Quantum computing with cavity QED

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In this article basic ideas of Quantum Computing (QC) using cavity quantum electrodynamics (QED) are reviewed in theory and experiment. The principle of a quantum gate is explained. Two proposals for experimental realizations with neutral atoms in cavities are presented and compared to reports of related experiments.

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I. INTRODUCTION

From a historical perspective Quantum computing (QC) combines two major developments of this century in its name, the rising of the quantum theory and the emerging of information technology. This seems to be a powerful combination [1]. But the challenges in combining both to a new synthesis are huge. In the following a motivation for quantum computing is given and its possible elementary realization at the present day is discussed using principles of cavity QED.

The interest in QC is fueled by the hope that new techniques of computing bring algorithms for prime factoring [2] and data search [3]. The quantum nature of the computing process gives hope to the idea that simulations of quantum systems become much more feasible than in classical computing [4]. As well one of the strong drivers to investigate more in that direction comes from the possibilities of quantum cryptography [5], which allows in theory one of the most secure communication means thinkable today using a technique called teleportation [6].

First concepts of QC were proposed more than 10 years ago [7]. But only in the last years the development gained momentum through experiments and is growing fast. Part of the reason is that original doubts of feasibility could be reduced by the proposal of error correction schemes [8]. Questions of scalability are addressed by ideas of quantum networks [9].

II. THE QC PRINCIPLE

In classical computing the smallest unit of information, a bit, is either 0 or 1. Replacing this smallest unit by a quantum bit or short *qubit*, is the first step towards quantum computing. A qubit allows besides 0 and 1 any superposition of these two values

$$\alpha|0\rangle + \beta|1\rangle ,$$

where the coefficients are allowed to be complex numbers. It measures the amount of information, which can be stored in a two state quantum system. As an example one may think of the state of a spin 1/2 particle. Several of these qubits can be thought to form a tensor product similar as bits form bytes

$$|0\rangle|1\rangle|1\rangle$$

But since superposition is allowed now, we can end up in a form like the following

$$\alpha|0\rangle|1\rangle|1\rangle + \beta|1\rangle|1\rangle|0\rangle$$

These so called entangled states connect the information of one qubit intimately with the information of another one. There is no possibility to factorize this state into its single qubit states. This was the major concern of interpretation in quantum mechanics since the famous paper of Einstein-Podolsky and Rosen [10]. Now it turns out to be at the heart of quantum computing concepts and one of the reasons for its theoretical potential and experimental challenges.

In order to perform QC the states have to undergo unitary transformations given by the Schrödinger equation

$$|\psi(t)\rangle = U|\psi(0)\rangle , \qquad \text{with} \qquad U = \exp\left(-\frac{i}{\hbar}\int_0^t dt' H\right).$$

As long as the quantum system is not in contact with its environment, the states evolve reversibly. The possible entanglement is preserved. But in order to do a measurement or any kind of manipulation the system has to be coupled to the macroscopic world. This will introduce decoherence of the state, which may lead to a decay of the state before the computation is finished. Performing the required logical operation before the state decays is the art and challenge of QC.

The basic logical operations turn out to be the one qubit transformation, where the phase of the one qubit state is changed, and a two qubit gate. The two qubit gate operates similar as a gate in classical computing. The first qubit controls the second one according to some logical relation. It was shown, that any two qubit gate allows one to realize any kind of logical gate in principle [11].

The gate which is proposed to be realized is the CNOT, controlled not gate and a generalized version of it. Here the first qubit serves as a controlling unit and the second one as the target one. Its logical relation is defined as:

$$\begin{array}{ccc} |0\rangle|0\rangle & \rightarrow & |0\rangle|0\rangle \\ |0\rangle|1\rangle & \rightarrow & |0\rangle|1\rangle \\ |1\rangle|0\rangle & \rightarrow & |1\rangle|1\rangle \\ |1\rangle|1\rangle & \rightarrow & |1\rangle|0\rangle \end{array}$$

