Lasing without inversion

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In this review the physical concept of lasing without inversion (LWI) is presented. LWI concept is discussed in bare states and dressed states. Both theoretical and experimental work is treated. The discussion will concentrate on the analysis of physical conditions under which a three level medium can lase when there is no population inversion between the lasing levels. The experimental work considered in this article focuses on experimental demonstration of laser oscillation without population inversion in Λ - type atomic configuration. At the end of this review a possible application of LWI for realization of a γ-ray laser is reported.

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I. INTRODUCTION

Many interesting phenomena in Optics, such as quantum beats, the Hanle effect and self –induced transparency originate from atomic coherence and interference of radiative processes. Under special conditions coherent atomic transitions can cancel absorption. This result of atomic coherence is used in concepts of coherent trapping, electromagnetically induced transparency (EIT), enhancement of the index of refraction in nonabsorbing medium and lasing without inversion.

The main idea of LWI is that absorption cancellation provides the possibility to obtain light amplification even if the population of the upper level is less than the population of the lower level. Such a situation can be realized, for instance, in a three-level system, when two coherent atomic transitions destructively interfere and, hence, cancel absorption. This phenomenon can have interesting application for developing sources of coherent radiation in a region of the electromagnetic spectrum where the implementation of traditional laser schemes is either too difficult or impractical.

To present the basic physics of LWI it is best to consider the theory of this effect in a three-level Λ- configuration and then to demonstrate how the concept of lasing without inversion can be realized experimentally.

II. CONCEPT OF LWI

The configuration of a Λ-three-level atomic system is presented in Fig.1. It is formed by an upper level |a> connected to lower levels |b> and |c> through interaction with electromagnetic fields E_1 and E_2, respectively, in such way that only atomic transitions |a>-|c> and |a>-|b> are allowed. The physical reason for canceling absorption in this system is the uncertainty in atomic transitions |c>-|a> and |b>-|a> which results in destructive interference between them. Since both of these transitions are directed to the same atomic state |a>, it is impossible to find out along which path, |c>-|a> or |b>-|a>, such a transition is made. This situation is similar to Young's double-slit problem, where interference is a consequence of uncertainty in determining through which of the two slits the photon passed [1].

The absorption probability will be equal to the squared sum of the probability amplitudes corresponding to |c>-|a> and |b>-|a> transitions. When there is a correlation between these probability amplitudes, it will lead to an interference term which, under appropriate phase conditions, can make the total absorption probability equal to zero. The emission probability is equal to the sum of the transition probabilities |a>-|c> and |a>-|b>, and is independent of their mutual correlation. This results from the different final states. It is known exactly along which path the atom makes its transition to the lower level |a>-|c> or |a>-|b>, so there is no uncertainty in atomic routes and as a consequence, there is no interference between these transitions. Such asymmetry in up and down transitions allows amplification in this atomic system with zero absorption losses.

To demonstrate this concept in a more rigorous mathematical way, it is helpful to follow the treatment of M.O. Scully and M.S. Zubairy [2]. Their approach is based on semiclassical treatment, where the electromagnetic field is considered classically but the atom is treated quantum mechanically. The idea is to calculate the time dependent probability amplitudes for each level and then show that the probability of a transition to the upper level can vanish for particular initial conditions but the transition probability to a lower level does not vanish.

The Hamiltonian for the atomic system in rotating-wave approximation is

$H = H_0 + H_1,$ (1)

where:

$$H_0 = \hbar w_a|a><a| + \hbar w_b|b><b| + \hbar w_c|c><c| \quad (2)$$

$$H_1 = -\frac{\hbar}{2}(\Omega_{R1} e^{-i\phi_1} e^{-iw_{L1}t} \mid a >< b \mid + \Omega_{R2} e^{-i\phi_2} e^{-iw_{L2}t} \mid a >< c \mid) + H.C. \quad (3)$$

H_0 and H_1 represent the unperturbed and interaction part of the Hamiltonian respectively.

$$\Omega_{R1}\, e^{-i\,\phi_1} = \wp_{ba}E_1 /\hbar; \; \Omega_{R2}\, e^{-i\,\phi_2} = \wp_{ca}E_2 /\hbar \quad (4)$$

$\Omega_{R1}\, e^{-i\,\phi_1}$ and $\Omega_{R2}\, e^{-i\,\phi_2}$ are the complex Rabi frequencies associated with the interaction of the electromagnetic fields E_1 and E_2 of frequencies w_{L1} and w_{L2} with the atomic transitions |a> - |b> and |a> - |c> , respectively. E_1 and E_2 are amplitudes of the fields E_1 and E_2, respectively. Matrix elements of the electric dipole moment corresponding to atomic transitions |a> - |b> and |a> - |c> are

$$\wp_{ba} = e<b|r|a> ; \; \wp_{ca} = e<c|r|a> \quad (5)$$

respectively.

