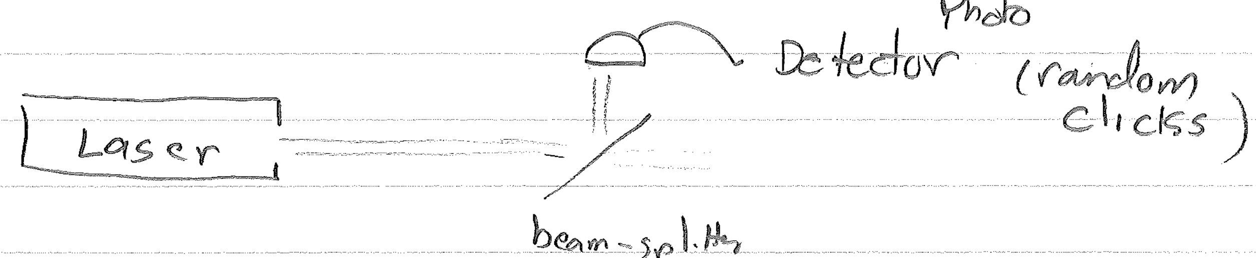
Physics 566: Quantum Optics

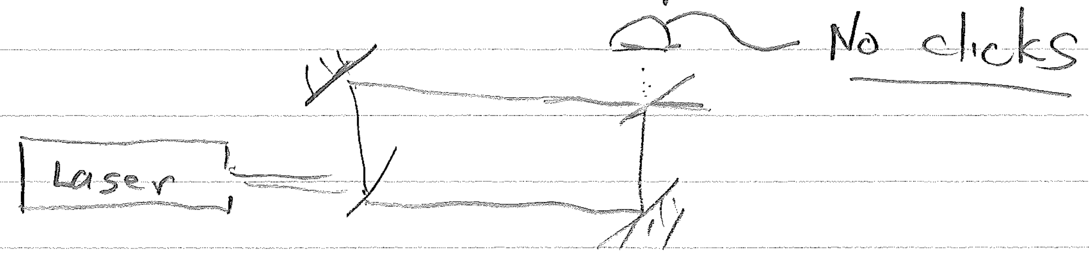
UNM: Fall 2010, Prof. I.H. Deutsch

Lecture 1: Introduction

• Quantum Optics: The study, manipulation, and control of quantum mechanical coherence associated with optical (electromagnetic) fields

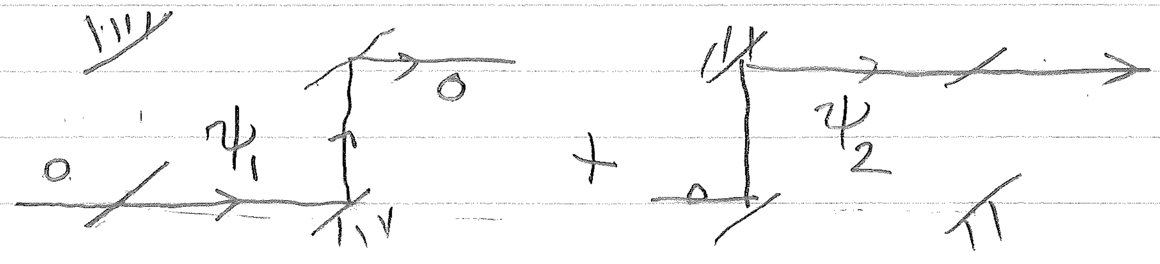
Two aspects of quantum physics:
- Waves – interference
- Particles – discrete events





Mach-Zender interferometer

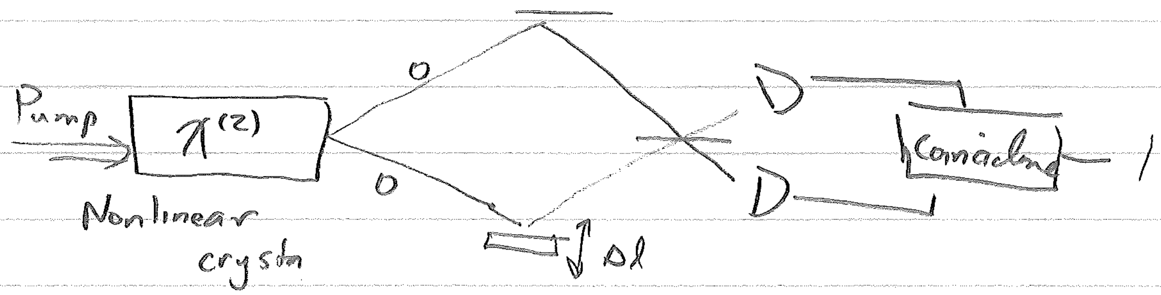
Interference of processes (photon by photon)



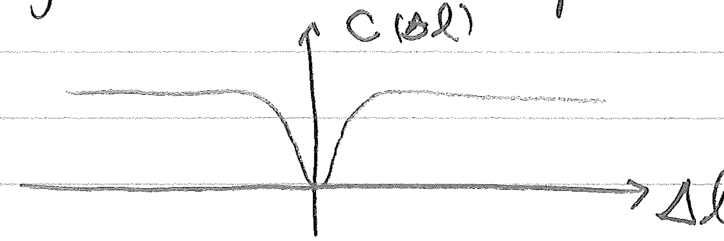
Classical vs. Nonclassical light

- Interference of classical waves $\vec{E}_{total} = \vec{E}_1 + \vec{E}_2$
- Stochastic clicks at detector - quantum Photon statistics!