III. PROPOSED SCHEMS AND TODAYS EXPERIMENTAL REALIZATION

A. Trapped atoms in a single mode cavity

1. The proposed scheme

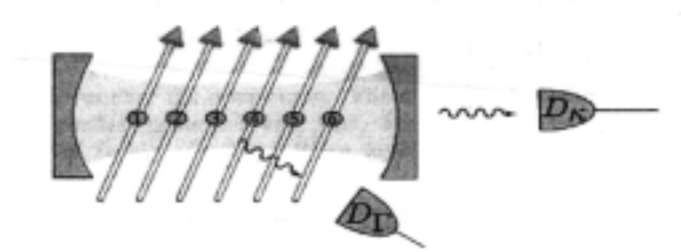

FIG. 1. Realization scheme of a cavity quantum computer using neutral atoms in an optical cavity. The arrows indicate laser beams addressing each atom individually. D_κ and D_Γ detect cavity photons and spontaneously emitted photons, respectively.

One proposal to realize this kind of gate using cavity QED was made by Pellizzari *et al.* [12]. The idea is to use neutral atoms in an optical cavity for the storage of the qubits. The qubits are stored in the Zeeman ground state levels of the trapped atoms. Different laser beams can address them individually in order to manipulate the qubits for writing and reading purposes as shown schematically in Fig.1. This is similar to the ideas of one of the first proposals of QC [13] where ions in a linear trap are the qubit carriers. But the Coulomb interaction does not serve as the coupling mechanism anymore; instead a single photon in the cavity is used.

For a theoretical description the manipulating laser beams may be treated classically, but the photon in the single mode cavity has to be treated according to QED. It provides the entanglement between the different atoms in the cavity. A two atom cavity with a single photon is sufficient to realize the CNOT gate. In this situation the source of decoherence is the spontaneous emission from the excited state of the atoms and from cavity decay during the gate operation. Using an adiabatic passage via a dark state of the strongly coupled two-atom and cavity system avoids spontaneous emission. Having a photon present only during the gate operation minimizes the cavity decay. In more detail the situation can be understood as following.

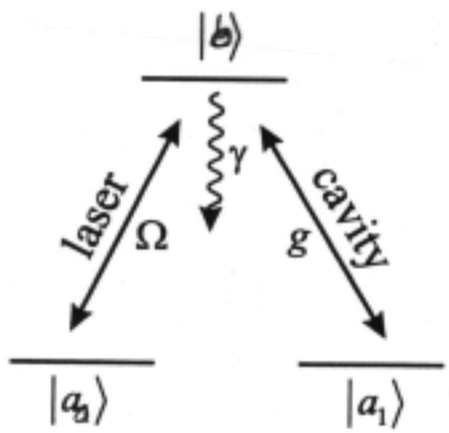

FIG.2. Basic three-level system for the cavity QED quantum computer. One transition is coupled to a laser field; the other is coupled to a quantized cavity mode.

(a) Manipulating one atom. The simplest energy system connecting one atom with the cavity can be understood with a three level lambda configuration (Fig.2). When no computation is performed the qubit is stored in the ground states $|a_0\rangle, |a_1\rangle$, which are not coupled directly. The energies between both ground levels $|a_0\rangle, |a_1\rangle$ and the excited level $|b\rangle$ are about the same and are represented by the

Rabi frequencies Ω_0, Ω_1 of the transitions. The dynamics for classical fields are given by the Hamiltonian

$$H = \hbar\Omega_0 |b\rangle\langle a_0| + \hbar\Omega_1 |b\rangle\langle a_1| + h.c..$$

Excitation from one of the states $|a_0\rangle$ with the laser allows to populate the upper state $|b\rangle$ and from there in principle the other one $|a_1\rangle$, which has a transition resonant to the cavity mode. But since the upper level is subject to spontaneous emission and consequently a source of strong decoherence, this is not the way to go. Instead one can take advantage of the dark state of the lambda configuration. The dark state is a superposition of the two ground levels given by

$$|D\rangle = N\left[\Omega_1 |a_0\rangle - \Omega_0 |a_1\rangle\right].$$

Here N is a normalization factor and Ω_0, Ω_1 are the Rabi frequencies of the two transitions. For a system in this state it is possible to make a so-called adiabatic passage from $|a_0\rangle$ to $|a_1\rangle$ or vice versa by changing the fields that control the Rabi frequencies from $\Omega_1 >> \Omega_0$ to $\Omega_0 >> \Omega_1$, or vice versa. This dark state is an eigenstate of the Hamiltonian with an eigenvalue 0. This means that the excited state won't be populated and no spontaneous emission is possible during the adiabatic passage. When the Rabi frequencies are fixed at a given ratio, the fields don't change the superposition of the state.