The wave function of this three-level atomic system is :

$$|\psi (t)> = C_a(t)\, |a> + C_b(t)|b> + C_c(t)|c> . \quad (6)$$

To find the probability amplitudes $C_a(t)$, $C_b(t)$,$C_c(t)$ it is necessary to solve the Schrodinger equation:

$$i\hbar \mid \dot{\psi}(t) >= H \mid \psi(t) > \quad (7)$$

It can be done by transforming the time dependent Hamiltonian - H and wave function $\psi(t)$ to new basis-"new picture", where the Hamiltonian will be time independent. Solving the Schrodinger equation (7) in "new picture", the probability amplitudes for each atomic state can be found for an arbitrary choice of initial conditions $C_a(t=0)$, $C_b(t=0)$, $C_c(t=0)$.

To calculate the absorption probability , it is convenient to consider the initial state of the system for which the population is equally distributed with fixed phases between the two lower states |b> and |c>, i.e. atomic system in low states at t =0. Mathematically this statement can be

$$C_a(0) = 0; \quad C_b(0) = \frac{1}{\sqrt{2}}; \quad C_c(0) = \frac{e^{-i\psi}}{\sqrt{2}} \tag{8}$$

expressed as:

The solution of the Schrodinger equation for this set of initial conditions under assumption of resonance $w_{ab}= w_{L1}$ and $w_{ac}=w_{L2}$, to the lowest order in time gives the following result for the probability amplitude of the upper level:

$$C_a(t) \approx i\frac{t}{2\sqrt{2}}(\Omega_{R1}e^{-i\phi_1} + \Omega_{R2}e^{-i(\phi_2+\Psi)}) \tag{9}$$

In this equation the first and the second terms of the sum represent the probability amplitudes corresponding to transitions from |b> to |a> and from |c> to |a>, respectively. The absorption probability, in such a case, equals the square of the probability amplitude of the upper level and for $\Omega_{R1} = \Omega_{R2} = \Omega_R$ can be written in the following form:

$$P_{absorption}=|C_a(t)|^2 \approx t^2\Omega_R^2 \, [1+ \cos(\phi_1 - \phi_2 -\psi)] / 4 \tag{10}$$

It is easy to see from this expression that absorption is cancelled ($|C_a(t)|^2 = 0$) when $\phi_1 - \phi_2 -\psi = \pm \pi$. The atomic system will stay at low energy levels |c> and |b> at all times for these specific phase conditions. Since there are no transitions to higher energy levels this system will have no absorption.

Now consider if the system is capable of emission. Suppose that initially the population is in the upper state, i.e. $C_a(0) = 1$; $C_b(0) = 0$; $C_c(0) = 0$. The solution of the Schrodinger equation (7) for these new initial conditions assuming $(\Omega^2_{R1} + \Omega^2_{R2})^{1/2}\, t = \Omega\, t \ll 1$ gives approximately the following result:

$$C_b(t) \approx i\, \Omega^*_{R1}\; t/2\,; \qquad C_c(t) \approx i\, \Omega^*_{R2}\; t/2 \tag{11}$$

The emission probability is equal to the sum of the squared probability amplitudes associated with atomic states |b> and |c>:

$$P_{emission} = P_b + P_c = |C_b(t)|^2 + |C_c(t)|^2 = \Omega^2\, t^2 /4. \tag{12}$$

Comparing the probability of emission with the absorption probability one can notice that the emission probability is independent of the relative phase between atomic states |b> and |c>, because first the probability amplitudes are squared and only then summed. However, for the probability of absorption, first the probability amplitudes are summed and only then squared. That is why the absorption probability is mathematically dependent on the relative phase between the atomic transitions.

