

# Physics 301

## Homework #2

Due Friday September 12.

1. The binomial distribution can be generalized to the case where you are interested two possible system states (We call it the trinomial distribution.). For this situation, assume we have probabilities  $p_1$  to be in state 1,  $p_2$  to be in state 2, and  $q$  to be in neither state ( $q = 1 - p_1 - p_2$ ). Find the probability,  $P_N(m, n)$ , that for  $N$  indistinguishable particles there are  $m$  particles in state 1 and  $n$  particles in state 2 (Notice that  $m = 0, 1, 2, \dots, N$ ,  $n = 0, 1, 2, \dots, N - m$ ).

Show that the random variables  $m$  and  $n$  are statistically independent.

A spin-3 particle has seven possible values for its  $\hat{z}$  spin component. ( $J_z = m\hbar$ ,  $m = -3, -2, -1, 0, 1, 2, 3$ ). In the absence of an external magnetic field, each component is equally likely. What is the probability that while measuring the  $\hat{z}$  component of spin for a sample of  $10^{23}$  spin-3 particles that  $1/4$  of them will have value  $-3\hbar$  and  $1/2$  of them will have value 0?

2. Using the Gaussian approximation, find the probability that in a gas with  $10^{23}$  particles that half of them are in the upper half of the container. Does this probability agree with the one calculated in class using the binomial distribution? Why is this probability so small? Is there a number of particles for which the probability is greater?
3. As shown in class, the binomial distribution,

$$P_N(m) = \frac{N!}{m!(N-m)!} p^m (1-p)^{N-m},$$

can, under the right conditions, be approximated by a Gaussian function. Under different conditions, it can also be approximated by a Poisson distribution. This situation is one in which the probability  $p$  becomes very small (practically zero) while the number  $N$  becomes large at the same rate, such that the product  $Np$  remains constant. To verify this fact, assume that  $p \ll 1$  and  $m \ll N$  (if  $N$  is very large then most values of  $m$  will be much less than  $N$ ) and show that

$$P_N(m) \approx e^{-\mu} \frac{\mu^m}{m!}$$

where  $\mu = Np$ .

**Hint:** Use the fact that  $\ln(1-p) \approx -p$  and the simple form of Stirling's approximation,  $\ln(m!) \approx m \ln(m) - m$ .

4. A classic example of the use of a Poisson distribution is in radioactive decay. For a sample of radioactive material, the probability of decay in a small time interval for any one atom is very tiny. The humongous number of atoms, however, ensures that the average rate of decay remains constant. Suppose that I give you 1 g of a radioactive material and a Geiger counter. Moreover, I inform you that there is an average of 5 decays/sec. What is the probability of measuring  $n$  decays in one second? Please explicitly calculate the probability for  $n$  ranging in value from 0 to 5.

What is the probability of measuring 6 or more decays in one second?

What is the probability of measuring  $n$  decays after 2 seconds? Find these probabilities for  $n = 0 \dots 10$ .