

Physics 405 Spring 2007

Problem Set #1: DUE Fri. 1/26/07

Read Griffiths, "Advertisement" and Chap. 1

(1) Gradients (Griffiths 1.12)

The height of a certain hill (in feet) is given by

$$h(x, y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$$

where y is the distance (in miles) north, x is the distance east of South Hadley.

- (a) Where is the top of the hill located?
- (b) How high is the hill?
- (c) How steep is the slope (in feet per mile) at a point 1 mile north and one mile east of South Hadley? In what direction is the slope steepest at that point?

(d) Check your answer with *Mathematica* by:

(i) **Define:** $h[x_, y_] := 10 * (2 * x * y - 3 * x^2 - 4 * y^2 - 18 * x + 28 * y + 12)$

(ii) **Plot $h(x, y)$:**

$$hPlot1 = \text{Plot3D}[h[x, y], \{x, xi, xf\}, \{y, yi, yf\}] \text{ (you choose xi, xf, etc.)}$$

(iii) **Plot contours:**

$$hPlot2 = \text{ContourPlot}[h[x, y], \{x, xi, xf\}, \{y, yi, yf\}]$$

(iv) **Get Graphics Package:** Needs["Graphics`PlotField`"]

(v) **Plot Gradient:**

$$\text{gradPlot} = \text{PlotGradientField}[h[x, y], \{x, xi, xf\}, \{y, yi, yf\}]$$

(vi) **Show gradient field and contours together:**

$$\text{Show}[hPlot2, \text{gradPlot}]$$

(2) Div and Curl

Calculate the divergence and curl of the following vector fields

(a) $\mathbf{v} = x^2 \hat{\mathbf{x}} + 3xz^2 \hat{\mathbf{y}} - 2xz \hat{\mathbf{z}}$

(b) $\mathbf{v} = \hat{\mathbf{x}} \cos(kz - \omega t)$,

(3) Practice with Gradient (fixed and variable points) (Griffiths 1.13)

(4) Notion of Flux: (Griffiths 1.29)