Eg. Correlated photons



Hong-Ou-Mandel Dip



Interfering processes

reflect - reflect

transmit - transmit

- History and connections: See Map of Q. Optics
- Review of the structure of Quantum Mechanics

States, Observables, Dynamics

$\Downarrow$ Our description of the system

$\Downarrow$ Physical Quantities

$\Downarrow$ Time evolution

Measurement Theory

Quantum theory gives a frame work to determine the probability of a given measurement outcome for a given preparation of the system and a given dynamics:

States: "Pure states" – Best possible description of the system

Vector in Hilbert space
(Complex inner-product vector space with possibly ∞ dimensions)

$$|\Psi\rangle$$

Actually "ray" since $e^{i\phi}|\Psi\rangle$ and $|\Psi\rangle$ are the same state (more later)

Observable: Hermitian operator $\hat{A} = \hat{A}^{\dagger}$

Possible outcomes of measurements $\{a\}$

= eigenvalues $\hat{A}|a\rangle = \underset{\text{real}}{\underset{\uparrow}{a}}|a\rangle$

Complete set $\sum_a' \underbrace{|a\rangle\langle a|}_{\hat{\mathbb{P}}_a} = \hat{\mathbb{1}}$

$\hat{\mathbb{P}}_a$: Projection operator $\hat{\mathbb{P}}_a^2 = \hat{\mathbb{P}}_a$

Probability of finding outcome "a", given state $|\Psi\rangle$

$$P(a|\Psi) = \frac{|\langle a|\Psi\rangle|^2}{\|\Psi\|^2}$$

Equivalence class: Ray in Hilbert space

Convention: restrict $\|\Psi\|^2 = \langle\Psi|\Psi\rangle = 1$

Overall phase of $|\Psi\rangle$ is not physical

Expected value of a measurement:

$$|\Psi\rangle = \sum_a' |a\rangle\underbrace{\langle a|\Psi\rangle}_{= c_a} \quad \text{(Prob. amp)}$$

$$|c_a|^2 = P(a|\Psi)$$

$$\Rightarrow \langle\hat{A}\rangle_\Psi = \sum_a' a\, P(a|\Psi) = \sum_a' a\,|\langle a|\Psi\rangle|^2$$

$$= \sum_a' a \langle\Psi|a\rangle\langle a|\Psi\rangle$$

$$= \langle\Psi|\left(\sum_a' a|a\rangle\langle a|\right)|\Psi\rangle$$

$$= \langle\Psi|\hat{A}|\Psi\rangle \quad \text{matrix element}$$

<u>Uncertainty</u>: $\Delta\hat{A} = \hat{A} - \langle\hat{A}\rangle$

Variance $\Delta A^2 = \langle (\hat{A} - \langle\hat{A}\rangle)^2 \rangle_\psi = \langle\hat{A}^2\rangle - \langle\hat{A}\rangle^2$

$$= \sum_a a^2 P(a|\psi) - \left(\sum_a a\, P(a|\psi)\right)^2$$

"Compatable" observables: Measurement of $\hat{A}$ does not affect the probability of outcome of $\hat{B}$

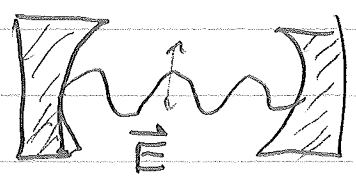
$$\Rightarrow \quad [\hat{A}, \hat{B}] = 0$$

Uncertainty principle: $\Delta A\, \Delta B \geq \frac{1}{2} |\langle [\hat{A}, \hat{B}] \rangle|$

