PHYSICS 301
HOMEWORK #4

Due Friday September 26.

1. (a) Two systems (call them system 1 and system 2) with total energy E are allowed to interact. What is the equilibrium energy of each system written in terms of the number of degrees of freedom and total energy? Assume that the these are classical systems with number of accessible states $\Omega_i = E_i^{(\frac{f_i}{2})}$ and that E_i can vary continuously between 0 and E.

   (b) In particular, show that if the two systems have an equal number of degrees of freedom that in equilibrium they will equally share the total energy, $E_1 = E_2 = E/2$.

2. Consider a system of N spin-1/2 particles (two possible values of any spin component) in an uniform magnetic field B. By wisely setting the zero point, the energy in this case can be written as $E = -smB$ where m is the magnetic moment of the spin-1/2 particles (a constant) and s is the number of spins pointing along the direction of the magnetic field (call this direction "up").

   (a) What is the number of accessible states available to this system when $E = -smB$, *i.e.*, when s spins are pointing up?

   (b) Now imagine two systems, system 1 with N_1 particles, s_1 of which are pointing up and system 2 with N_2 particles, s_2 of which are pointing up. Assume the two systems interact by exchanging energy which causes the number of up and down spins in both to change. Conservation of energy ensures that the total number of spins pointing up in the joint system, $s = s_1 + s_2$, remains constant.

   In equilibrium, what is s_1 and s_2? Express your answer in terms of s, N_1 and N_2.

   HINT: Use the fact that in equilibrium the entropy of the joint system is a maximum and Stirling's approximation (which is valid when N_1 and N_2 are large).

   (c) Let's investigate the sharpness of the most probable configuration. In this case, the number of spins can change by unit increments only (If a spin in system one flips up there must be a corresponding spin flip down in system two and vice-versa.). If we say in equilibrium that $s_1 = \hat{s}_1, s_2 = \hat{s}_2$, how likely is the subset $s_1 = \hat{s}_1 + 1, s_2 = \hat{s}_2 - 1$?

   HINT: To simplify the calculation, write s as its percentage of the total number of spins $N = N_1 + N_2$, *i.e.*, $s = pN$, $0 \leq p \leq 1$.

(d) Is there a value for p which would maximize the probability to be in this non-equilibrium state (which would violate the second law)? What is this greatest probability?