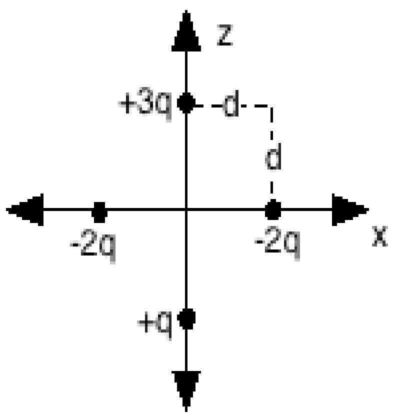
Physics 511

Problem Set #3: DUE Wed. Feb. 8, 2006

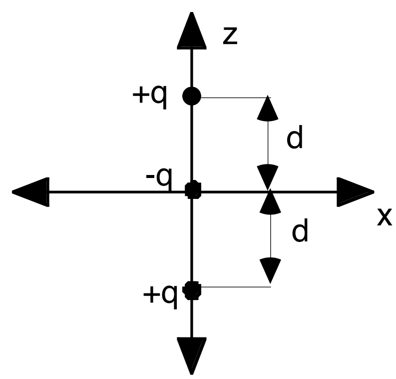
Read Jackson: Chap. 3.1-3.3,3.5,3.6
Low: Appendix B, 2.1-2.3

Problem 1 (10 points)

(a) For each the charge distributions drawn below, explicitly calculate the first three (monopole, dipole, quadrapole) Cartesian and Spherical multipole moments (all components).



(i)



(ii)

(b) Using these expressions, write the potential as a function of x,y, and z to order $(d/r)^3$

(c) Write the exact potential and expand to the same order to check your result.

(d) Plot the equipotential contours in the x-z plane for the exact potential and the approximate potential found in part (b) (set q=d=1). Plot for regions for r/d~1 and r>>d. Comment on these plots.

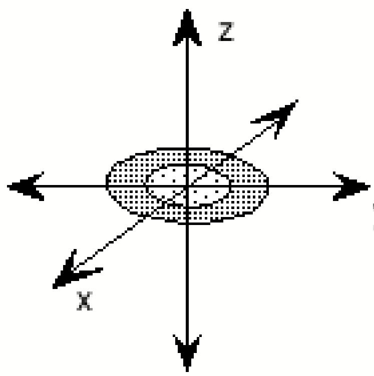
Problem 2: Jackson 4.4 (10 points)

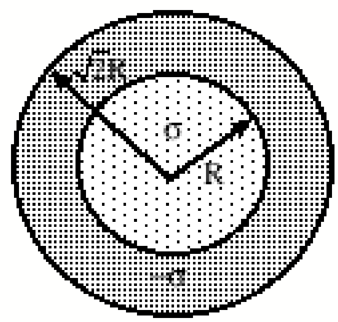
Problem 3: Jackson 4.6 (10 points)

Problem 4 (10 points)

Multipole moments for an azimuthally symmetric charge distribution.

Consider a disk of radius R and surface charge/area σ surrounded by an annulus of charge with outer radius $\sqrt{2}$ R, inner radius R, and surface charge/area –σ as sketched below.





Top View

(a) Find the multipole expansion of the potential up to the quadrapole moment. Express your final answer for the potential in spherical coordinates in terms of Legendre polynomials.

(b) Use direct integration to show that on the z-axis, the potential is

$$\Phi(z) = 2\pi\sigma\left(2\sqrt{z^2 + R^2} - \sqrt{z^2 + 2R^2} - z\right)$$

(c) Check your result. Since the charge distribution is azimuthally symmetric, outside the charge distribution the potential satifies Laplace's equation as given in in the form, Jackson Eq. (3.33). For r >$\sqrt{2}$ R use the answer to part (b) as a boundary condition at θ=0, (i.e. $\Phi(r,\theta=0) = 2\pi\sigma\left(2\sqrt{r^2 + R^2} - \sqrt{r^2 + 2R^2} - r\right)$), together with the b.c. $\lim_{r\to\infty}\Phi(r,\theta) \to 0$, to find the expansion coefficients, $\{A_l, B_l\}$ up to order $(R/r)^3$.