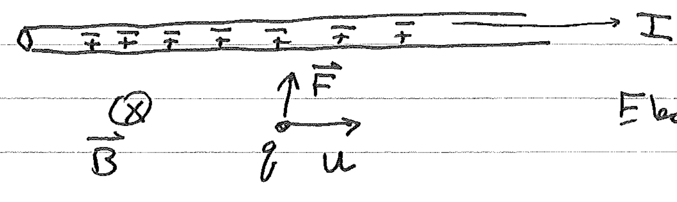
Lecture: #24 Transformation of Fields

- Magnetism as a relativistic phenomena

Is the Lorentz force law consistent with relativity



Electrically neutral
$\vec{E} = 0$

What if we go to the instaneous rest frame of q?
In this frame charge velocity is zero, no
no force due to mag. field. But charge is
attracted to wire

$\Rightarrow$ There must be an effective $\vec{E}$-field seen by q in its own rest frame

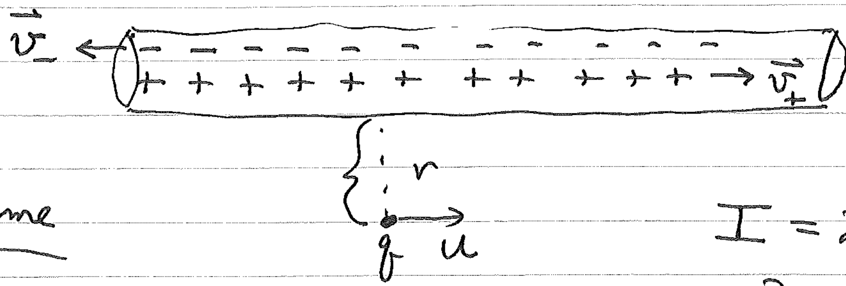
$\Rightarrow$ Magnetic phenomenon in one frame is electric phenomenon in another

$\Rightarrow$ One physical quantity: the electromagnetic field

We seek transformation laws

$$(\vec{E}', \vec{B}') \Leftrightarrow (\vec{E}, \vec{B})$$

Model: Two line charges moving in opposite directions



$|\vec{v}_+| = |\vec{v}_-| = v$

$|\lambda_+| = |\lambda_-| = \lambda$

Lab Frame

$I = 2\lambda v \, \hat{e}_x$

$\lambda_{total} = 0$

$$\vec{E} = 0, \quad \vec{B} = \frac{2I}{cr} = \frac{4\lambda v}{rc}, \quad F_\perp = q\frac{u}{c}B = q\frac{4\lambda}{r}\frac{uv}{c^2}$$

Transform to the charges instaneous rest frame

$$c\lambda'_{total} = -\gamma(u)\frac{u}{c}I \Rightarrow \lambda'_{Total} = -\gamma(u)\frac{2\lambda uv}{c^2}$$

Net negative charge on wire in q's rest frame!

> Physical picture: Length contraction is more for oppositely moving charges compared to coving charges
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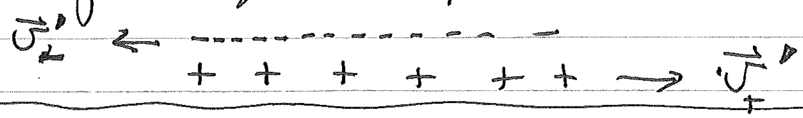
> 
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$\Rightarrow$ Net electric field in q's rest frame

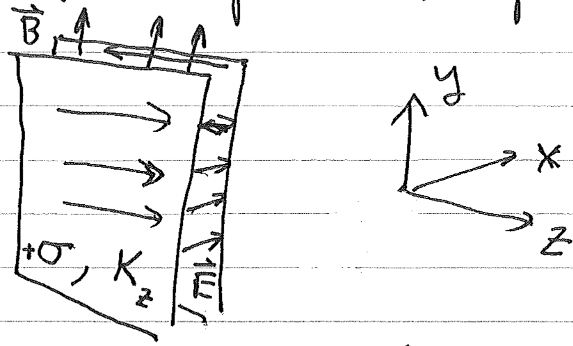
$$E' = \frac{2\lambda'_{Total}}{r} = -\gamma(u)\frac{4\lambda}{r}\frac{uv}{c^2}$$

