \mathbf{B} + i\mathbf{C}$, where $\mathbf{B}$ and $\mathbf{C}$ are the real and imaginary parts of $\mathbf{A}$.

(a) *Show* that the condition for A to be a normal operator is that $0 = \mathbf{A}\times\mathbf{A}^* = -2i\mathbf{B}\times\mathbf{C}$, i.e., that $\mathbf{C} = \mu\mathbf{B}$, for some (real) constant μ.

(b) *Find* the conditions on A_0 and $\mathbf{A}$ for A to be (i) a Hermitian operator and (ii) a unitary operator.

(c) *Show* that A has a spectral decomposition if and only if A is normal.