Squeezed Light

Thomas Rotter

University of New Mexico, Department of Physics and Astronomy, Albuquerque, NM 87131, USA

(Dated: January 28, 2000)

Abstract

This paper gives an overview on *squeezed* light with an emphasis on its generation. Squeezing means, that the fluctuation in one of two conjugate components (e.g. amplitude and phase) is suppressed while enhanced in the other. Consequently squeezing can be generated by phase sensitive processes. Such processes are known from nonlinear optics, e.g. second harmonic generation, parametric down– and upconversion and four wave mixing, and they have been successfully used to generate squeezed light. Application of squeezed light is still rare because it is difficult to maintain the squeezed character.

PACS number: 42.50

I. INTRODUCTION

A classical electromagnetic field consists of waves with well defined amplitude and phase. However, in a quantum treatment, fluctuations are associated with both conjugate variables. Equivalently the field can be described in two conjugate quadrature components and the uncertainties in the two conjugate variables satisfy the Heisenberg uncertainty principle. A coherent state, that most nearly describes a classical electromagnetic field, has an equal amount of uncertainty in the two variables, and the product is the minimum uncertainty. This case is usually called the shot noise limit.

While the product of the uncertainties in the two conjugate quadrature components cannot be minimized further, it is possible to reduce the fluctuations in one of them well below the quantum limit. The canonically conjugate quadrature component must then display enhanced fluctuations in order to fulfill the Heisenberg uncertainty principle. Those states are called *squeezed states*.

Squeezed light with fluctuations below the standard quantum limit in one of the quadrature components has many attractive applications, e.g. in optical communication, in precision and sensitive measurements such as gravitational wave detection, or in noise free amplification. Squeezed light is however extraordinarily fragile and may be degraded even by a beam splitter or a mirror, as those admit the vacuum fluctuations from outside to enter, which exceed the squeezed fluctuations.

In this paper I will briefly describe the theory of squeezed states and their properties. The main focus will be on generation of squeezed light by a variety of processes, including $\chi^{(2)}$- and $\chi^{(3)}$- nonlinearities. I will present experiments that have been successfully carried out, as well as briefly describe theoretical approaches to investigate the generation processes. Comparisons between theoretical and experimental results will be given.

II. SQUEEZED STATES

A Theoretical considerations

Consider a quantized single-mode electric field of frequency ν [1]:

$$\vec{E}(t) = E\vec{\epsilon}(\hat{a}e^{-i\nu t} + \hat{a}^\dagger e^{i\nu t}) \tag{1}$$

where $\hat{a}$ and $\hat{a}^\dagger$ are the non-Hermitian annihilation and creation operators, respectively, obeying the commutation relation $[\hat{a}, \hat{a}^\dagger] = 1$. The field can also be written in terms of Hermitian quadratures $\hat{X}$ and $\hat{Y}$,

$$\vec{E}(t) = 2E\vec{\epsilon}(\hat{X}\cos\nu t + \hat{Y}\sin\nu t). \tag{2}$$

Here $\hat{X} = \frac{1}{2}(\hat{a} + \hat{a}^\dagger)$ and $\hat{Y} = \frac{1}{2i}(\hat{a} - \hat{a}^\dagger)$ are dimensionless versions of the position and momentum operators $\hat{x}$ and $\hat{p}$. The uncertainty relation

$$\Delta\hat{X}\Delta\hat{Y} \geq \frac{1}{4} \tag{3}$$

follows from the commutation relation $[\hat{X}, \hat{Y}] = \frac{i}{2}$. If the equality in the uncertainty relation (3) holds, then the minimum amount of fluctuation, the shot noise limit, is reached. Further reduction of fluctuation, *squeezing*, is possible only in one of the two canonically conjugate variables

$$(\Delta\hat{X}_i)^2 < \frac{1}{4} \qquad (\mathrm{i} = 1 \text{ or } 2) \tag{4}$$

at the expense of increasing fluctuation, *antisqueezing* [11], in the other, so that (3) is still satisfied. An ideal squeezed state is obtained if in addition to (4) the equality in the uncertainty relation (3) also holds.