As one can recognize from the equation (12), the emission probability is always larger than zero for $t > 0$. Therefore, if the atomic system is prepared with phase conditions as discussed above, the net gain can be observed even when there is no population inversion in the atomic scheme.

III. LWI IN BARE STATES AND DRESSED STATES

The three-level Λ- atomic system is not the only configuration where lasing without inversion is possible. LWI was reported for many different configurations of three and four level atomic systems [3-10] and, for some of them, was proved experimentally [5,11-14]. However, in most of these laser schemes, although there is no population inversion in bare –state basis (i.e. the eigenstate basis of the isolated atomic system), there is a population inversion in the dressed states (i.e. the eigenstate bases of the coupled atom-field system) [7,19]. So the question of noninversion and inversion in these atomic systems depends on the selected state basis. Such a situation can give some skepticism about the reality of the pure noninversion amplification because true noninversion should be independent of the state bases.

It is interesting to find an atomic configuration where LWI exists in any state basis. Several such schemes were reported [16-18]. One of these systems was demonstrated by A. Imamoglu, J.E. Field and S.E. Harris [16]. It is shown in Fig.2a. This system can be considered as a three-level Λ atomic scheme. The state |3> is pumped incoherently from both the lower states |2> and |1>. In addition, a coherent field interacts with the transition |3>-|2>. The laser transition is |1> -|3>, on this transition the probe field is amplified.

To find the condition of the amplification without inversion for this system in any atomic state basis, it is possible to use a density matrix approach and solve the master equation

$$\frac{\partial\tilde{\rho}}{\partial t} = \frac{1}{i\hbar}[\tilde{H},\tilde{\rho}] + L_{relax}[\tilde{\rho}] \qquad (13)$$

and hence, obtain the equation of motion for atomic density matrix elements in a frame rotating at the probe frequency. Then considering the steady state solution of these equations it is straightforward to find stimulated absorption and emission rates. Analyzing these rates, assuming resonance for coupling and probe fields(i.e. $\Delta w_{21}= w_2-w_c-w_p-w_1 = \Delta w_{31}= w_3-w_p-w_1=0$), it is possible to derive the necessary and sufficient condition for amplification without population inversion in

$$\frac{\Gamma_{32}}{\gamma_{21}} > \frac{\Gamma_{31}}{R_{13}}\frac{\Omega_{23}^2 + R_{23}\gamma_{32}}{\Omega_{23}^2} > \frac{\Omega_{23}^2 + (\Gamma_{32} + R_{23})\gamma_{23}}{\Omega_{23}^2} \qquad (14)$$

any atomic state basis, this condition is

Where decay rates: γ_{21}=R_{23} +R_{13} ; γ_{32}=Γ_{23}+ 2R_{23} +R_{13} +Γ_{31} ;

Γ_{ij} -spontaneous emission rate from |i> to |j>; R_{ij} - pumping rate; Ω_{ij} – Rabi frequency.

The first inequality is the condition for the net gain, the second one is the requirement of no population inversion. The system will satisfy the condition (14) if the spontaneous decay rate from state |3> to state |2> exceeds that from state |3> to state |1>, i.e. $\Gamma_{32} > \Gamma_{31}$ and the average number of thermal photons per mode in the |1>-|3> transition exceeds that in the |2>-|3> transition, i.e. $R_{13}\Gamma_{32} > R_{23}\Gamma_{31}$.

By performing a dressed-state analysis [20], it is possible to show that in the presence of a strong coherent field on the transition |3>-|2>, the states |3> and |2> turn into the dressed states $|\psi_{\pm}>$. The bare state |1> remains unchanged , and is coupled to $|\psi_{\pm}>$ by the weak probe field. It can be shown [20] that amplification can take place even if the total population of states $|\psi_{+}>$ and $|\psi_{-}>$ is less than that of |1>. This gain arises not from the population inversion, but from the coherence between dressed states $|\psi_{+}>$ and $|\psi_{-}>$, which is induced by the coherent pump on the transition |2>-|3>.

LWI in this and other atomic systems, which was independent of state bases, was also demonstrated experimentally [5,14].

I would like to point out that even atomic schemes in which there is not inversion in the bare state basis, but there is in dressed states, are important and interesting because these systems use coherence and interference effects of atomic transitions to provide gain in an active medium. This fact makes them principally different from conventional laser schemes, which do not utilize these effects. Therefore these LWI configurations can potentially provide performance and characteristics which are not reachable by traditional laser schemes.