For the situation in which one of the transitions is coupled to the cavity mode only one Rabi frequency is accessible via a laser pulse while the other one is replaced by the coupling terms $gc^\dagger$ and $g^* c$ leading to the Hamiltonian

$$H = \hbar\Omega_0 |b\rangle\langle a_0| + \hbar g c^\dagger |b\rangle\langle a_1| + h.c. = H_{laser1-atom1} + H_{cavity-atom1}.$$

(b) Manipulating two atoms. So far only one atom was considered. In the case of two atoms coupled to each other by the cavity (Fig. 3) we have

$$H = H_{laser1-atom1} + H_{cavity-atom1} + H_{laser2-atom2} + H_{cavity-atom2}.$$

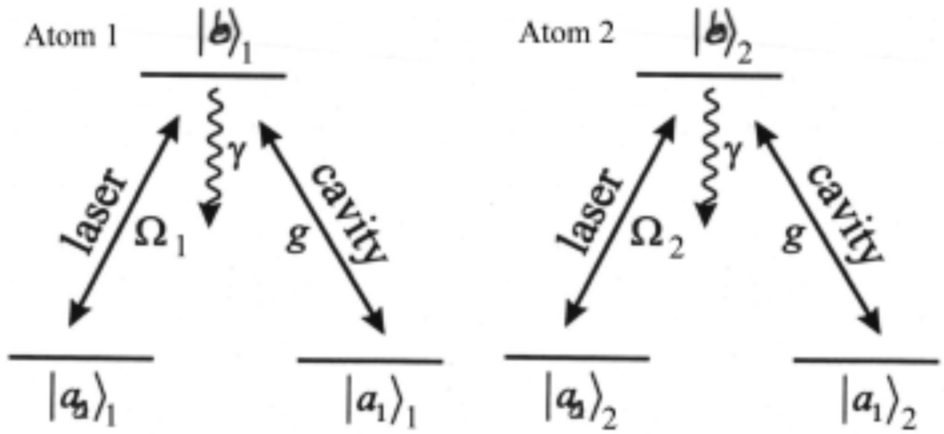

FIG. 3. Two atoms each having a three-level system are coupled to the same cavity, but different laser beams resulting in two Rabi frequencies Ω_1 and Ω_2.

Here there are two dark states possible,

$$|D_0\rangle = |a_1\rangle_1 |a_1\rangle_2 |0\rangle,$$
$$|D_1\rangle = N\left[\Omega_2 g|a_0\rangle_1 |a_1\rangle_2 |0\rangle + \Omega_1 g|a_1\rangle_1 |a_0\rangle_2 |0\rangle - \Omega_1\Omega_2 |a_1\rangle_1 |a_1\rangle_2 |1\rangle\right],$$

where Ω_1, Ω_2 are now the Rabi frequencies of each atom. Superposition of the two dark states allows an adiabatic passage of superposed ground states from one atom to the other without spontaneous emission. For $\Omega_2 >> \Omega_1$ we can write

$$\alpha|D_1\rangle_1 + \beta|D_0\rangle_1 = \left[\alpha|a_0\rangle_1 + \beta|a_1\rangle_1\right]|a_1\rangle_2$$

and for $\Omega_1 >> \Omega_2$ we get

$$\alpha|D_1\rangle_1 + \beta|D_0\rangle_1 = |a_1\rangle_1\left[\alpha|a_0\rangle_2 + \beta|a_1\rangle_2\right].$$

By tuning the Rabi frequencies from $\Omega_2 >> \Omega_1$ to $\Omega_1 >> \Omega_2$ we can swap the qubit (quantum state) from between the atoms

$$\left[\alpha|a_0\rangle_{atom1} + \beta|a_1\rangle_{atom1}\right]|a_1\rangle_{atom2} \Rightarrow |a_1\rangle_{atom1}\left[\alpha|a_0\rangle_{atom2} + \beta|a_1\rangle_{atom2}\right].$$