Figure II A illustrates a coherent state in the phasor plane. The expectation value of the annihilation operator $\hat{a} = \hat{X} + i\hat{Y}$ has amplitude α and phase ϕ. The hatched areas indicate the probability distribution for an event yielding a phasor terminating in one of the points in the phasor plane, the circle indicating the root-mean-square deviation of the distribution. This is a schematic illustration of the Wigner distribution.

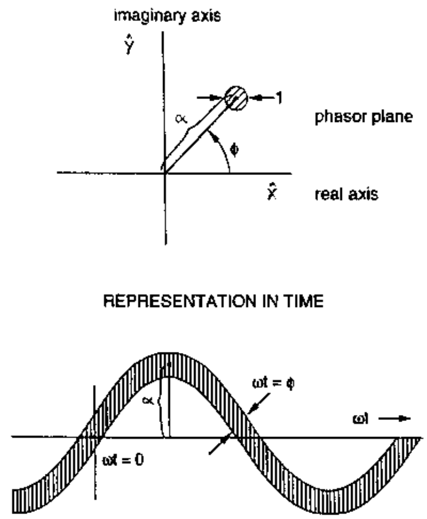

FIG. 1: Representation of a coherent state in the complex phasor plane with X along the real axis and Y along the imaginary, and below Y versus phase $\phi = \omega t$. [2]

Figure II A shows a squeezed state. The Wigner distribution is represented by an ellipse. There are two phases to be considered, that of the expectation of the phasor ϕ and that of the orientation of the ellipse of the Wigner distribution ψ (or θ in Fig. II A). The area of the ellipse is the same as the area of the circle of the phase-independent zero-point fluctuations, since the uncertainty principle must be obeyed. As shown in Fig. II A, the major and minor axes of the ellipse depend on each other as $\frac{1}{2}\mathrm{e}^{+s}$ and $\frac{1}{2}\mathrm{e}^{-s}$, where s is a squeezing parameter. $s = 0$ corresponds to a coherent state with no squeezing, as depicted in Fig. II A.

For the general case of Fig. II A and Fig. II A the unitary squeeze operator [7] is given by

$$\hat{S}(\zeta) = \mathrm{e}^{\frac{1}{2}\zeta^* \hat{a}^2 - \frac{1}{2}\zeta \hat{a}^{\dagger 2}}, \tag{5}$$

where ζ is the complex squeeze parameter, which depends on s and θ from Fig. II A,

$$\zeta = s\mathrm{e}^{i\theta}, \qquad 0 \le s < \infty, \qquad 0 \le \theta \le 2\pi. \tag{6}$$

The *vacuum* coherent state $|0\rangle$ corresponds to Fig. II A with $\alpha = 0$, likewise *squeezed vacuum* $|0_s\rangle = \hat{S}(\zeta)|0\rangle$ corresponds to Fig. II A with $\alpha = 0$, or to Fig. II A. In contrast to

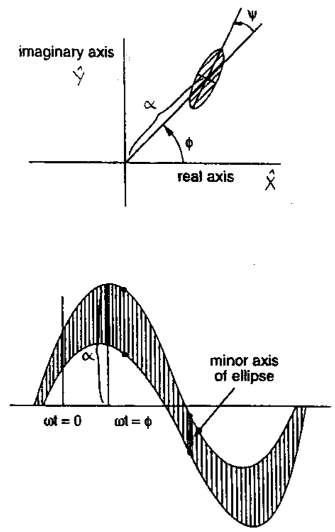

FIG. 2: Representation of a squeezed state in the complex phasor plane. X along the real axis and Y along the imaginary, and below Y versus phase $\phi = \omega t$. [2]

squeezed vacuum, the situation $\alpha \neq 0$ is called bright squeezed light. Every squeezed state can then be described as

$$|\alpha, \zeta\rangle = \hat{D}(\alpha)\hat{S}(\zeta)|0\rangle \tag{7}$$

where

$$\hat{D}(\alpha) = \mathrm{e}^{\alpha \hat{a}^\dagger - \alpha^* \hat{a}} \tag{8}$$

is a coherent displacement operator [7].

