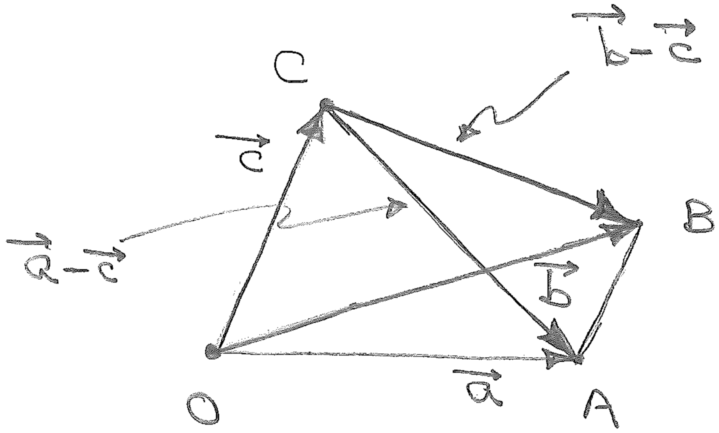
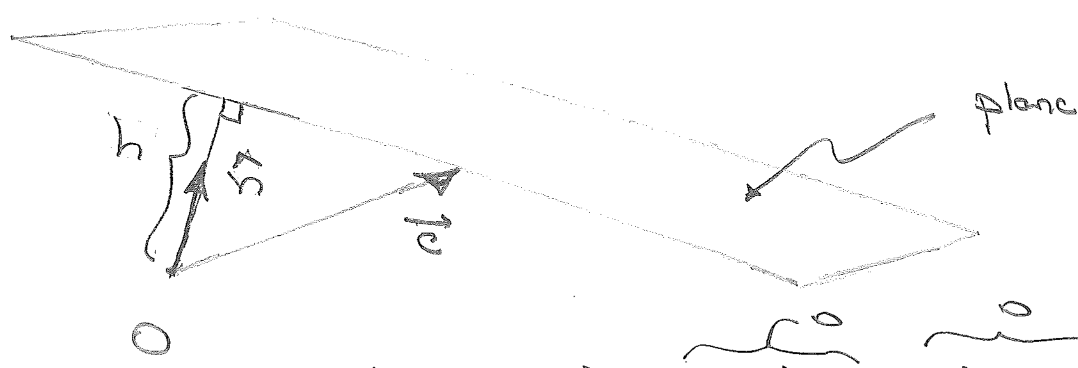
R.R



1 The vectors $\vec a - \vec c$ and $\vec b - \vec c$ span the plane defined by A, B, and C, so the unit vector $\perp$ to the plane is

$$\hat n = \frac{(\vec a - \vec c)\times(\vec b - \vec c)}{|(\vec a - \vec c)\times(\vec b - \vec c)|} = \frac{\vec a\times\vec b + \vec b\times\vec c + \vec c\times\vec a}{|\vec a\times\vec b + \vec b\times\vec c + \vec c\times\vec a|}.$$

Now we can draw the following picture



$$\text{So}\quad h = |\vec c\cdot\hat n| = \frac{|\vec c\cdot\vec a\times\vec b + \overbrace{\vec c\cdot\vec b\times\vec c}^{0} + \overbrace{\vec c\cdot\vec c\times\vec a}^{0}|}{|\qquad\qquad|}$$

this could be negative, so we take the absolute value.

$$\boxed{h = \frac{|\vec{c}\cdot\vec{b}\times\vec{a}|}{|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}}$$

Notice that we would get the same answer if we used $h = |\vec{a}\cdot\hat{n}|$ or $h = |\vec{b}\cdot\hat{n}|$.

2) The area of the triangle ABC is half the area of the parallelepiped spanned by $\vec{a}-\vec{c}$ and $\vec{b}-\vec{c}$, so

$$A = \frac{1}{2}\left|(\vec{a}-\vec{c})\times(\vec{b}-\vec{c})\right|$$

$$\boxed{A = \frac{1}{2}\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|}$$