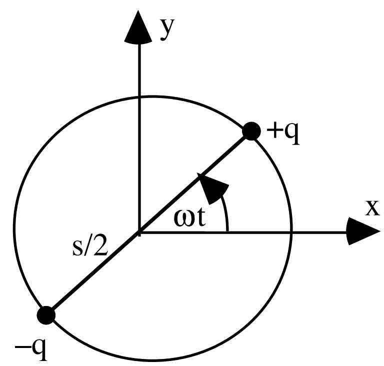
Physics 511 Spring

Problem Set #8: Due Friday April 28, 2006

Problem 1 Radiation by a rotating electric dipole (15 points)

Consider an electric dipole rotating in the x-y plane



The motion of this dipole can be model by the superposition of two linearly oscillating dipole which are 90° out of phase with one another:
$\mathbf{p}(t) = p_0 \cos(\omega t)\hat{\mathbf{x}} + p_0 \sin(\omega t)\hat{\mathbf{y}}, \quad p_0 \equiv qs$.

(a) An observer is located at position (r, θ, ϕ), with respect to the origin at the center of the ring. If we write the electric field associated with the dipole radiation as $\mathbf{E}(\mathbf{x}, t) = \mathrm{Re}\left[\tilde{\mathbf{E}}(\mathbf{x})e^{-i\omega t}\right]$, show that the complex amplitude in the radiation zone is,

$$\tilde{\mathbf{E}}(r, \theta, \phi) = k^2 p_0 \left(\cos\theta\ \hat{\theta} + i\hat{\phi}\right) \frac{e^{i(kr+\phi)}}{r}$$

What is the polarization of the wave at an arbitrary point?

(b) As a check show that on the x, y, and z axes the real part of the electric fields are

On x-axis (i.e. θ=π/2, φ=0) $\qquad \mathbf{E}(\mathbf{x}, t) = k^2 p_0 \dfrac{\sin(\omega t - kx)}{x}\ \hat{\mathbf{y}}$

On y-axis (i.e. θ=π/2, φ=π/2) $\qquad \mathbf{E}(\mathbf{x}, t) = k^2 p_0 \dfrac{\cos(\omega t - ky)}{y}\ \hat{\mathbf{x}}$

On z-axis (i.e. θ=0, φ=0) $\quad \mathbf{E}(\mathbf{x},t) = k^2 p_0 \left(\dfrac{\cos(\omega t - kz)\hat{\mathbf{x}} + \sin(\omega t - kz)\hat{\mathbf{y}}}{z} \right)$

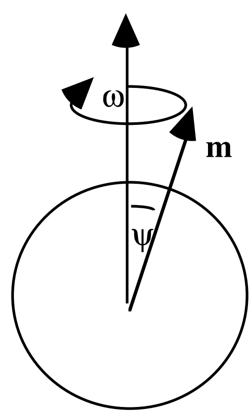
(Remember that the values of the spherical unit vectors depend on position.
e. g. Along x-axis (i.e. θ=π/2, φ=0) $\hat{\theta} = \hat{\mathbf{z}}$, $\hat{\phi} = \hat{\mathbf{y}}$) Please explain why these are what you expect.

(c) Find the time-averaged rate at which electromagnetic energy, momentum, and angular momentum, are radiated per solid angle. Sketch the angular distribution of both the radiated power, and the z-component of the angular momentum.

(d) Find the total time-averaged rate at with energy, momentum, and angular momentum are radiated to infinity. Comment on your result.

Problem 2. Magnetic Dipole Radiation (10 points)

Pulsars are thought to be rotating neutron stars, which have a strong magnetic moment, but which rotate about an axis different than the magnetic north pole (as does the earth)



Given a neutron star with typical radius of 10 km, a rotational period of 10^{-3} s, a surface magnetic field of 10^8 Tesla, an a tilt of ψ=10 degrees. Calculate the energy radiated (as magnetic dipole radiation) per year and estimate the relative change per year (use the solar mass as an estimate of the star's mass). How long will such a pulsar "live"?

Problem 3. Radiation from the hydrogen atom (10 Points)
(This is problem 9.4 from Jackson 2nd edition)

Consider the hydrogen atom in a superposition of the ground state ($1s$) and the first exited state ($2p$, m=0), where m is so-called the "magnetic quantum number". The effective charge density, associated with the electron motion relative to the nucleus, is given by product wave functions

$$\rho(r,\theta,t) = -e\psi^*_{2p,0}(\mathbf{x},t)\psi_{1s}(\mathbf{x},t) + c.c. = 2\mathrm{Re}\left(-e\psi^*_{2p,0}(\mathbf{x},t)\psi_{1s}(\mathbf{x},t)\right)$$
$$= \mathrm{Re}\left(\frac{4e}{\sqrt{6}}a_0^{-4}\ re^{-3r/2a_0}\,Y_{00}Y_{10}(\theta)\,e^{-i\omega t}\right),$$

where $a_0 = \hbar^2 / me^2$ is the Bohr radius and $\hbar\omega_0 = 3e^2\ /8a_0$ is the energy difference between the two levels.

(a) Evaluate all the radiation multipoles in the long wavelength limit.

(b) In the electric dipole approximation, calculate the total time-average power radiated. Express your answer in units of $\alpha^4(\hbar\omega_0)c / a_0$ where $\alpha = e^2 / \hbar c$ is the fine-structure constant.

(c) Estimate the transition rate between energy levels as the classical power divided by the photon energy $\hbar\omega_0$.

(d) If, instead of the semiclassical charge density used above, we described the $2p$ state by a *circular* Bohr orbit of radius $2a_0$, with the electron moving at the angular velocity ω_0, what would the power be? Express your answer in the same units as in (b) and evaluate the ratio of the two powers numerically. Please comment on your result.