The squeeze operator (5) transforms the annihilation operator according to the Bogoliubov-transformation

$$\hat{a} \to \hat{S}^{-1}(\zeta)\hat{a}\hat{S}(\zeta) = \hat{a}\cosh s - \hat{a}^\dagger \mathrm{e}^{i\theta} \sinh s. \tag{9}$$

The mean of the quadrature operators $\langle \hat{X} \rangle$, $\langle \hat{Y} \rangle$ correspond then to the real and imaginary part of α, respectively, as shown in Fig. II A. The variances are

$$\langle (\Delta X)^2 \rangle = \frac{1}{4}(\mathrm{e}^{-2s} \cos^2 \frac{1}{2}\theta + \mathrm{e}^{2s} \sin^2 \frac{1}{2}\theta), \tag{10}$$

$$\langle (\Delta Y)^2 \rangle = \frac{1}{4}(\mathrm{e}^{-2s} \sin^2 \frac{1}{2}\theta + \mathrm{e}^{2s} \cos^2 \frac{1}{2}\theta), \tag{11}$$

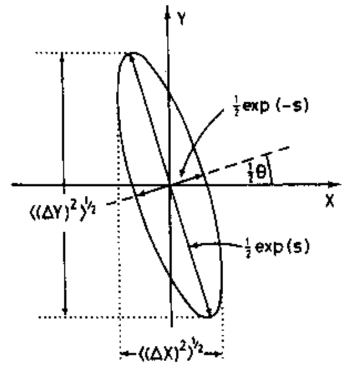

FIG. 3: Uncertainty ellipse of the ideal squeezed vacuum state. [7]

corresponding to Fig. II A.

Detection of squeezed states requires a phase sensitive scheme that measures the variance of a quadrature (4) of the field. In *homodyne* detectors the squeezed input state is superposed by a beamsplitter on the mode from a local oscillator with the same frequency, and this superposition is measured by a photodetector. The principle is somewhat similar to that of a *lock in amplifier*, which is familiar to every experimentalist. Squeezing in the input mode is then revealed by sub-Poissonian photocount statistics. In a *balanced* homodyne detector both output ports of the beamsplitter are measured and subtracted from each other. This technique removes the noise contributions that are made by the input signal and the local oscillator *alone*. A thorough discussion of these and other detection schemes can be found e.g. in [7] or [1].

III. GENERATION OF SQUEEZED STATES

To generate squeezed light from coherent light is to reduce fluctuation in one quadrature component, while enhancing it in the canonically conjugate component. As can be seen from (2), this is a phase sensitive process. Such phase sensitive processes are known from the nonlinear interaction of light with matter, e.g. *second harmonic generation*, *parametric down-* and *upconversion* and *four wave mixing*. All of these processes have in fact been successfully employed to generate squeezed light.

A Second harmonic generation

Although most of the earlier experiments on squeezing have concentrated on the frequency downconversion process, the reverse process, namely second harmonic generation (SHG), has also attracted considerable attention. It is one of the simplest $\chi^{(2)}$- or second order nonlinear optical processes and it can create nonclassical light for both the fundamental and the harmonic fields [3, 4, 5]. Substantial squeezing arises if the nonlinear crystal is suitably long or if it is placed in an optical cavity to resonate the fundamental or the second-harmonic mode or both. Although noise reduction as high as 52% was observed with an actively stabilized doubly resonant cavity [10], it is quite difficult to maintain doubly resonant condition for a sufficiently long time. Therefore singly resonant cavities, in which only the fundamental mode is confined, have been proposed [9] to generate squeezed light. A theoretical analysis by Paschotta *et al.* [9] predicted that such a system can produce 9.5 dB of squeezing in the second harmonic output and demonstrated this in an experiment featuring a monolithic nonlinear device. The analysis by Paschotta *et al.* is based on a two-photon loss model. Here the intensity of the fundamental is treated so that it remains constant during interaction with the second harmonic. This assumption is reasonable if the single-pass conversion efficiency is quite small. To estimate the maximum amount of squeezing, which is reached at high pump powers and high conversion efficiencies, Maeda *et al.* [4] has performed an analysis under consideration of fundamental depletion in the crystal. It is found that a singly resonant system can produce an arbitrary amount of squeezing in the second harmonic. This high noise suppression is explained by an optical negative feedback mechanism realized by the fundamental cavity mode. Because of the large conversion efficiency, the fundamental field is strongly correlated with the second harmonic. Since the fundamental is reflected at the cavity mirrors, it carries information on fluctuations of the output second harmonic over to the next circulation, thus providing the negative feedback. The reflectivity of the cavity mirrors is therefore a limiting factor of squeezing.