Force in S': $F'_\perp = qE' = -\gamma(u)\, q\frac{4\lambda}{r}\frac{uv}{c^2}$

Agree's with Minkowski Force: $F_\perp^{(rest)} = \gamma F_\perp^{(moving)}$

Transformation Law for fields

Consider a parallel plate with surface current flowing



In Lab

$$\vec{E} = 4\pi\sigma\,\vec{e}_x$$

$$\vec{B} = \frac{4\pi}{c} K_z\,\vec{e}_y$$

Consider transformations

- Frame moving with v along $\vec{e}_z$

$$K_z' = \gamma(K_z - v\sigma) \qquad \sigma' = \gamma\left(\sigma - \frac{v}{c^2} K_z\right)$$

$$\Rightarrow \quad E_x' = 4\pi\sigma' = \gamma\left(4\pi\sigma - \frac{v}{c}\frac{4\pi}{c}K_z\right) = \gamma\left(E_x - \frac{v}{c}B_y\right)$$

$$B_y' = \frac{4\pi}{c}K_z' = \gamma\left(\frac{4\pi}{c}K_z - \frac{v}{c}4\pi\sigma\right) = \gamma\left(B_y - \frac{v}{c}E_x\right)$$

- Frame moving along $\vec{e}_y$

$$K_z' = K_z, \qquad \sigma' = \gamma\sigma, \qquad K_y' = -\gamma v\sigma$$

$$E_x' = 4\pi\sigma' = \gamma E_x, \qquad B_z = -\frac{4\pi}{c}K_y' = \gamma\frac{v}{c}E_x$$

$$B_y' = \frac{4\pi}{c}K_z' = B_y$$

(Cannot consider third transformation given two infinite planes)

$\Rightarrow$ General transformation law

Given $\vec{E}$ and $\vec{B}$ in frame S
seek $\vec{E}'$ and $\vec{B}'$ in frame S'

$$\boxed{\begin{aligned} \vec{E}_\perp' &= \gamma\left(\vec{E}_\perp + \frac{\vec{v}}{c}\times\vec{B}_\perp\right) \qquad & \vec{E}_\parallel' &= \vec{E}_\parallel \\ \vec{B}_\perp' &= \gamma\left(\vec{B}_\perp - \frac{\vec{v}}{c}\times\vec{E}_\perp\right) \qquad & \vec{B}_\parallel' &= \vec{B}_\parallel \end{aligned}}$$

Special cases:

- $\vec{B} = 0$ in S $\Rightarrow$ in S' $\vec{E}_\perp' = \gamma(\vec{E}_\perp)$, $\vec{B}_\perp' = -\frac{\vec{v}}{c}\times\gamma\vec{E}_\perp$
  $\vec{E}_\parallel' = \vec{E}_\parallel$, $\vec{B}_\parallel' = 0$

$$\Rightarrow \boxed{\vec{B}' = -\frac{\vec{v}}{c}\times\vec{E}'}$$

- $\vec{E} = 0$ in S $\Rightarrow$ in S' $\vec{B}_\perp' = \gamma\vec{B}_\perp$, $\vec{E}_\perp' = \frac{\vec{v}}{c}\times\gamma\vec{B}_\perp$
  $\vec{B}_\parallel' = \vec{B}_\parallel$, $\vec{E}_\parallel' = 0$

$$\Rightarrow \boxed{\vec{E}' = \frac{\vec{v}}{c}\times\vec{B}'}$$

If $\exists$ a frame in which either $\vec{E}$ or $\vec{B}$ is zero, then in any other frame $\vec{E}'$ and $\vec{B}'$ are simply related, $\perp$, and

Note Pure F[illegible]