

1.4

$$\text{Let } \tanh^{-1} z = w$$

$$\Rightarrow \tanh w = z$$

$$\text{ie, } \frac{e^w - e^{-w}}{e^w + e^{-w}} = z$$

$$\Rightarrow \frac{e^w}{e^{-w}} = \frac{1+z}{1-z}$$

$$\text{ie, } e^{2w} = \frac{1+z}{1-z}$$

$$\text{Log } \Rightarrow 2w = \ln\left(\frac{1+z}{1-z}\right) \text{ ie, } w = \frac{1}{2} \ln\left(\frac{1+z}{1-z}\right)$$

$$= \frac{1}{2} \left(\text{Ln}\left(\frac{1+z}{1-z}\right) + i2m\pi \right),$$

$$\text{ie, } \boxed{\tanh^{-1} z = \frac{1}{2} \text{Ln}\left(\frac{1+z}{1-z}\right) + im\pi, \quad m=0, \pm 1, \dots}$$

Similarly, if $\coth^{-1} z = w$ then $\coth w = z$

$$\Rightarrow e^{2w} = \frac{z+1}{z-1} \text{ or, } w = \frac{1}{2} \ln\left(\frac{z+1}{z-1}\right)$$

$$\text{ie, } \coth^{-1} z = \frac{1}{2} \text{Ln}\left(\frac{z+1}{z-1}\right) + im\pi, \quad m=0, \pm 1, \pm 2, \dots$$