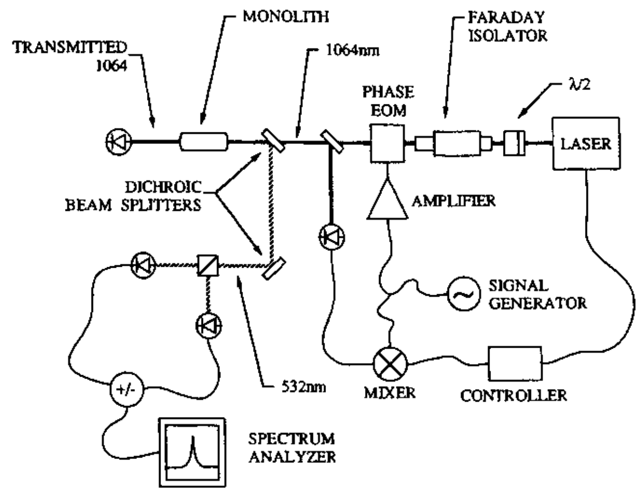

FIG. 4: Schematic representation of the experimental setup of Ralph *et al.* [6]. A diode pumped Nd:YAG laser was used to pump the monolithic doubler. The Faraday isolator prevents significant backreflection from entering the laser. A half-wave plate ($\lambda/2$) permits variable attenuation. The electrooptic modulator (EOM) phase modulates the driving field at 88 MHz. An error signal for locking the laser is derived from the 88 MHz signal driving the EOM and the fundamental light reflected from the monolith by use of a double balanced mixer. The second harmonic beam exits the front face of the monolith, where it is separated from the fundamental by use of two dichroic beam splitters. It is then incident upon a balanced detector, the summed and differenced outputs of which are monitored with a spectrum analyzer. [6]

As an example of an experimental realization of squeezing by SHG, consider the experiment performed by Ralph *et al.* [6]. Here squeezing from a singly resonant second harmonic generating crystal was observed. The experimental setup is depicted in Fig. III A. The summed photocurrent of the two detectors yields the intensity noise of the light, and the differenced photocurrent yields the quantum noise level.

Figure III A shows comparisons of the experimental results with theoretical calculations. In the theoretical analysis a technique was employed that permits the noise characteristics of the pump source used in the experiment to be explicitly modelled. The pump laser and the SHG crystal were treated as single quantum systems. As can be seen from Fig. III A,

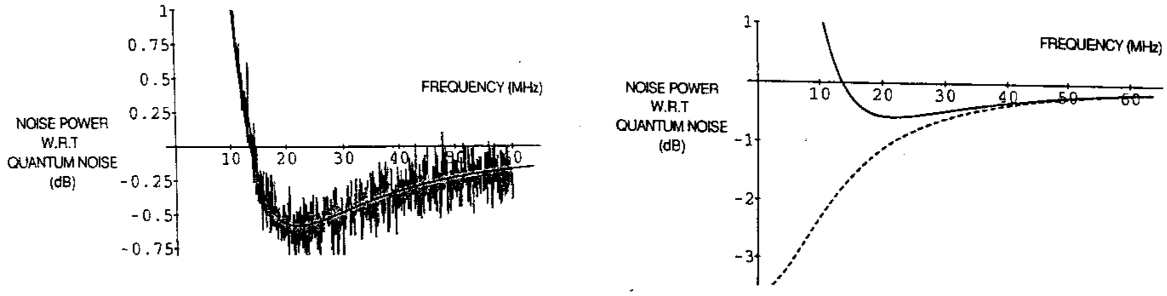

FIG. 5: left: Comparison of experimental and theoretical noise spectra of the second harmonic light. Right: Comparison of squeezing spectra with (solid curve) and without (dashed curve) pump noise. [6]

excellent quantitative agreement was obtained between theory and experiment. Squeezing emerges well below 20 MHz, because the fundamental cavity acts as a noise filter, reducing the amount of laser noise that enters the SHG crystal. From the graph on the right hand side of Fig. III A it is clear, that a major factor limiting the squeezing is the noise of the laser pump.