IV. EXPERIMENT

To show how LWI can be realized experimentally, I will concentrate on continuos wave amplification in a three-level Λ system [14], similar to the Imamoglu's concept [16] of the atomic scheme with amplification without inversion in any state basis, which was discussed in the previous section.

In this experiment, a sodium atomic beam with transitions within the D_1 line was used as the active medium. By using a weak probe laser, it first demonstrated complete transparency and then inversionless gain. Next, a laser cavity was installed and aligned. With the probe blocked, it was found that the laser started spontaneously from vacuum fluctuations.

The energy levels for Na are shown in fig.2b. The 2-1' transition was used for the driving field. The laser transition chosen was 1-1', since it had the slowest decay rate which was required by the gain condition (14), i.e. the radiative decay on drive transition(Γ_{32}) must be faster than on the lasing transition (Γ_{31}). The incoherent pumping field was on the same 1-1' transition.

The experimental setup is shown in Fig.3. To provide a coherent drive field, a CW ring dye laser was used with line width ~1MHz. A part of this beam passed through an acousto-optic modulator (AO) and was used as the probe beam. The AO provides modulation with frequency 1770 MHz, which was necessary to overlap the probe beam with intervals of time when the active medium had absorption cancellation. It is interesting to mention that the absence of absorption was temporal, due to destructive interference between the contribution of the two low levels, which have different frequencies: $w_{23} \neq w_{13}$. That is why it is required to modulate the probe beam with the frequency $(E_{13}\text{-}E_{23})/\hbar$.

The output of the second CW ring dye laser passed through another AO, which destroyed its phase coherence and produces an incoherent pump beam. The AO was driven by a white noise generator with a 40 MHz bandwidth. It was important because a coherent pump would produce additional atomic coherences that would lead to the population trapping and no excitation to the upper lasing level.

Fig.4a. gives the probe transmission through the Na atomic beam as a function of a frequency in the neighborhood of the 1-1' transition. Curve P is the transmission when only the probe laser is present. Curve D is the transmission in the presence of the strong drive laser. It can be seen that, at resonance nearly complete EIT is achieved. Curve D&I shows the gain in the presence of the incoherent pump. Fig.4b. shows a density matrix calculation for this laser scheme, which is in a good agreement with the experimental results. It is seen that the observed gain depends on the incoherent pump laser intensity, i.e. on the number of atoms pumped into an excited state. Two percent gain had been obtained, using an atomic beam producing over six percent absorption in the absence of the drive.

The data of Fig.4a. show, there was no population inversion because away from resonance, in the wings of the lines, the intensity of the probe beam in the absence of the drive field is larger than the intensity of the probe beam when there was a drive or drive plus an incoherent field.

Finally, I would like to mention one of the possible applications of LWI. It is known that it is an extremely difficult problem to develop lasers in Gamma- spectral range ($\lambda \leq 1$ A) although such sources of radiation would make a revolutionary impact in basic science and technology, medicine. In particular, due to the extremely short wave of coherent radiation, they would establish completely new records in all fields where high precision is required such as lithography, microscopy, tomography, holography and etc..

One of the major difficulties in realizing γ-ray laser is the fact that lifetime progressively decreases with increase of radiation frequency(lifetime scales as w^{-3}). Hence it becomes progressively difficult to provide a population inversion, because a more intense pump is required. One approach to realize a γ-ray laser can be LWI. Olga Kocharovskaya in her recent paper [15] showed the way in which LWI will be able to give a principle solution to this problem. The solution is based on the possibility of pumping active nuclei in a solid host. The possibility in turn appears due to cancellation of the resonance absorption and direct resonant pumping at the operating transition.

V. SUMMARY

The concept of lasing without inversion has been considered using the semiclassical treatment. It has been shown there are atomic schemes where LWI can occur in any state basis. The experimental realization of amplification without population inversion in one of these schemes was presented which clearly demonstrates that such atomic system really exists. In conclusion, an application of LWI for developing a laser in γ –spectral range was discussed.

