A review of quantum nondemolition

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Abstract

Quantum nondemolition measurements seek to evade the precision limit set by the Heisenberg uncertainty relation by forcing the measurement back action into one of a pair on noncommuting observables. This allows very precise measurements of the observable of interest. In effect, one measures an observable while maintaining the value of that observable through the measurements. In this article, I review the theoretical criteria for quantum nondemolition measuementsand examine recent experimental results.

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I. INTRODUCTION

Every physicist is familiar with the Heisenberg uncertainty relation: the product of the uncertainties for two operators is related to the commutator by

$$\Delta\widehat{X}\Delta\widehat{Y} \geq \frac{1}{2}\left|\left\langle\left[\widehat{X},\widehat{Y}\right]\right\rangle\right|. \tag{1}$$

For noncommuting operators, this product is non zero. It is a common misconception that this relationship implies a limit to the precision of a measurement of $\widehat{X}$ or $\widehat{Y}$. In fact, the Heisenberg uncertainty relation only applies to a simultaneous measurement of both $\widehat{X}$ and $\widehat{Y}$. If one is willing to accept an arbitrarily large uncertainty in one variable, then one can measure the other variable with arbitrary precision. However, this measurement changes the state of the system. Often, the state changes such that a second measurement of the observable of interest is different than the original measurement. A classic example of this is the Stern-Gerlach experiment. One can measure the spin state of a particle, but the state is destroyed by the detector. Condsider a state which is an eigenstate of an observable. In order to gain informataion about this state, one must couple it to another system. Even though the original state was a stationary state of the original Hamiltonian, it is likely not a stationary state of the combined Hamiltionian. Therefore, the measurement process changes the original state.

The field of quantum nondemolition (QND), seeks to make precise measurements of an observable in ways that preserve the existence of the system and the measured value of the observable. With such measurements, the back action is shunted onto the noncommuting variable. These measurements were originally considered for mechanical oscillators. [1]

QND measurements are made by coupling a meter to the system of interest, also called the signal. The meter and system are allowed to interact and become correlated. An observable of the meter is measured by some destructive method. Because of the correlation, the measurement of the meter observable gives information about the system. Ideally, the meter would have no impact on the observable of interest and would only contain information about the observable of interest.

II. THEORY

A. Ideal QND measurements

Consider a system of two non-commuting operators: $\widehat{X}_{\mathsf{s}}$ and $\widehat{Y}_{\mathsf{s}}$. Let this system interact with a meter: $\widehat{X}_{\mathsf{m}}$ and $\widehat{Y}_{\mathsf{m}}$. After the interaction period, the system observables are unchanged, and the meter is an exact duplicate of the system.

$$\widehat{X}_{\mathsf{s}} \longrightarrow \widehat{X}_{\mathsf{s}} \quad \widehat{Y}_{\mathsf{s}} \longrightarrow \widehat{Y}_{\mathsf{s}} \tag{2}$$

$$\widehat{X}_{\mathsf{m}} \longrightarrow \widehat{Y}_{\mathsf{s}} \quad \widehat{Y}_{\mathsf{m}} \longrightarrow \widehat{X}_{\mathsf{s}} \tag{3}$$

Now, a measurement of $\widehat{Y}_{\mathsf{m}}$ is also a measurement of $\widehat{X}_{\mathsf{s}}$. However, any measurement of $\widehat{Y}_{\mathsf{m}}$ will perturb $\widehat{X}_{\mathsf{m}}$. Since $\widehat{X}_{\mathsf{m}} = \widehat{Y}_{\mathsf{s}}$, any perturbation of $\widehat{X}_{\mathsf{m}}$ will also perturb $\widehat{Y}_{\mathsf{s}}$. The measurement has changed the state of the system, but not the measured value of the observable $\widehat{X}_{\mathsf{s}}$.