In another experiment performed by Bell *et al.* [8], bright tunable ultraviolet squeezed light was generated. Squeezing of ultraviolet light is more difficult than at longer wavelengths, because beta-barium borate (BBO), which has a small nonlinear coefficient, must be used for SHG instead of the $LiNbO_3$ or $KNbO_3$. This wavelength region is however of particular interest for atomic spectroscopy, as many atoms have strong absorption lines here. The experimental setup is in principle not very different from that of [6], Fig. III A. The SHG crystal was also placed in a singly resonant cavity in order to generate maximum squeezing on the second harmonic beam rather than the fundamental. The squeezed light was detected by a balanced homodyne detector. Squeezing was detected here at much lower frequencies than in [6], $3.5\ldots18$ MHz. These are smaller frequencies than the repetition rate of the laser, which means that not every pulse is squeezed, instead the integration over a group of pulses is considered.

This experiment was theoretically modelled by the above mentioned theory of Paschotta *et al.* [9], because the small nonlinear coefficient of BBO justifies the neglection of fundamental depletion. The results show that for all frequencies the measured noise spectrum lies above the theoretical spectrum, but there are minima in the experimental spectrum that approach the theoretical values. The extra noise can at least partially be attributed to the argon laser that pumps the Ti:Sapphire laser, which results in extra noise at the fundamental. Furthermore, the dependence of the squeezing on the laser power and on the input coupling of the fundamental field into the SHG cavity was investigated. In agreement with the theory, the observed squeezing increases with increasing laser power. As for the dependence on the input coupling, the theory predicts that the amount of squeezing will increase continuously as the input coupling is decreased, since small input coupling reduces the fluctuation of the fundamental in the SHG crystal. This applies for the case of no other losses than from the SHG. However, for a finite loss there will be an optimum input coupling to achieve maximum squeezing. This behavior was confirmed by the experiment.

B Degenerate parametric amplification

The degenerate parametric amplifier (DPA) was the earliest candidate suggested to produce squeezed light. As SHG, parametric amplification is also a $\chi^{(2)}$-process and generates downconverted light at one half of the pump wave frequency. (Here signal and idler beam have the same wavelength, they are *degenerate*.) The strong quantum correlations between the two downconverted photons are responsible for the squeezing. A theoretical analysis [1], treating the pump field classically and neglecting pump depletion, shows that the output of a DPA can in principle be squeezed to 100% (which means zero uncertainty in one component and infinite in the other) and is in an ideal squeezed state, as described in section II. This makes it a particularly important source of squeezed radiation. However, this result holds only for a perfectly coherent, monochromatic pump with a stabilized intensity. Amplitude and phase fluctuations in the pump field due to noise in the laser radiation will degrade the squeezing [1, 7].

C Optical parametric oscillator

The parametric amplifier becomes an optical parametric oscillator (OPO), when placed in an optical cavity. This is the preferred method to generate squeezing since the optical cavity helps increase the interaction of the field with the nonlinear medium, as mentioned in section III A for the case of SHG. A theoretical analysis [1] shows, that the maximum amount of squeezing of the cavity field is only 50%. This is due to vacuum fluctuations that enter through the out-coupling mirror. The field emitted from the OPO can however be almost perfectly squeezed, since the cavity field and the field due to the vacuum fluctuations entering the cavity through the out-coupler become correlated over time, thus making it possible for the residual fluctuations in the out-coupled cavity field to cancel out with the corresponding fluctuations of the vacuum field, which is reflected on the out-coupling mirror. This leads to almost perfect multi-mode, or spectral squeezing at an appropriate frequency.

In an experiment performed by Breitenbach *et al.* [11] a frequency doubled Nd:YAG laser pumps a monolithic degenerate type-I OPO. Its output is analysed in a balanced homodyne detector. The OPO consists of the parametric amplifier, being a $LiNbO_3$-crystal doped with MgO, whose end faces are polished and appropriately coated, thus providing the cavity. The monolithic design has the advantage of good mechanical stability and small losses. Furthermore, because of the short length of the cavity, the linewidth of the resonator is broad, which implies a broad bandwidth in the noise reduction. In the frequency range of $1 \ldots 30\,\mathrm{MHz}$ the squeezing spectrum as well as the antisqueezing spectrum was recorded. The maximal squeezing level was 5.5 dB below the vacuum noise level at a frequency of 2 MHz.