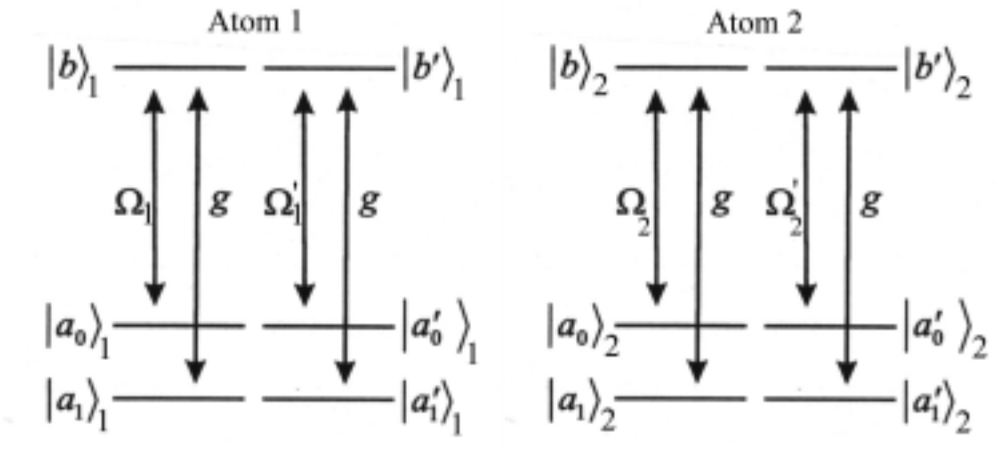

FIG. 4. Two parallel three-level systems in each atom are necessary to perform quantum gates in the cavity QED quantum computer.

(c) Performing a controlled-NOT gate. In order to create a controlled-NOT gate it is necessary to allow *two* three-level systems for each atom (Fig. 4). We assign for atom-1

$$|0\rangle_1 = |a_0\rangle_1$$
$$|1\rangle_1 = |a_1\rangle_1$$

and for atom-2

$$|0\rangle_2 = |a_1\rangle_2$$
$$|1\rangle_2 = |a_1'\rangle_2 \, .$$

Now it is possible to perform the following sequence: adiabatic passage (swapping) from atom-1 to atom-2, inversion of the cavity ground states of the second atom and finally adiabatic passage back to atom-1.

The first step switches the states in from one atom to the other atom in the same way as described above via a dark state. We get

$$\begin{array}{ll} |0\rangle_1|0\rangle_2 = |a_0\rangle_1|a_1\rangle_2 & |a_1\rangle_1|a_0\rangle_2 \\ |0\rangle_1|1\rangle_2 = |a_0\rangle_1|a_1'\rangle_2 & |a_1\rangle_1|a_0'\rangle_2 \\ |1\rangle_1|0\rangle_2 = |a_1\rangle_1|a_1\rangle_2 & |a_1\rangle_1|a_1\rangle_2 \\ |1\rangle_1|1\rangle_2 = |a_1\rangle_1|a_1'\rangle_2 & |a_1\rangle_1|a_1'\rangle_2 \end{array} \Rightarrow$$

This operation transfers the states from one atom to the other one and vice versa. The next step is to switch the ground states, which are coupled to the cavity in atom 2, while keeping the ones from atom 1 (Raman transition with two lasers)

$$\begin{matrix} |a_1\rangle_1 |a_0\rangle_2 \\ |a_1\rangle_1 |a_0'\rangle_2 \\ |a_1\rangle_1 |a_1\rangle_2 \\ |a_1\rangle_1 |a_1'\rangle_2 \end{matrix} \Rightarrow \begin{matrix} |a_1\rangle_1 |a_0\rangle_2 \\ |a_1\rangle_1 |a_0'\rangle_2 \\ |a_1\rangle_1 |a_1'\rangle_2 \\ |a_1\rangle_1 |a_1\rangle_2 \end{matrix} .$$