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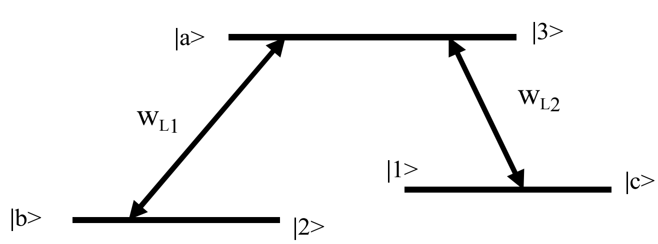
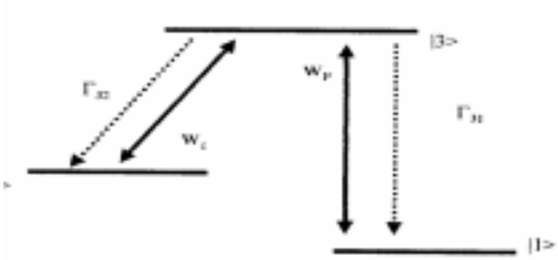
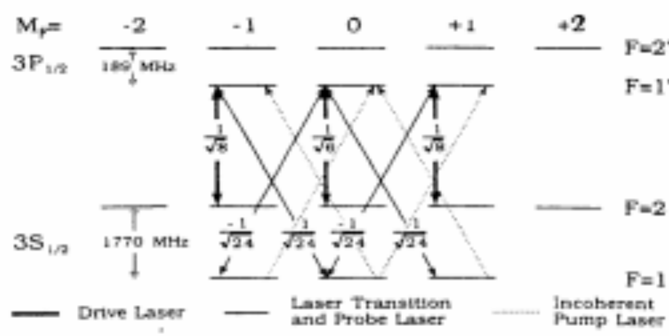


Fig.1.Three-level atom in the Λ- configuration interacting with two fields of frequencies w_{L1} and w_{L2}.





g.2a. Three level Λ atomic system for LWI any state bases.

Fig.2b.Relevant energy levels of Na. For comparison the levels |1>,|2>,|3> of the Λ scheme of the fig.2a. would each correspond to an appropriate level of F=1,F=2, and F=1', respectively.

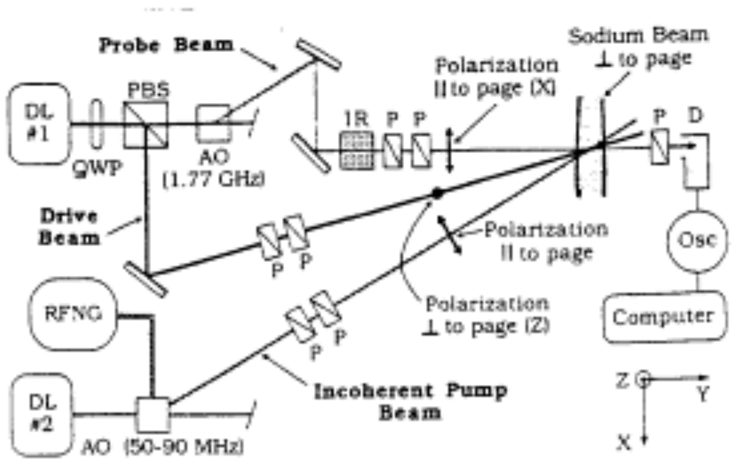


Fig.3. Schematic of the experiment to observe inversionless gain and cw laser oscillation. DL –dye laser, AO – acousto-optic modulator, RFNG – rf noise generator, D – detector.

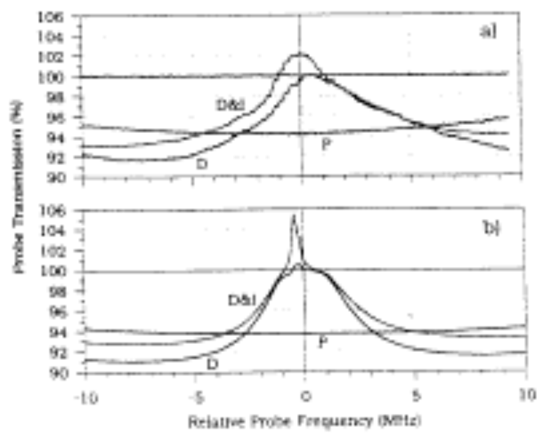


Fig.4. Frequency dependence of the probe laser transmission near the center of the 1-1' transition: a) experiment ; b) theory.

P-probe laser only; D-probe laser + drive laser; D&I –probe laser + drive laser + incoherent pump.