From the measurements of pairs of rotated quadrature components the Wigner function of the system was reconstructed. This technique is called the optical homodyne tomography. It represents the first step in a complete experimental characterization of a squeezed state.

D Four-wave mixing

In contrast to the previously discussed processes four-wave mixing is a $\chi^{(3)}$- or third order nonlinear process [1]. Here two planar counterpropagating pump waves interact in the nonlinear medium with a probe field entering at an arbitrary angle to the pump waves and yield a fourth (output) wave. The latter is proportional to the complex conjugate of the probe wave. This possibility of generating phase conjugate waves has many applications in adaptive optics. Four-wave mixing is also an important source of squeezed light, as seen in the considerable number of experiments that have been successfully performed.

Squeezed states of light were observed using both near-resonant and nonresonant optical nonlinearities. For a theoretical analysis of the latter case, the medium can be characterized by a classical susceptibility χ. This is the case for four wave mixing in glass fiber and for many nonlinear crystals, e.g. MgO:$LiNgO_3$, which can also be used as a parametric amplifier, as described in section III C. In four wave mixing experiments in atomic beams and vapors, however, the pump is tuned close to an atomic resonance. Here losses are inherent and spontaneous emission must be considered. In this case the medium must be quantized and can no longer be characterized by a classical χ.

For near-resonant experiments there are still two choices to be made: degenerate or non-degenerate four-wave mixing. In the degenerate case the pump and signal fields are at the same frequency. Squeezing is measured here only at a weak field at the pump frequency. *Nondegenerate* four-wave mixing provides a great advantage as for minimizing the effects of spontaneous emission [12]. Nondegenerate in this case means that each of the two weak intensity modes is equally and oppositely detuned in frequency from the central high intensity pump mode.

Slusher *et al.* [12] have investigated squeezed light generated by four wave mixing near the Na atomic resonance, both experimentally and theoretically. The experimental apparatus is shown in Fig. III D.

It consists of a cw ring dye laser, that is tuned to near the weaker hyperfine group

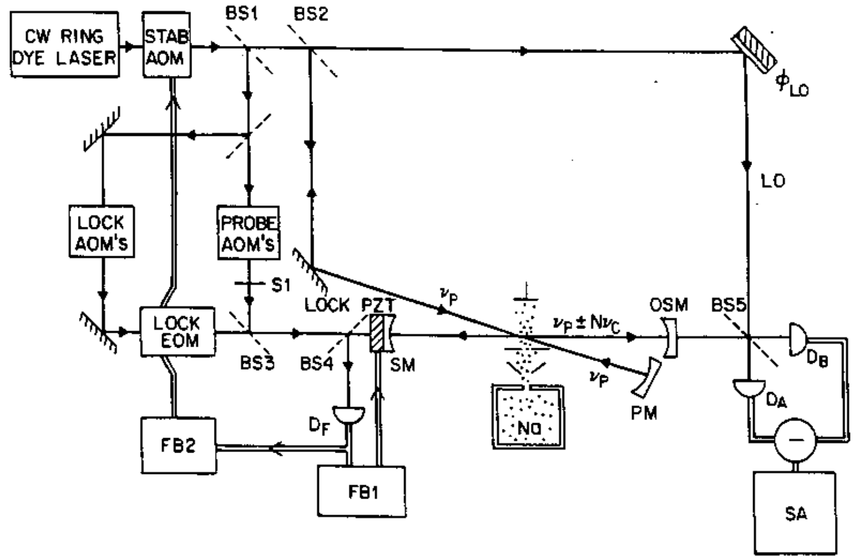

FIG. 6: Schematic diagram of the experimental arrangement for generating squeezed light by four-wave mixing near the Na atomic resonance, used by Slusher *et al.* [12]. Explanation see text.