B. Non-ideal QND measurements

Because experiments never achieve the theoretical limit, we require some criteria to determine how well the experiment approximates ideal QND. We must consider three quantities: (1) the correlation between the system and meter, (2) the impact of the meter on the system, and (3) the information contained in the measured meter observable which does not come from the observable of interest. I will follow the analyses of Grangier et. al. [2], Roch et. al. [3], and Holland et. al. [4]

We define the state of the combined system as a four element vector

$$\mathcal{F} = \begin{bmatrix} \widehat{X}_{\mathsf{s}} \\ \widehat{Y}_{\mathsf{s}} \\ \widehat{X}_{\mathsf{m}} \\ \widehat{Y}_{\mathsf{m}} \end{bmatrix}. \tag{4}$$

The observable of interest is $\widehat{X}_{\mathsf{s}}$; the measured observable is $\widehat{Y}_{\mathsf{m}}$. The fluctuations of an observable are given by

$$\delta\widehat{X} = \widehat{X} - \left\langle \widehat{X} \right\rangle . \tag{5}$$

The fluctuations after the measurement are then given by

$$\delta\mathcal{F}^{\mathsf{out}} = \lambda \cdot \delta\mathcal{F}^{\mathsf{in}} + \delta\mathcal{F}^{\mathsf{ad}} \tag{6}$$

where λ is a matrix which defines the interaction of the meter and system and $\mathcal{F}^{\mathsf{ad}}$ is an additional noise term. The correlations between the fluctuations of each operator are defined as a covariance matrix W, given by the outer product of $\delta\mathcal{F}$ with itself.

$$W = \left\langle \delta\mathcal{F}\delta\mathcal{F}^{\mathsf{T}} \right\rangle \tag{7}$$

Note that $W_{\mathsf{AbAb}} = (\Delta A_{\mathsf{b}})^2$ is the square of the uncertainty or noise in the observable $\widehat{A}$. This is a directly measurable quantity. Take the photon number, $\widehat{n}$, as an example. The photon number is proportional to the intensity of the electromagnetic field. $(\Delta n)^2$ is then related to the power spectral density of amplitude noise. The output covariance matrix in terms of the input is

$$W^{\mathsf{out}} = \lambda \cdot W^{\mathsf{in}} \cdot \lambda^{\mathsf{T}} + W^{\mathsf{ad}}. \tag{8}$$

For the ideal measurement considered in the previous section,

$$W^{\mathsf{in}} = \begin{bmatrix} \left(\Delta\widehat{X}_{\mathsf{s}}^{\mathsf{in}}\right)^2 & 0 & 0 & 0 \\ 0 & \left(\Delta\widehat{Y}_{\mathsf{s}}^{\mathsf{in}}\right)^2 & 0 & 0 \\ 0 & 0 & \left(\Delta\widehat{X}_{\mathsf{m}}^{\mathsf{in}}\right)^2 & 0 \\ 0 & 0 & 0 & \left(\Delta\widehat{Y}_{\mathsf{m}}^{\mathsf{in}}\right)^2 \end{bmatrix}, \tag{9}$$

$$\lambda = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \tag{10}$$

and

$$W^{\mathsf{out}} = \begin{bmatrix} \left(\Delta\widehat{X}_{\mathsf{s}}^{\mathsf{in}}\right)^2 & 0 & 0 & \left(\Delta\widehat{X}_{\mathsf{s}}^{\mathsf{in}}\right)^2 \\ 0 & \left(\Delta\widehat{Y}_{\mathsf{s}}^{\mathsf{in}}\right)^2 & \left(\Delta\widehat{Y}_{\mathsf{s}}^{\mathsf{in}}\right)^2 & 0 \\ 0 & \left(\Delta\widehat{Y}_{\mathsf{s}}^{\mathsf{in}}\right)^2 & \left(\Delta\widehat{Y}_{\mathsf{s}}^{\mathsf{in}}\right)^2 & 0 \\ \left(\Delta\widehat{X}_{\mathsf{s}}^{\mathsf{in}}\right)^2 & 0 & 0 & \left(\Delta\widehat{X}_{\mathsf{s}}^{\mathsf{in}}\right)^2 \end{bmatrix}. \tag{11}$$

The off diagonal elements of W^{in} are zero since the initial observables are uncorrelated with each other. However, W^{out} has non zero off diagonals because of the correlations between the signal and meter cerated by the interaction.