Then again the states between the two atoms are exchanged, so all together we get

$$\begin{matrix} |0\rangle_1 |0\rangle_2 = |a_0\rangle_1 |a_1\rangle_2 \\ |0\rangle_1 |1\rangle_2 = |a_0\rangle_1 |a_1'\rangle_2 \\ |1\rangle_1 |0\rangle_2 = |a_1\rangle_1 |a_1\rangle_2 \\ |1\rangle_1 |1\rangle_2 = |a_1\rangle_1 |a_1'\rangle_2 \end{matrix} \Rightarrow \begin{matrix} |a_1\rangle_1 |a_0\rangle_2 \\ |a_1\rangle_1 |a_0'\rangle_2 \\ |a_1\rangle_1 |a_1\rangle_2 \\ |a_1\rangle_1 |a_1'\rangle_2 \end{matrix} \Rightarrow \begin{matrix} |a_1\rangle_1 |a_0\rangle_2 \\ |a_1\rangle_1 |a_0'\rangle_2 \\ |a_1\rangle_1 |a_1'\rangle_2 \\ |a_1\rangle_1 |a_1\rangle_2 \end{matrix} \Rightarrow \begin{matrix} |a_0\rangle_1 |a_1\rangle_2 = |0\rangle_1 |0\rangle_2 \\ |a_0\rangle_1 |a_1'\rangle_2 = |0\rangle_1 |1\rangle_2 \\ |a_1\rangle_1 |a_1'\rangle_2 = |1\rangle_1 |1\rangle_2 \\ |a_1\rangle_1 |a_1\rangle_2 = |1\rangle_1 |0\rangle_2 \end{matrix} . \qquad (1)$$

In summary the quantum information has been switched from atom-1 to atom-2 where it has been manipulated and then distributed back again on atom-1.

For storage purpose all the quantum bits are transferred to the ground states that are resonant with the driving laser fields $|a_0\rangle, |a_0'\rangle$.

The advantage of this scheme is that the excited states are never populated and therefore no spontaneous emission will take place. The interaction stays resonant so that no phase shift will be introduced, and the interaction times don't need to be as accurate as long as the adiabaticity is fulfilled. Finally the cavity decay can occur only for the short time when the intermediate state is populated.

2. *Today's experimental realization*

In order to realize the above situation where the atoms are trapped in a cavity but still behave as anticipated, one has to come up with a complex setup. One recent realization coming close is Ref. [14]. The gate is not implemented yet; there is no switching data available. But for the first time it was possible to trap a single cesium atom in a cavity (Fig. 5).

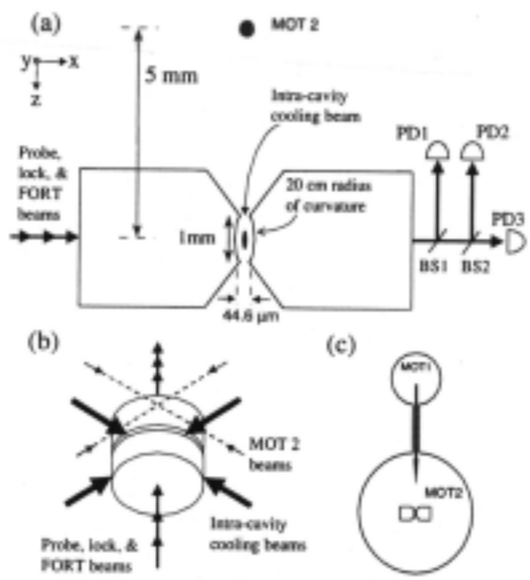

Fig. 5. Schematic of the experimental apparatus. (a) The dichroic beam splitter BS1 sends the cavity-length-stabilizing beam to PD1. BS2 separates the FORT (sent to PD2 for locking) and cavity QED beams (sent to PD3 for balanced heterodyne detection). (b) Beam geometry for intra-cavity cooling and MOT-2. (c) Differentially pumped chamber and two-stage MOT.

Two magneto optical traps are used to store the atoms before they enter the cavity. The lower one is located in an ultra high vacuum (UHV) chamber, just above the optical cavity, which is also located in UHV. From the lower magneto optical trap the cesium atoms may fall in the cavity, where they are stopped and cooled down to about 2μK by optical intra-cavity laser cooling. A probe beam crosses the cavity longitudinal and is detected with a heterodyne detection scheme behind it, to monitor the cavity. If a cesium atom detected, a far off resonance trap (FORT) is switched on to trap the atom in the cavity. Finally a lock beam locks the cavity length to its set value.

