WHAT'S WRONG WITH THESE ELEMENTS OF REALITY?

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The subject of Einstein–Podolsky–Rosen correlations—those strong quantum correlations that seem to imply "spooky actions at a distance"—has just been given a new and beautiful twist. Daniel Greenberger, Michael Horne and Anton Zeilinger have found a clever and powerful extension of the two-particle EPR experiment to *gedanken* decays that produce more than two particles.[1] In the GHZ experiment the spookiness assumes an even more vivid form than it acquired in John Bell's celebrated analysis of the EPR experiment, given over 25 years ago.[2] The argument that follows is my attempt to simplify a refinement of the GHZ argument given by the philosophers Robert Clifton, Michael Redhead and Jeremy Butterfield.[3]

Consider *three* spin-½ particles, named 1, 2 and 3. They have originated in a spin-conserving *gedanken* decay and are now *gedanken* flying apart along three different straight lines in the horizontal plane. (It's not essential for the *gedanken* trajectories to be coplanar, but it makes it easier to describe the rest of the geometry.) I specify the spin state Ψ of the three particles in a time-honored manner, giving you a complete set of commuting Hermitian spin-space operators of which Ψ is an eigenstate.

Those operators are assembled out of the following pieces (measuring all spins in units of $\frac{1}{2}\hbar$): σ_z^i, the operator for the spin of particle i along its direction of motion; σ_x^i, the spin along the vertical direction; and σ_y^i, the spin along the horizontal direction orthogonal to the trajectory. (Any three orthogonal directions independently chosen for each particle would do. But we're going to be *gedanken* measuring x and y components of each particle's spin, so it's nice to think of the x and y directions as orthogonal to the direction of motion, since the components can then be straightforwardly measured by passage through a conventional Stern–Gerlach magnet.) The complete set of commuting Hermitian operators consists of

$$\sigma_x^1\sigma_y^2\sigma_y^3,\quad \sigma_y^1\sigma_x^2\sigma_y^3,\quad \sigma_y^1\sigma_y^2\sigma_x^3. \qquad (1)$$

Even though the x and y components of a given particle's spin anticommute—a fact of paramount importance in what follows—all three of the operators in (1) do indeed commute with one another, because the product of any two of them differs from the product in the reverse order by an even number of such anticommutations. Because they all commute, the three operators can be provided with simultaneous eigenstates. Since the square of each of the three is unity, the eigenvalues of each are $+1$ or -1, and the 2^3 possible choices are indeed just what we need to span the eight-dimensional space of three spins-½.

For simplicity of exposition let's focus our attention on the symmetric eigenstate in which each of the operators (1) has the eigenvalue $+1$. (Its state vector is $\Psi = (1/\sqrt{2})(|1,1,1\rangle - |-1,-1,-1\rangle)$, where 1 or -1 specifies spin up or down along the appropriate z axis, but you don't need to know this. I'm only telling you because discussions of EPR always write down an explicit form for the state vector and I wouldn't want you to think you were missing anything.) Because the spin vectors of distinct particles commute component by component, we can simultaneously measure the x component of one particle and the y components of the other two (using three Stern–Gerlach magnets in three remote regions of space). Since the three particles are in an eigenstate of all three operators (1) with eigenvalue unity, the product of the results of the three spin measurements has to be $+1$, regardless of which particle we single out for the x-spin measurement.

This affords an immediate application of the EPR reality criterion[4]: "If, without in any way disturbing a system, we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity." The "element of physical reality" is that predictable value, and it ought exist whether or not we actually carry out the procedure necessary for its prediction, since that procedure in no way disturbs it. Because the product of the results of measuring one x component and two y components is unity in the state Ψ, we can predict with certainty the result of measuring the x component of the spin of any one of the three particles by measuring the y components of the two other, far away particles. For if both y components turn out to be the same then the x component, when measured, must yield the value $+1$; if the two y components turn out to be different, the subsequently measured x component will necessarily yield the value -1. In the absence of spooky actions at a distance or the metaphysical cunning of a Niels Bohr, the two far away y-component measurements cannot "disturb" the particle whose x component is subsequently to be measured. The EPR reality criterion therefore asserts the existence of elements of reality m_x^1, m_x^2 and m_x^3, each having the value $+1$ or -1, each waiting to be revealed by the appropriate pair of far away y-component measurements.

In much the same way, we can also predict the result of measuring the y component of the spin of any particle with certainty, by measuring one x component and one y component of the spins of the other two. There are thus elements of reality m_y^1, m_y^2 and m_y^3, with values $+1$ or -1, also waiting to be revealed by far away measurements. All six of the elements of reality m_x^i and m_y^i have to be there, because we can predict in advance what any one of the six values will be by measurements made

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so far away that they cannot disturb the particle that subsequently does indeed display the predicted value.