Our three criteria can be found in the elements of the covariance matrix. The correlation of $\widehat{X}_{\mathsf{s}}^{\mathsf{out}}$ with $\widehat{Y}_{\mathsf{m}}^{\mathsf{out}}$ is given by $W_{\mathsf{XsYm}}^{\mathsf{out}}$. The impact of the meter on the system is contained in $W_{\mathsf{XsXs}}^{\mathsf{out}}$. The noise on the meter is in $W_{\mathsf{YmYm}}^{\mathsf{out}}$.

C. Correlation

Instead of working with $W_{\mathsf{XsYm}}^{\mathsf{out}}$ directly, it is more convenient to define

$$W_{\mathsf{QSP}} = W_{\mathsf{XsXs}}^{\mathsf{out}} - \frac{|W_{\mathsf{XsYm}}^{\mathsf{out}}|^2}{W_{\mathsf{YmYm}}^{\mathsf{out}}} = W_{\mathsf{XsXs}}^{\mathsf{out}} - \frac{|W_{\mathsf{XsYm}}^{\mathsf{out}}|^2}{(W_{\mathsf{YmYm}}^{\mathsf{out}})^2} W_{\mathsf{YmYm}}^{\mathsf{out}} = W_{\mathsf{XsXs}}^{\mathsf{out}} - g^2 W_{\mathsf{YmYm}}^{\mathsf{out}}. \tag{12}$$

QSP stand for "Quantum State Preparation." W_{QSP} indicates how well the meter was prepared as an exact quantum duplicate of the system. For suitably normalized operators and shot noise limited states, $\Delta X = \Delta Y = 1$. With no correlation, $g = 0$, $W_{\mathsf{QSP}} = 1$. However, if the interaction causes a correlation ($W_{\mathsf{QSP}} < 1$) our first criterion is met. Using the elements of the output matrix from eq. 11, $W_{\mathsf{QSP}} = 0$ for the ideal measurement. If our initial states are not in a minimum noise state initially, then $W_{\mathsf{QSP}} > 1$. This quantity is referred to as the conditional variance.

D. Meter noise

$W^{\text{out}}_{\text{YmYm}}$ written in terms of the input is given by

$$W^{\text{out}}_{\text{YmYm}} = |\gamma_{\text{ms}}|^2 \, [W^{\text{in}}_{\text{XsXs}} + \frac{1}{|\gamma_{\text{ms}}|^2}(|\delta_{\text{ms}}|^2 \, W^{\text{in}}_{\text{YsYs}} + \gamma_{\text{ms}}\delta^*_{\text{ms}}W^{\text{in}}_{\text{XsYs}} + \gamma^*_{\text{ms}}\delta_{\text{ms}}W^{\text{in}}_{\text{YsXs}} + |\gamma_{\text{mm}}|^2 \, W^{\text{in}}_{\text{XmXm}} + |\delta_{\text{mm}}|^2 \, W^{\text{in}}_{\text{YmYm}} + W^{\text{ad}}_{\text{YmYm}})], \quad (13)$$

where $\gamma_{\text{ms}}, \delta_{\text{ms}}, \gamma_{\text{mm}}$, and δ_{mm} are elements of the coupling matrix λ. Clearly, the output signal is not solely a function of the input signal. All of the extra terms contribute to additional noise on the output. The sum of these terms is relabeled N^{eq}_{m}, which is a "noise referred to the input."