With this setup it is possible to trap a single atom inside the cavity. The average lifetime for single atoms trapped within the FORT is τ=(28±6) ms. Nevertheless there is an issue with the FORT since it shifts the energy level of the excited state, which in consequence leads to (dipole heating and) expulsion of the atom. The authors are confident to be able to resolve this issue.

B. Atoms flying through a single mode cavity

1. The proposed scheme

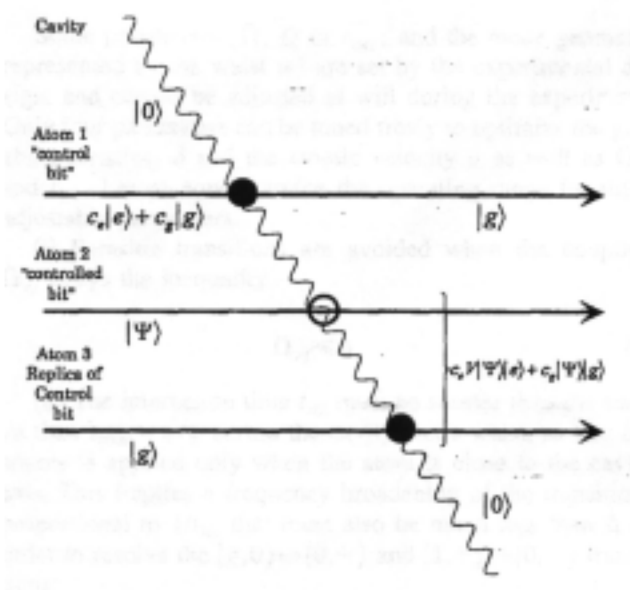

Fig. 6. Scheme of the quantum gate using only atoms as the q-bit carriers. The first atom's state is copied onto the cavity state through a resonant interaction depicted by a solid black circle. The second atom experiences a resonant interaction (open circle) and undergoes the required conditional dynamics. A third atom, interacting resonantly again with the cavity, carries away a replica of the first atom's state and leaves the cavity empty. Applying a dc voltage between the superconducting cavity mirrors changes the atom-cavity resonance condition.

A different approach to QC in cavity QED was proposed by Domokos et al. [15]. Instead of trapping the atoms in the cavity, they propose to use flying Rydberg atoms as the carriers of the qubits (Fig. 6). Manipulation of their states then can be performed before or after the cavity while the interaction of the atom with the field takes place when they cross the cavity. The proposal uses three atoms in order to perform a universal two qubit gate. The exact type of gate can be determined by the experimental parameters. The general form of which is:

$$\hat{U} = \begin{pmatrix} \hat{1} & 0 \\ 0 & \hat{V} \end{pmatrix} \qquad \hat{V} = \begin{pmatrix} \cos\varphi & e^{-i\theta}\sin\varphi \\ -e^{i\theta}\sin\varphi & \cos\varphi \end{pmatrix}$$

where $\hat{U}$ acts on the vector of the two qubits in the basis $|0\rangle = |0\rangle|0\rangle; |1\rangle = |0\rangle|1\rangle; |2\rangle = |1\rangle|0\rangle; |3\rangle = |1\rangle|1\rangle$. For $\theta = -\pi/2, \varphi = \pi/2$ it becomes the controlled NOT gate as before (up to a common phase factor in the off-axis elements). For the sake of simplicity we focus on this situation, even if the other cases have possible realizations too.

The operations needed to perform a CNOT gate are exactly the same as given before in Eq. (1), if the states are associated with the states of the flying Rydberg rubidium atoms. The first atom carries the control qubit. It is prepared in a superposition state of the two quantum numbers 51 and 50 corresponding to an excited $|e\rangle$ and a ground $|g\rangle$ state of the atom. This atom crosses the initially empty cavity mode. While crossing the cavity, the atom can be tuned with an electric field, using the Stark effect in exact resonance for a controlled amount of time. This interaction time is adjusted so that the atom experiences, when in state $|e\rangle$, an exact π pulse and releases one photon in the cavity, while it is not affected if initially in state $|g\rangle$. Writing the states of the cavity with and without a photon as $|1\rangle$ $|0\rangle$ respectively, we get the following transformation due to the passage

$$|0\rangle(c_e|e\rangle + c_g|g\rangle) \Rightarrow (-ic_e|1\rangle + c_g|0\rangle)|g\rangle \; . \tag{2}$$

This means that the state of the atom is transferred to the cavity. This is the first step of the CNOT gate. (The entanglement of the first atom with the exciting photon is transferred to the cavity.)