of the D_2 Na resonance. The laser crosses an atomic beam of Na and is reflected back into the same Na interaction region, resulting in a standing wave pattern. This drives a polarization in Na to generate pairs of photons at a frequency shift of $\pm\,595\,\mathrm{MHz}$ from the pump frequency. These pair frequencies are are resonant in a cavity oriented at a small angle ($0.86°$) with respect to the pump beam. To detect the squeezed light from this cavity, a balanced homodyne detector is used. The theoretical description involves a fully quantum mechanical treatment in which the medium is modelled as N two level atoms, and which includes losses and dephasing due to spontaneous emission near the atomic resonance. The theory predicts large squeezed-noise reductions of the order of 10 at pump intensities of $0.1 \ldots 0.5$ times the saturation intensity of the atomic resonance. The experimental results compare well with theoretical predictions at small pump intensities and large detuning Δ of the pump frequency with respect to the center of the atomic resonance. At small detuning and large pump intensity an extra phase insensitive noise is observed that is not predicted by the theory at hand. At high pump intensities, the four wave mixing process reaches the threshold for oscillation, in agreement with the theoretical model.

IV. SUMMARY

The processes mentioned in section III are but a small sampling of a great variety of processes that can be employed for the generation of squeezed light. Other successfully applied techniques include photon number squeezing in semiconductors [13], amplitude squeezing in semiconductor diode lasers [18], utilizing the Kerr-nonlinearity as another $\chi^{(3)}$- or third order nonlinear process, e.g. in optical fibers [14, 15], using a Q-switched laser to pump a parametric downconverter, which exhibits squeezing [16], producing amplitude-squeezed optical solitons [17], using phase-sensitive parametric amplifiers to compensate for the propagation losses incurred by solitons in optical fibers, which squeezes fluctuations in the soliton frequency, thus reducing the Gordon-Haus jitter [19], etc. However, the application of squeezed light in any of the areas, that were mentioned at the beginning, is still rare. This is due to the fragility of squeezed states, that makes technological application difficult.

REFERENCES

[1] Marlan O. Scully and M. Suhail Zubairy, *Quantum optics*. (Cambridge University Press, 1997)

[2] Hermann A. Haus, J. Opt. Soc. Am. B **12**, 2019 (1995)

[3] L. A. Lugiato, G. Strini, and F. De Martini, Opt. Lett. **8**, 256 (1983)

[4] Joji Maeda and Kazuro Kikuchi, Opt. Lett. **21**, 821 (1996)

[5] Ruo-Ding Li and Prem Kumar, Opt. Lett. **18**, 1961 (1993)

[6] T. C. Ralph, M. S. Taubman, A. G. White, D. E. McClelland, and H.-A. Bachor, Opt. Lett. **20**, 1316 (1995)

[7] R. Loudon and P. L. Knight, J. Mod. Opt. **34**, 709 (1987)

[8] A. S. Bell, E. Riis, and A. I. Ferguson, Opt. Lett. **22**, 531 (1997)

[9] R. Paschotta, M. Collet, P. Kürz, K. Fiedler, H.-A. Bachor, and J. Mlynek, Phys. Rev. Lett. **72**, 3807 (1994)

[10] H. Tsuchida, Opt. Lett. **20**, 2240 (1995)

[11] G. Breitenbach, T. Müller, S. F. Pereira, J.-Ph. Poizat, S. Schiller, and J. Mlynek, J. Opt. Soc. Am. B **12**, 2304 (1995)

[12] R. E. Slusher, B. Yurke, P. Grangier, A. LaPorta, D. F. Walls, and M. Reid, J. Opt. Soc. Am. B **4**, 1453 (1987)

[13] Malvin C. Teich, Frederico Capasso, and Bahaa E. A. Saleh, J. Opt. Soc. Am. B **4**, 1663 (1987)

[14] G. J. Milburn, M. D. Levenson, R. M. Shelby, S. H. Perlmutter, R. G. DeVoe, and D. F. Walls, J. Opt. Soc. Am. B **4**, 1476 (1987)

[15] M. D. Levenson and R. M. Shelby, J. Mod. Opt. **34**, 775 (1987)

[16] Orhan Aytür and Prem Kumar, Opt. Lett. **17**, 529 (1992)

[17] Dmitriy Krylov, Keren Bergman, and Yinchieh Lai, Opt. Lett. **24**, 774 (1999)

[18] Yong-qing Li and Min Xiao, Opt. Express **2**, 110 (1998)

[19] I. H. Deutsch and I. Abram, J. Opt. Soc. Am. B **11**, 2303 (1994)