$$N^{\text{eq}}_{\text{m}} = \frac{1}{|\gamma_{\text{ms}}|^2}[|\delta_{\text{ms}}|^2 \, W^{\text{in}}_{\text{YsYs}} + (\gamma_{\text{ms}}\delta^*_{\text{ms}}W^{\text{in}}_{\text{XsYs}} + \gamma^*_{\text{ms}}\delta_{\text{ms}}W^{\text{in}}_{\text{YsXs}} + |\gamma_{\text{mm}}|^2 \, W^{\text{in}}_{\text{XmXm}} + |\delta_{\text{mm}}|^2 \, W^{\text{in}}_{\text{YmYm}} + W^{\text{ad}}_{\text{YmYm}})] \quad (14)$$

Essentially, these terms make our observable of interest appear more noisy. If we assume that $W^{\text{in}}_{\text{XsYs}} = 0$, then N^{eq}_{m} is positive or zero. For our QND measurement, $N^{\text{eq}}_{\text{m}} < 1$, preferably 0. No noise is good noise. The assumption $W^{\text{in}}_{\text{XsYs}} = 0$, means that the input signal operators are not correlated: a requirement for QND. If the inputs are correlated, $W^{\text{out}}_{\text{YmYm}}$ measures the whole state instead of $\widehat{X}_{\text{s}}$. The definition of N^{eq}_{m} may also be written in terms of measurable quantities,

$$N^{\text{eq}}_{\text{m}} = \frac{1}{|\gamma_{\text{ms}}|^2}W^{\text{out}}_{\text{YmYm}} - W^{\text{in}}_{\text{XsXs}}. \quad (15)$$

In the literature, a more common measure is the noise transfer coefficient T_{m} defined as

$$T_{\text{m}} = \frac{1}{1 + (N^{\text{eq}}_{\text{m}})^2}. \quad (16)$$

Using the elements of the output matrix from eq. 10 and eq. 11, $N^{\text{eq}}_{\text{m}} = 0$ and $T_{\text{m}} = 1$ for the ideal measurement.

E. Signal noise

Just as we defined a noise referred to the input for $W^{\mathrm{out}}_{\mathrm{Y\,mY\,m}}$, we define a similar term for additional noise added to the signal.

$$N^{\mathrm{eq}}_{\mathrm{s}} = \frac{1}{|\alpha_{\mathrm{ss}}|^2}(|\beta_{\mathrm{ss}}|^2 W^{\mathrm{in}}_{\mathrm{Y\,sY\,s}} + \alpha_{\mathrm{ss}}\beta^*_{\mathrm{ss}} W^{\mathrm{in}}_{\mathrm{X\,sY\,s}} + \alpha^*_{\mathrm{ss}}\beta_{\mathrm{ss}} W^{\mathrm{in}}_{\mathrm{Y\,sX\,s}} + |\alpha_{\mathrm{sm}}|^2 W^{\mathrm{in}}_{\mathrm{X\,mX\,m}} + |\beta_{\mathrm{sm}}|^2 W^{\mathrm{in}}_{\mathrm{Y\,mY\,m}} + W^{\mathrm{ad}}_{\mathrm{X\,sX\,s}}). \quad (17)$$

In terms of measurable quantities,

$$N^{\mathrm{eq}}_{\mathrm{s}} = \frac{1}{|\alpha_{\mathrm{ss}}|^2} W^{\mathrm{out}}_{\mathrm{X\,sX\,s}} - W^{\mathrm{in}}_{\mathrm{X\,sX\,s}}. \quad (18)$$

As before, a more common measure is the noise transfer function defined as

$$T_{\mathrm{s}} = \frac{1}{1 + (N^{\mathrm{eq}}_{\mathrm{s}})^2}. \quad (19)$$

With the same assumption as before, $N^{\mathrm{eq}}_{\mathrm{s}}$ is positive or zero. $N^{\mathrm{eq}}_{\mathrm{s}} = 0$ and $T_{\mathrm{s}} = 1$ is the ideal so that input observable $\widehat{X}_{\mathrm{s}}$ is not disturbed. The "classical" limit for the noises is $N^{\mathrm{eq}}_{\mathrm{m}} N^{\mathrm{eq}}_{\mathrm{s}} \geq 1$. [3] The corresponding relation for the transfer coefficients is $T_{\mathrm{s}} + T_{\mathrm{m}} \leq 1$. In the quantum regime, $N^{\mathrm{eq}}_{\mathrm{m}} N^{\mathrm{eq}}_{\mathrm{s}} \leq 1$ and $1 \leq T_{\mathrm{s}} + T_{\mathrm{m}} \leq 2$.