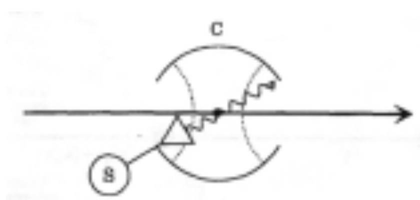

Fig. 7. Scheme of the cavity-QED two bit gate. The control qubit is the state of a superconducting cavity C, either empty or containing one photon. A two-level circular Rydberg atom, slightly detuned from the cavity frequency, carries the controlled bit. The state of the atom is manipulated by a classical field source S. The atom interacts noticeably with S only if the single photon field in C light shifts the transition in resonance with S.

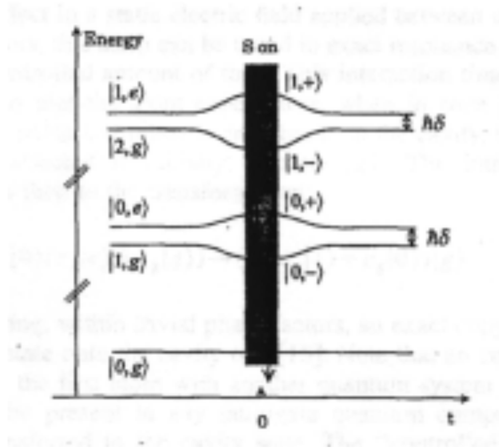

Fig. 8. Position of the atom-cavity energy levels ("dressed levels") as a function of time. The atom is on cavity axis at t=0. The classical source S is switched on during a small time interval around t=0. It is resonant with the transition between the dressed states $|0,-\rangle$ (originating from $|1,g\rangle$) and $|1,+\rangle$ (originating from $|1,e\rangle$). It is therefore out of resonance on the transition between the state $|0,g\rangle$ (not affected by the atom-field coupling) and $|0,+\rangle$.

Next the target atom enters the cavity. It experiences the conditional dynamic depending on the state transferred from the first atom to the cavity. During its passage through the cavity it is subject to an auxiliary classical millimeter-wave field, radiated in a transverse mode not belonging to the cavity (Fig.7). The effect of the auxiliary field depends upon the state of the field in the cavity and the state of the second atom entering it. Figure 8 sketches the atom-field energy levels as a function of the position of the atom along its path across the cavity mode (i.e. as a function of time). Far away from the cavity the atom-cavity states are uncoupled and paired in nearly degenerate pairs $\{|n,e\rangle,|n+1,g\rangle\}$. Inside the cavity, these levels are coupled and form "dressed states" of the atom-field system $\{|n,+\rangle,|n,-\rangle\}$. Their splitting increases with the coupling between atom and cavity and the energy level modification is proportional to the excitation number. If the atomic motion through the cavity is slow enough, the system will follow the dressed energy levels adiabatically. If there was no auxiliary millimeter-field, nothing would change the second atom passing the cavity.

But with the classical field, radiated by S, tuned into resonance with the $|1,+\rangle \leftrightarrow |0,-\rangle$ transition at the center of the cavity, the second atom will undergo a Rabi flopping, if there is a control photon in the cavity. If there is no photon in the cavity, the atom cavity system does not experience a Rabi flopping since the $|0,+\rangle \leftrightarrow |g,0\rangle$ transition is far from resonance with S. Depending on amplitude and phase of S any conditional unitary transformation (given by U and V) of the atomic qubit can be realized, in principle.

The third atom undergoes the inverse of the transformation of atom 1. This will copy the state of the cavity back on the third atom, which entered the cavity in $|g\rangle$. It will exit the cavity as a replica of the initial state of the control bit. The cavity is left empty, ready for the next gate cycle. As well it would be possible to use the first atom as the third one. Then the first carries the control qubit, and the second one the target qubit.

The advantage of this technique is it's relative simplicity. As well it is possible in principle to start with a pair of entangled atoms, but the decoherence times will decrease for systems of entangled atoms.

