Prob 31, Sec 2.10, Boas

n^{th} roots of a number z are, if $z = re^{i\theta}$,

$$z_m = \left(re^{i\theta + i2m\pi}\right)^{1/n} = z_0 \; e^{i\frac{2m\pi}{n}}, \quad m = 0, 1, 2, \cdots, (n-1)$$

with $z_0 = r^{1/n} e^{i\theta/n}$

$$\sum_{m=0}^{n-1} z_m = z_0 \sum_{m=0}^{n-1} e^{i\frac{2m}{n}\pi}$$

$$= z_0 \left(\frac{1 - e^{i\frac{2\pi}{n}\cdot n}}{1 - e^{i\frac{2\pi}{n}}} \right) = z_0 \left(\frac{1 - e^{i2\pi}}{1 - e^{i2\pi/n}} \right)$$

$e^{i2\pi} \to 1$

$$= 0 \text{ for all } n \geq 2$$

Prob 32, Sec 2.10, Boas

$$1^{1/3} = \{ e^{i2m\pi/3}, \quad m = 0, 1, 2 \}$$

$$= \{ 1, e^{i2\pi/3}, e^{i4\pi/3} \} = \{ 1, \omega, \omega^2 \},$$

where $\omega = e^{i2\pi/3}$ or $e^{i4\pi/3}$

$\omega^2 = e^{i4\pi/3}$ or $e^{i8\pi/3}$

$= e^{i4\pi/3}$ or $e^{i2\pi/3}$,

since $e^{i8\pi/3} = e^{i8\pi/3 - i2\pi}$

$= e^{i2\pi/3}$