F. Measurement regimes

We now have our criteria for QND measurements: $W_{\mathrm{QSP}} \leq 1$ and $T_{\mathrm{s}} + T_{\mathrm{m}} \geq 1$. The ideal QND measurement is one for which $W_{\mathrm{QSP}} = 0$ and $T_{\mathrm{s}} + T_{\mathrm{m}} = 2$. These conditions define four regions, as shown in Fig. 1.

A measurement is QND when both condition are met. The meter is correlated with the signal, and the noise added to the signal and meter is less than the calssical limit. This is the lower right quandrant in Fig. 1. The measurement is QSP (lower left quadrant) when only the correlation condition is met. In this case, the meter is prepared as a duplicate of the signal, but there is excess noise in either the meter or the signal. This happens when the initial signal state observable of interst is disturbed or destroyed. A measurement is in the upper qight quadrant when is does not introduce excessive additional noise but does not build correlations between the signal and meter. this is sometimes refered to as a quantum tap. The meter has coupled into the signal while adding additional noise below the classical limit. The measurement is "classical" when neither condition is met.

G. Ideal parametric coupler

The coupling of two light beams is an illuminating example of the QND criteria. This example is treated in references [2] and [3]. Consider two beams coupled with the cross-Kerr effect. The phase of one beam is affected only by the amplitude of the other beam. The observable for each beam are the photon number, $\widehat{n}$, and the phase $\widehat{\phi}$. In the limit of large average photon number, the observable abey the uncertainty relation $\Delta\widehat{n}\Delta\widehat{\phi} \geq 1/2$. I'll use the normalized variables defined by

$$\widehat{X} = \frac{\widehat{n}}{\sqrt{\langle n\rangle}} \qquad \widehat{Y} = 2\sqrt{\langle n\rangle}\widehat{\phi}. \tag{20}$$

The coupling between beams is given by the transforms

$$\widehat{X}_{\mathsf{s}}^{\mathsf{out}} = \widehat{X}_{\mathsf{s}}^{\mathsf{in}} \qquad \widehat{Y}_{\mathsf{s}}^{\mathsf{out}} = \widehat{Y}_{\mathsf{s}}^{\mathsf{in}} + G\widehat{X}_{\mathsf{m}}^{\mathsf{in}} \tag{21}$$

and

$$\widehat{X}_{\mathsf{m}}^{\mathsf{out}} = \widehat{X}_{\mathsf{m}}^{\mathsf{in}} \qquad \widehat{Y}_{\mathsf{m}}^{\mathsf{out}} = \widehat{Y}_{\mathsf{m}}^{\mathsf{in}} + G\widehat{X}_{\mathsf{s}}^{\mathsf{in}}. \tag{22}$$

The observable of interest is $\widehat{X}_{\mathsf{s}}$. The measured meter observable is $\widehat{Y}_{\mathsf{m}}$. Note that the coupling leaves $\widehat{X}_{\mathsf{s}}$ unaffected. However, the measured meter observable has two terms: the desired $\widehat{X}_{\mathsf{s}}$ and an extraneous $\widehat{Y}_{\mathsf{m}}$. This extra term will add some unwanted noise. Given this coupling, W_{QSP} and the transfer coefficients become

$$W_{\mathsf{QSP}} = \frac{1}{1+G^2}, \qquad T_{\mathsf{s}} = 1, \qquad T_{\mathsf{m}} = \frac{G^2}{1+G^2}. \tag{23}$$