2. Today's experimental realization

Experimentally Hagley et al. [16] could demonstrate the preparation of entangled Rubidium atoms. They use a superconducting microwave cavity at 0.6K. The principle sequence of their setup is shown in Fig.9. The oven (O) emits the Rubidium atoms, the two crossed laser beams L_1 and L'_1 serve as a Doppler effect selector while the excitation in one of the two circular Rydberg states takes place in B. The high Q cavity C may store the single photon for the entanglement. The low Q cavity allows in connection with the ionization detectors D_e and D_g the state selective detection of the atoms.

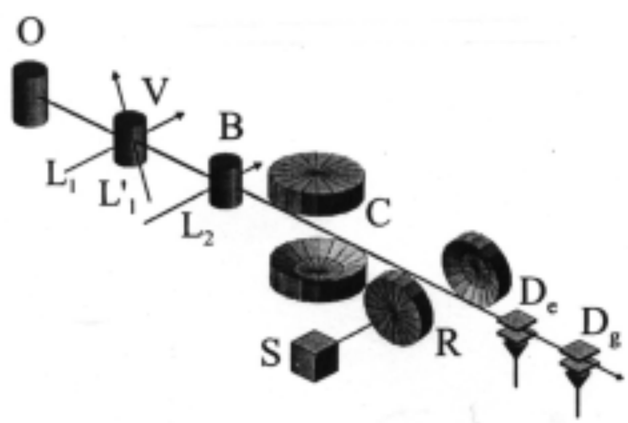

Fig. 9. Sketch of the experimental setup

This setup allows one to excite the first atom before it enters the cavity.

$$|\psi\rangle = |e_1, g_2, 0\rangle$$

There it will undergo a Rabi flopping, depending on its lifetime in the cavity. Adjusting it right, makes the atom lose the photon to the cavity with probability 1/2. It will leave the system in the state

$$|\psi'\rangle = \frac{1}{\sqrt{2}}(|e_1, g_2, 0\rangle - |g_1, g_2, 1\rangle) .$$

Now the second atom is introduced in the cavity where the photon still might be but the first atom is not present anymore. If the first atom has emitted a photon, the second one absorbs it with unity probability and ends up in e. The result for the combined system is

$$|\psi_{EPR}\rangle|0\rangle = \frac{1}{\sqrt{2}}(|e_1, g_2\rangle - |g_1, e_2\rangle)|0\rangle .$$

The result is the entangled state of the two atoms, while the cavity is empty. As a proof of the entanglement the conditional probabilities of measuring the second atom in level e when the first one has been found in e or g, respectively are shown (Fig.10).

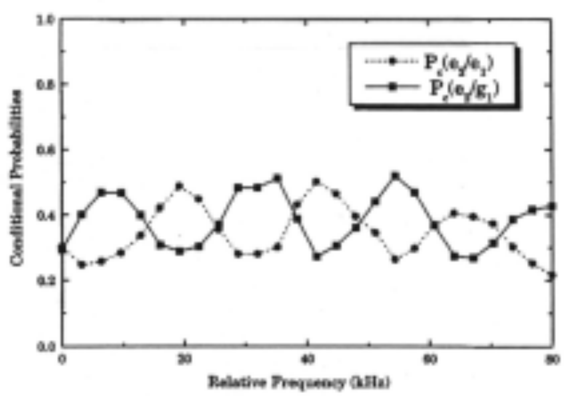

Fig. 10. Conditional probabilities $P_c(e_2/e_1)$ (circles) and $P_c(e_2/g_1)$ (squares) of measuring the second atom in level e when the first one has been found in e or g, respectively, plotted versus the frequency ν of the pulses in R. The lines connecting the experimental points have been added for visual convenience

This result is an important step to QC using some of the setup, which is necessary for the CNOT gate described above. Besides there is hope to be able to test Bell's inequalities with atoms and to prepare correlated triplets (Greenberger-Horne-Zeilinger states).

IV. CONCLUSIONS

In summary QC with cavity QED had a vivid start and the first experimental results look promising. Even though there are still several technical improvements necessary, a quantum gate seems to be feasible. The fragility of quantum states puts high demands on the experiments, so a more complex computing structure than a single gate seems still some time away. Nevertheless these studies allow a better understanding and experience of the general physics of decoherence, which gives reason in itself to continue the work.

ACKNOWLEGMENTS

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