This conclusion is, of course, highly heretical, because σ_x^i does not commute with σ_y^i—in fact the two *anti*-commute—and therefore they cannot have simultaneous values. (The operators (1) are nicely chosen to hide this failure to commute, since the anticommutations always occur in pairs.) But heresy or not, since the result of either measurement can be predicted with probability 1 from the results of other measurements made arbitrarily far away, an open-minded person might be sorely tempted to renounce quantum theology in favor of an interpretation less hostile to the elements of reality.

In the GHZ experiment, however, as in Bell's version of the EPR, the elements of reality are demolished by the straightforward quantum mechanical predictions for some additional experiments, entirely unencumbered by accompanying metaphysical baggage.

In the GHZ case the demolition is spectacularly more efficient. Suppose, heretically, that the elements of reality really do exist in each run of the experiment. While we cannot know all six of their values, those values are constrained by the fact that the values of $\sigma_x^1\sigma_y^2\sigma_y^3$, $\sigma_y^1\sigma_x^2\sigma_y^3$ and $\sigma_y^1\sigma_y^2\sigma_x^3$, all unity in the state Ψ, are given by the values of the corresponding products $m_x^1 m_y^2 m_y^3$, $m_y^1 m_x^2 m_y^3$ and $m_y^1 m_y^2 m_x^3$. But if these latter three quantities are unity, so is their combined product. Since each individual m_y^i is either $+1$ or -1 and each occurs twice in the combined product, that combined product is just $m_x^1 m_x^2 m_x^3$. So the existence of the elements of reality implies that should we choose to measure the x components of all three spins in the state Ψ, the product of the three resulting values must once again be $+1$.

The value of that product can also be determined without invoking disreputable elements of reality by a simple quantum mechanical calculation, since it is just the result of measuring the Hermitian operator

$$\sigma_x^1\sigma_x^2\sigma_x^3. \qquad (2)$$

You can easily check that this operator also commutes with all of the operators (1): Once again the number of anticommutations is always even. This is encouraging, for if the value of the operator (2) in the state Ψ is invariably to be $+1$, it had better also have Ψ for an eigenstate, a requirement that is guaranteed by its commuting with all three members of the complete set of commuting operators (1) whose eigenvalues define Ψ.

However:

Not only does (2) commute with each of the operators (1), but you can easily check that it is a simple explicit function of them, namely, *minus* the product of all three. The (crucial) minus sign arises because here, at last, in bringing the pairs of operators σ_y^i together to produce unity, one runs up against an *odd* number of anticommutations of σ_y^i's with σ_x^i's. Since Ψ is an eigenstate with eigenvalue $+1$ of each of the operators (1), it is therefore indeed an eigenstate of the operator (2), but with the *wrong* eigenvalue, opposite in sign to the one required by the existence of the elements of reality.

So farewell elements of reality! And farewell in a hurry. The compelling hypothesis that they exist can be refuted by a *single* measurement of the three x components: The elements of reality require the product of the three outcomes *invariably* to be $+1$; but *invariably* the product of the three outcomes is -1.

This is an altogether more powerful refutation of the existence of elements of reality than the one provided by Bell's theorem for the two-particle EPR experiment. Bell showed that the elements of reality inferred from one group of measurements are incompatible with the *statistics* produced by a second group of measurements. Such a refutation cannot be accomplished in a single run, but is built up with increasing confidence as the number of runs increases. Thus in one simple version of the two-particle EPR experiment (which I described in PHYSICS TODAY, April 1985, page 38) the hypothesis of elements of reality requires a class of outcomes to occur at least 55.5% of the time, while quantum mechanics allows them to occur only 50% of the time. In the GHZ experiment, on the other hand, the elements of reality require a class of outcomes to occur *all* of the time, while quantum mechanics *never* allows them to occur.

It is also appealing to see the failure of the EPR reality criterion emerge quite directly from the one crucial difference between the elements of reality (which, being ordinary numbers, necessarily commute) and the precisely corresponding quantum mechanical observables (which sometimes anticommute).

I was surprised to learn of this always-vs-never refutation of Einstein, Podolsky and Rosen. After all, quantum magic generally flows from the fact that it is the amplitudes that combine like probabilities rather than the probabilities themselves. But when the probabilities are zero, so are the amplitudes. Guided by such woolly thinking, and the failure of anybody to strengthen Bell's result in this direction in the ensuing 25 years, I recently declared in writing[5] that no set of experiments, real or *gedanken*, was known that could produce such an all-or-nothing demolition of the elements of reality. With a bow of admiration to Greenberger, Horne and Zeilinger, I hereby recant.

References

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