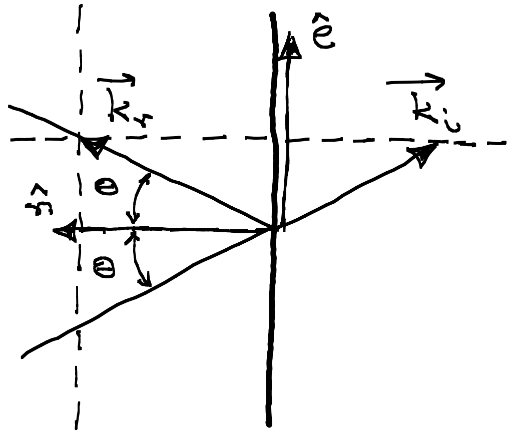
2.3



(a) $\vec{k}_r = k(\hat{n}\cos\theta + \hat{e}\sin\theta)$

$\vec{k}_i = k(-\hat{n}\cos\theta + \hat{e}\sin\theta)$

$$\Rightarrow \boxed{\vec{k}_r - \vec{k}_i = 2k\hat{n}\cos\theta}$$

(b) $k\cos\theta = \vec{k}_r \cdot \hat{n} = -\vec{k}_i \cdot \hat{n}$

$$\Rightarrow \boxed{\vec{k}_r = \vec{k}_i - 2\hat{n}(\hat{n}\cdot\vec{k}_i)}$$

Given $\hat{n}$ and $\vec{k}_i$, this formula gives us $\vec{k}_r$.

(c) Dotting (b) into $\hat{n}$ gives ① $\hat{n}\cdot\vec{k}_r = -\hat{n}\cdot\vec{k}_i$, which means that the component of $\vec{k}_r$ along $\hat{n}$ is equal and opposite to the component of $\vec{k}_i$. So this condition means that the tip of $\vec{k}_r$ is in the plane defined by the vertical dashed line above and extending orthogonally out of the page.

Crossing (b) into $\hat{n}$ gives ② $\hat{n}\times\vec{k}_r = \hat{n}\times\vec{k}_i$. This is a vector condition, which is actually two conditions. The component out of the page says $\vec{k}_r$ and $\vec{k}_i$ have the same component along $\hat{e}$. The zero component along $\vec{k}_r$ says that $\vec{k}_r$ lies in the plane spanned by $\hat{n}$ and $\vec{k}_i$, i.e., $0 = \vec{k}_r \cdot \hat{n}\times\vec{k}_r = \vec{k}_r\cdot\hat{n}\times\vec{k}_i$ ($\vec{k}_r$ is orthogonal to the direction orthogonal to $\hat{n}$ and $\vec{k}_i$ and so is in their plane).

So we should be able to derive (b) from ① and ②:

Cross $\hat{n}$ into ②: $\hat{n}\times(\hat{n}\times\vec{k}_r) = \hat{n}\times(\hat{n}\times\vec{k}_i)$

$$\hat{n}\underbrace{(\hat{n}\cdot\vec{k}_r)}_{① \; -\hat{n}\cdot\vec{k}_i} - \vec{k}_r \qquad \hat{n}(\hat{n}\cdot\vec{k}_i) - \vec{k}_i \leftarrow \text{BAC-CAB}$$

So $\vec{k}_r = \vec{k}_i - 2\hat{n}(\hat{n}\cdot\vec{k}_i)$