The dashed line in Fig. 1 described the parametric coupler as a function of the gain. For $G = 0$, there is no coupling. For any non zero G, this system meets the QND criteria. As G gets very large the parametric coupler approaches a perfect QND measurement. Experimentally, W_{QSP}, T_{s}, T_{m} and are relatively easy to measure. While we have defined the elements of W in terms of normalized operators, the measurable quantities are the fluctuation in photon number and the fluctuation in phase. $W^{\mathsf{out}}_{\mathsf{XsXs}}$ is proportional to the amplitude noise power of the signal beam; $W^{\mathsf{out}}_{\mathsf{YmYm}}$ is proportional to the phase noise power of the meter beam. One can measure W_{QSP} simply by multiplying the output of the meter phase detector by some gain and subtracting it from the output of the signal photodetector. When the gain is equal to g as defined in eq. (12), the noise of the combined signal is less than the shot noise of the signal photodetector signal alone. W_{QSP} is the ratio of the measured noise to the shot noise. The noise transfer coefficients can also be measured by looking at the appropriate noise levels of the input and output beams (eqs. (15) and (18)).

III. RECENT EXPERIMENTAL RESULTS

While QND was originally conceived of in terms of mechanical oscillators, the experimental successes have come in optics. Most of these experiments have used some combination of photons and atoms. Experimental results are included in Fig. 1.

A. QND of a single photon

Nogues **et. al.** [6] have devised a novel QND experiment to detect the presence of a single photon using a single atom as the meter. The experiment relies on partial cycles of Rabi oscillations. The experiment consists of an interaction region to prepare the atom, a high Q cavity to hold the photon, and a second interaction region to measure the atom's final state (see Fig. 2). The atom can be considered a three level system: the initial state $|i\rangle$, the ground state $|g\rangle$, and the excited state $|e\rangle$. The first interaction region prepares the atom in a superposition of the initial and ground states by applying a $\pi/2$ pulse of light tuned to the $i \rightarrow g$ transition. $|\Psi\rangle = 1/\sqrt{2}(|i\rangle + |g\rangle)$. The atom then enters the cavity which may or may not contain a photon tuned to the $g \rightarrow e$ transition. If the cavity has a photon, the atom receives a 2π pulse. This leaves the photon number in the cavity unchanged, but changes the phase of the ground state by π. If the cavity contains a photon, the resulting state is $|\Psi\rangle = 1/\sqrt{2}(|i\rangle - |g\rangle)$. The state is unchanged if there are no photons present. The second interaction regions again applies a $\pi/2$ pulse of light tuned to the $i \rightarrow g$ transition. If the atom interacted with a photon, the second $\pi/2$ pulse leaves the atom in the ground state. If there was no photon, the pulse leaves the atom in the initial state. This is a Ramsey interferometer with a phase shift provided by the atom-photon interaction. The state of the atom is then detected. The preceding explanation is simplified and applies only when the interaction regions are exactly at the $i \rightarrow g$ resonance. As the light is tuned away from resonance, the measurement of the atom oscillates between the excited and the ground state. The presence of a photon in the cavity shifts the oscillation by π.

B. QND with trapped atoms

Roch et. al. [7] use a 4 wave mixing process to couple signal and meter beams. The coupling mechanism is an interaction with trapped rubidium atoms at very low temperature. The low temperature of the atoms minimizes additional noise coupled into the optical beams. The rubidium D1 line is a Λ-type three level system. See Fig. 3. The signal is slightly detuned from the $|1\rangle \rightarrow |2\rangle$ transition, and the meter is slightly detuned from the $|0\rangle \rightarrow |2\rangle$ transition. The strong signal beam saturates the $|1\rangle \rightarrow |2\rangle$ transition. The weak meter beam picks up additional phase due to the change in the energy of level 2 caused by the saturation. This change is directly related to the photon number of the signal. The phase change is detected with a balanced homodyne scheme. Theoretical treatment of this experiment predicts nearly perfect QND ($W_{\mathsf{QSP}} < 0.1$, $T_{\mathsf{s}} > 0.9$, and $T_{\mathsf{m}} > 0.9$) when the meter is tuned near one of the Rabi split levels. [8] However, the theory also predicts instability if the meter is nearly resonant with the shifted level.

