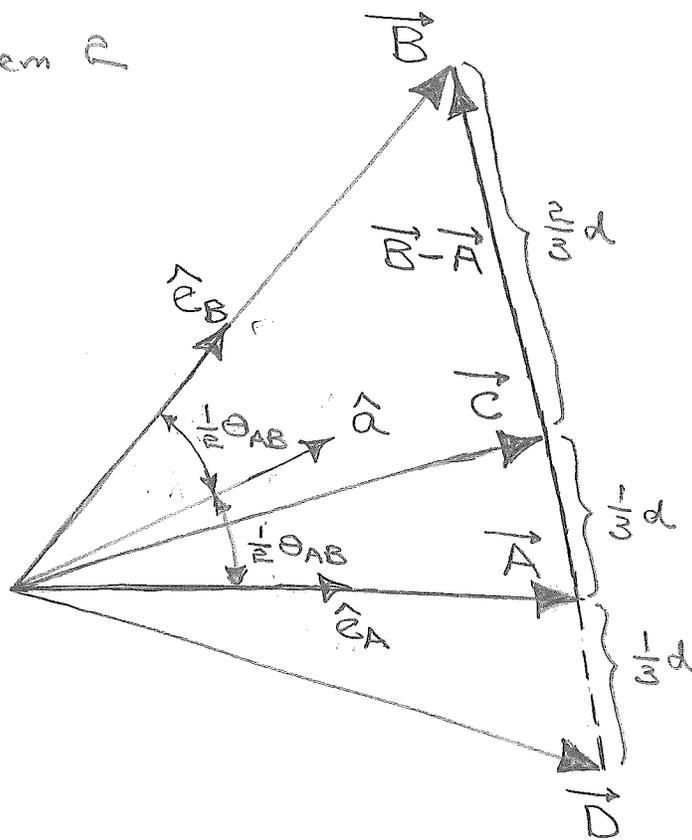Quiz #1, Problem 2

(a) $\vec{B} - \vec{A}$ is the vector that points from the tip of $\vec{A}$ to the tip of $\vec{B}$. Its length is

$$d = |\vec{B} - \vec{A}| = [(\vec{B} - \vec{A}) \cdot (\vec{B} - \vec{A})]^{1/2}$$

$$= (A^2 + B^2 - 2\vec{A} \cdot \vec{B})^{1/2}$$

$$\boxed{d = (A^2 + B^2 - 2AB\cos\theta_{AB})^{1/2}}$$

(b) $\vec{C} = \vec{A} + \frac{1}{3}(\vec{B} - \vec{A}) = \boxed{\frac{2}{3}\vec{A} + \frac{1}{3}\vec{B} = \vec{C}}$

$\vec{D} = \vec{A} - \frac{1}{3}(\vec{B} - \vec{A}) = \boxed{\frac{4}{3}\vec{A} - \frac{1}{3}\vec{B} = \vec{D}}$

(c) $\hat{e}_A = \dfrac{\vec{A}}{A}$, $\hat{e}_B = \dfrac{\vec{B}}{B}$ are unit vectors along $\vec{A}$ and $\vec{B}$.

$\hat{a}$ is an equal linear combination of $\hat{e}_A$ and $\hat{e}_B$, normalized to have unit length.

$$\hat{\rho} = \frac{\hat{e}_A + \hat{e}_B}{\left(|\hat{e}_A + \hat{e}_B|\right)} = \boxed{\frac{1}{\sqrt{2}\,(1+\cos\theta_{AB})^{1/2}}\left(\frac{\vec{A}}{A} + \frac{\vec{B}}{B}\right) = \hat{\rho}}$$

$$|\hat{e}_A + \hat{e}_B| = \left[(\hat{e}_A + \hat{e}_B)\cdot(\hat{e}_A + \hat{e}_B)\right]^{1/2}$$

$$= \left(\underbrace{\hat{e}_A\cdot\hat{e}_A}_{1} + \underbrace{\hat{e}_B\cdot\hat{e}_B}_{1} + \underbrace{2\,\hat{e}_A\cdot\hat{e}_B}_{\cos\theta_{AB}}\right)^{1/2}$$

$$= \sqrt{2}\left(1 + \cos\theta_{AB}\right)^{1/2}$$