1.3 Probs 17, 18, Sec 2.11, Boas

$$\int e^{(a+ib)x}\,dx = \frac{e^{(a+ib)x}}{(a+ib)}$$ from simple exponential f'n integration

Real part $\Rightarrow \int e^{ax}\,\mathrm{Re}(e^{ibx})\,dx = e^{ax}\,\mathrm{Re}\left(\frac{e^{ibx}}{a+ib}\right)$

ie, $\int e^{ax}\cos bx\,dx = e^{ax}\,\frac{1}{2}\left(\frac{e^{ibx}}{a+ib} + \frac{e^{-ibx}}{a-ib}\right)$

$[\mathrm{Re}(z) = (z+z^*)/2]$ $\quad = \frac{e^{ax}\left[(a-ib)e^{ibx} + (a+ib)e^{-ibx}\right]}{2(a^2+b^2)}$

$$= e^{ax}\left(\frac{a\cos bx + b\sin bx}{a^2+b^2}\right)$$

Similarly, imaginary part

$$\Rightarrow \int e^{ax} \sin bx \, dx = e^{ax} \, \mathrm{Im}\left(\frac{e^{ibx}}{a+ib}\right)$$

$$= e^{ax} \, \mathrm{Im}\left(\frac{(a-ib)e^{ibx}}{a^2+b^2}\right)$$

$$= \frac{e^{ax}}{a^2+b^2} \, \mathrm{Im}\left[(a-ib)(\cos bx + i \sin bx)\right]$$

$$= \frac{e^{ax}}{a^2+b^2}(a \sin bx - b \cos bx)$$