This results of the experiment are remarkable: $W_{\mathsf{QSP}} = 0.45$, $T_{\mathsf{s}} = 0.90$, and $T_{\mathsf{m}} = 0.65$. This experiment was well within the QND region.

C. QND with semiconductor receivers

High efficiency photodiodes can transform a sub-shot noise limited beam of light into a sub-shot noise limited electrical signals. High quality LEDs can perform the inverse transformation. This leads to experiments which satisfy the QND criteria. The experiment of Roch et. al. [9] measured $W_{\mathrm{QSP}} = 0.76$ and $T_{\mathrm{s}} + T_{\mathrm{m}} = 1.8$. Goobar et. al. [10] achieved similar results with $W_{\mathrm{QSP}} \approx 0.7$ and $T_{\mathrm{s}} + T_{\mathrm{m}} = 1.8$.

However, these experiments are not truly "nondemolition" experiments. They actually destroy the photons to make the measurement. However, the electronics are capable of reproducing the states with very high fidelity. These are more like quantum resurrection experiments.

D. QND with optical parametric amplifiers

Levenson et. al. [11] use a degenerate parametric down converter to make QND measurements of an input signal. The input signal is horizontally polarized. The meter is initially in the vacuum state and is the orthogonal polarization. The measurement device consists of a frequency doubled laser which acts as the pump for the nonlinear process. The residual unconverted laser light is the incident signal. The signal passes though a half wave plate and a KTP crystal. The half wave plate rotates the polarization of the input signal so that the OPA acts as a phase sensitive amplifier. The signal then passes through polarizing beam splitter cube to a detector. The meter is reflected from the beam splitter to another detector. The detector output are mixed to measure the relative noise between the signal and meter. The experimental results are clearly in the QND regime with $W_{\mathrm{QSP}} \approx 0.77$ and $T_{\mathrm{s}} + T_{\mathrm{m}} = 1.32$.

A later experiment by Bencheikh and Levenson [12] used two down converters to make repeated measurements of the same signal beam. There are five important quadratures: $\widehat{X}^{\mathrm{in}}_{\mathrm{S1}}$, $\widehat{X}^{\mathrm{out}}_{\mathrm{M1}}$, $\widehat{X}^{\mathrm{in}}_{\mathrm{S2}}$, $\widehat{X}^{\mathrm{out}}_{\mathrm{M2}}$, $\widehat{X}^{\mathrm{out}}_{\mathrm{S2}}$. $\widehat{X}^{\mathrm{in}}_{\mathrm{S1}}$ is the signal incident on the first device. $\widehat{X}^{\mathrm{out}}_{\mathrm{M1}}$ is the output meter of the first device. $\widehat{X}^{\mathsf{in}}_{\mathsf{S2}}$ is the signal output of the first device which is incident on the second device. $\widehat{X}^{\mathsf{out}}_{\mathsf{M2}}$ and $\widehat{X}^{\mathsf{out}}_{\mathsf{S2}}$ are the meter and signal output of the second device. For the first device, $W_{\mathsf{QSP1}} \approx 0.66$ and $T_{\mathsf{s1}} + T_{\mathsf{m1}} = 1.34$. This indicates that the first device is a QND measurement. More importantly, the conditional variance between $\widehat{X}^{\mathsf{out}}_{\mathsf{M1}}$ and $\widehat{X}^{\mathsf{out}}_{\mathsf{S2}}$ was measured at 0.66. This shows that the signal output from the second device remains strongly correlated with the meter from the first device. Plainly, the second device did not destroy the quantum state of the signal.