Quintescential example: $[\hat{x}, \hat{p}] = i\hbar$

$\Delta x\, \Delta p \geq \hbar/2$ position   momentum

Quantum Optic analog: Single mode oscillations of an electric field



Optical cavity

$$\hat{\vec{E}} = E_0 \vec{\epsilon} \, (\hat{X} \cos\omega t + \hat{P} \sin\omega t)$$

quadrature operators

$$[\hat{X}, \hat{P}] = i$$

Other examples

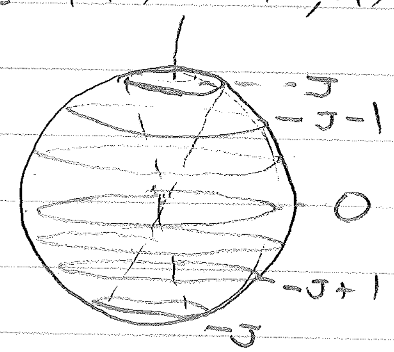
Components of angular momentum

$$[\hat{J}_i, \hat{J}_j] = i\hbar\, \epsilon_{ijk}\, \hat{J}_k$$

$$\Rightarrow \Delta J_x \Delta J_y \geq \frac{1}{2} |\langle \hat{J}_z \rangle| \leftarrow \text{Depends on state}$$

Example $|\psi\rangle = |J, M\rangle$

$$\begin{cases} \hat{J}^2 |J,M\rangle = J(J+1)\,|JM\rangle \\ \hat{J}_z\,|J,M\rangle = J\,|JM\rangle \end{cases}$$



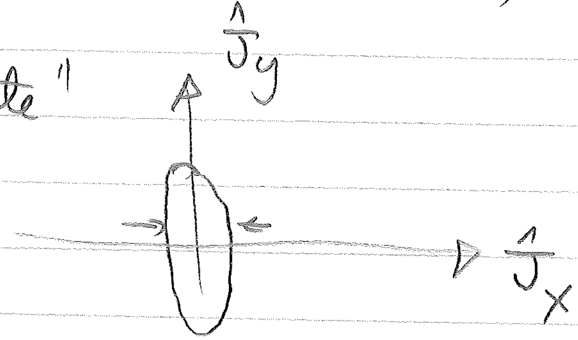
Homework: $|\psi\rangle = |J, J\rangle$

$$\Rightarrow \Delta J_x = \Delta J_y = \sqrt{\frac{J}{2}}$$

$$\Delta J_x \Delta J_y = \frac{J}{2} = \frac{|\langle J_z \rangle|}{2}$$

Minimum Uncertainty State
("Spin Coherent State")

"Spin Squeezed State"



$$\Delta J_x < \sqrt{\frac{J}{2}}$$

$$\Delta J_x \Delta J_y = \frac{J}{2}$$

Quantum Superposition and Interference

Though the *overall* phase of the state has no physical meaning, the *relative phase* in a superposition is crucial:

$$|\Psi\rangle = c_1|\phi_1\rangle + c_2|\phi_2\rangle = |c_1|\,|\phi_1\rangle + e^{i\phi}|c_2|\,|\phi_2\rangle$$

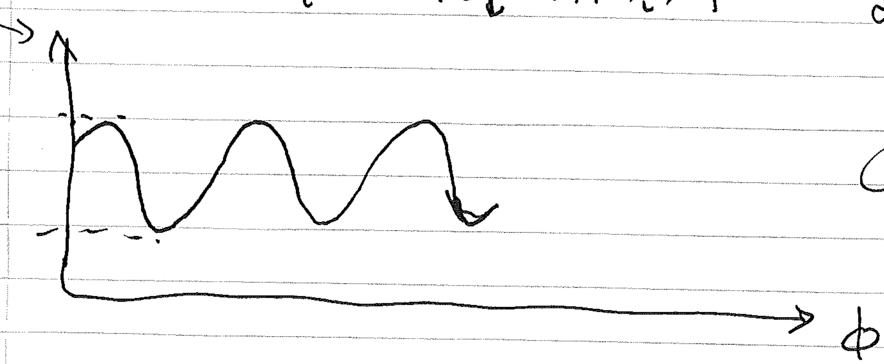
where $\phi = \mathrm{Arg}(c_2) - \mathrm{Arg}(c_1)$

$$\boxed{\begin{aligned} P(a|\Psi) &= P(a|\phi_1)\,P(\phi_1) + P(a|\phi_2)\,P(\phi_2) \\ &\quad \underbrace{+2|c_1||c_2|\,|\langle\phi_1|a\rangle|\,|\langle\phi_2|a\rangle|\cos(\phi+\delta)}_{\text{interference!}} \end{aligned}}$$

All that's strange and wonderful in quantum mech.

$$P(a|\Psi) = A_1^2 + A_2^2 + 2A_1A_2\cos(\phi+\delta)$$

$$A_i = |c_i\langle a|\phi_i\rangle| \qquad \delta = \mathrm{Arg}(\langle a|\phi_2\rangle\langle\phi_1|a\rangle)$$



Coherence = Interference

Visibility: $\dfrac{\max - \min}{\max + \min} = \dfrac{2A_1A_2}{A_1^2 + A_2^2}$

Distinguishability: If $\langle\phi_1|a\rangle\langle a|\phi_2\rangle = 0$

$\Rightarrow$ No interference