1.4.

$$P_{X,Y}(0,0) = P_{X,Y}(1,1) = p/2 \qquad p+q=1$$

$$P_{X,Y}(0,1) = P_{Y,X}(1,0) = q/2$$

(a) $\underbrace{P_Y(1) = P_X(1) = P_Y(0) = P_X(0)}_{\text{Symmetry}} = P_X(0,0) + P_X(0,1) = \frac{p+q}{2} = \frac{1}{2}$

$\underbrace{P_{Y|X}(1|1) = P_{Y|X}(0|0) = P_{X|Y}(1|1) = P_{X|Y}(0|0)}_{\text{Symmetry}} = \frac{P_{X,Y}(0,0)}{P_Y(0)} = p$

$\underbrace{P_{Y|X}(1|0) = P_{Y|X}(0|1) = P_{X|Y}(1|0) = P_{X|Y}(0|1)}_{\text{Symmetry}} = \frac{P_{X,Y}(0,1)}{P_Y(1)} = q$

(b) $H(Y) \underset{\substack{\uparrow \\ \text{symmetry}}}{=} H(X) = -P_X(0)\log P_X(0) - P_X(1)\log P_X(1) = 1$ bit

$$H(Y|X) = H(X|Y) = \sum_y P_Y(y)\left(-\sum_x P_{X|Y}(x|y)\log P_{X|Y}(x|y)\right)$$

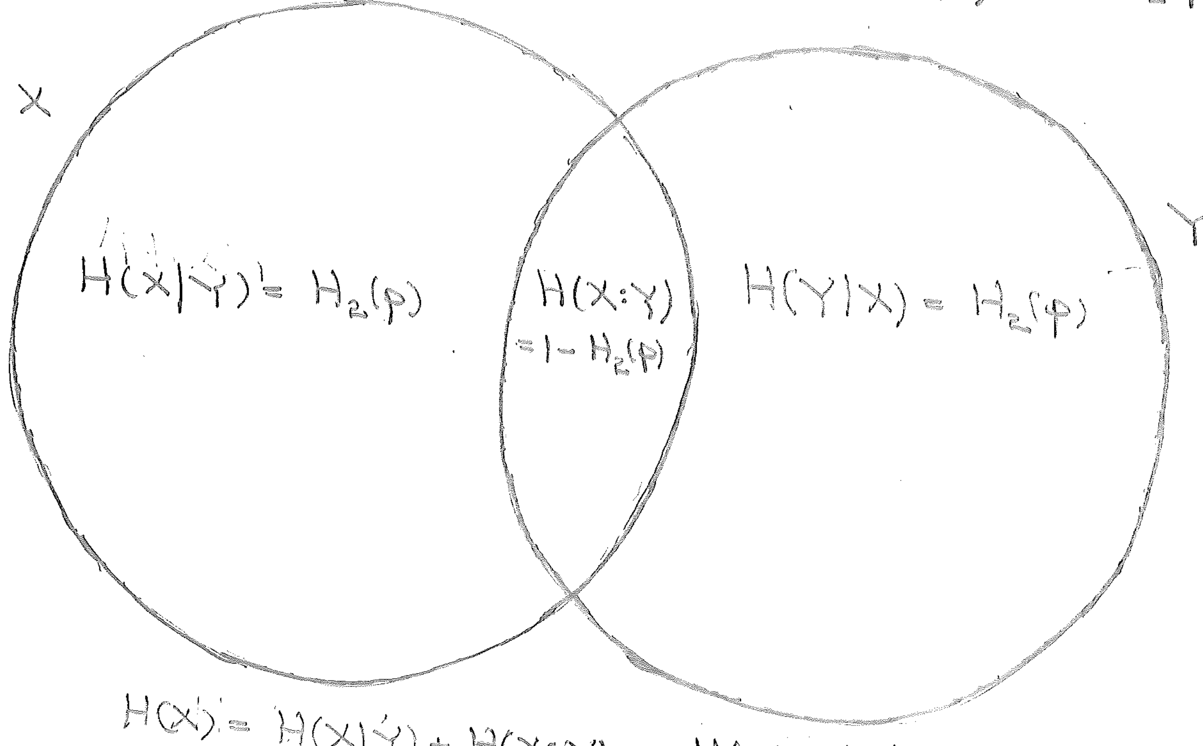
$$= -p\log p - q\log q$$

$$= H_2(p) = H_2(q)$$

$$H(X,Y) = H(Y) + H(X|Y) = 1 + H_2(p)$$

$$H(X:Y) = H(X) - H(X|Y) = 1 - H_2(p)$$

$$H(X,Y) = H(X|Y) + H(X:Y) + H(Y|X) = 1 + H_2(p)$$



$$H(X) = H(X|Y) + H(X:Y) = 1 \qquad H(Y) = H(Y|X) + H(X:Y) = 1$$

$p=1,\ q=0$: The two circles lie on top of one another; the one bit of information — the answer to the question, "Is it 00 or 11?" — is shared in common by X and Y.

$p=0,\ q=1$: The two circles lie on top of one another; the one bit of information — the answer to the question, "Is it 01 or 10?" — is shared in common by X and Y.

$p=q=1/2$: The two circles are disjoint; the two variables are uncorrelated and thus share no information. For each variable, the one bit of information is the answer to the question, "Is it 0 or 1?"

Generally, both variables store one bit of information. Finding out the one-bit value of Y leaves one with probabilities p and $q = 1-p$ for the two values of X, corresponding to conditional information

$$H(X|Y) = H_2(p).$$