IV. SUMMARY

While is it impossible to measure a system observable without perturbing the state, one can measure an observable without changing subsequent measurements of that observable. Because of the potentially strong coupling between atoms and photons, the most effective of these measurements have been made using optical systems interacting with atomic systems or optical systems interacting with other optical systems through an atomic mediator.

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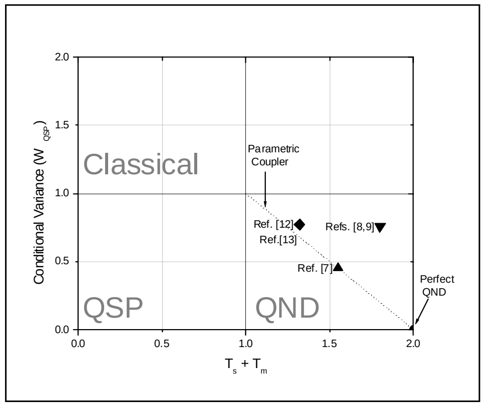

Figure 1: Measurement regions and recent experimental results. Measurements in the top left quadrant are classical. The bottom left quadrant represents Quantum State Preparation where the meter is closely correlated to the signal but the measurement has introduced excess noise. Measurements in the bottom right quadrant are true Quantum Non-demolition measurements. The Points show the experimental results for the cited sources. The dotted line is for the parametric coupler. Reproduced from ref. 3.

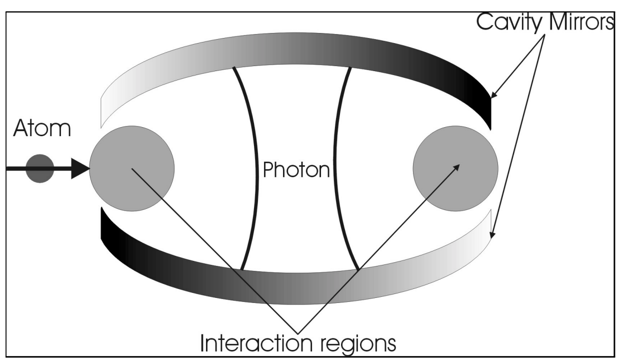

Figure 2: CND of a single photon. An atom enters the apparatus and is put into a superposition $|\mathsf{i}\rangle$ and $|\mathsf{g}\rangle$ by the first interaction region. The atom then interacts with the photon in the cavity which is tuned to the $|\mathsf{g}\rangle \rightarrow |\mathsf{e}\rangle$ transition. The atom then sees another interaction region tuned to the $|\mathsf{i}\rangle \rightarrow |\mathsf{g}\rangle$ transition. If there was no photon in the cavity, the atome leaves in the initial state. If there was a photon, the atom leaves in the ground state. Reproduced from ref. 5.

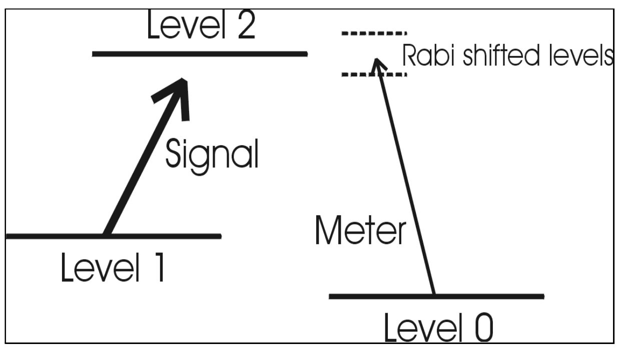

Figure 3: Λ energy level configuration. The strong signal saturates the $|1\rangle \rightarrow |2\rangle$ transition and shifts the energy of level 2. The weak probe tuned to the $|0\rangle \rightarrow |2\rangle$ transition picks up a phase shift which can be measured. The measure phase shift depends on the amount of energy shift which depends on the intensity of the signal. Reproduced from ref. 7.