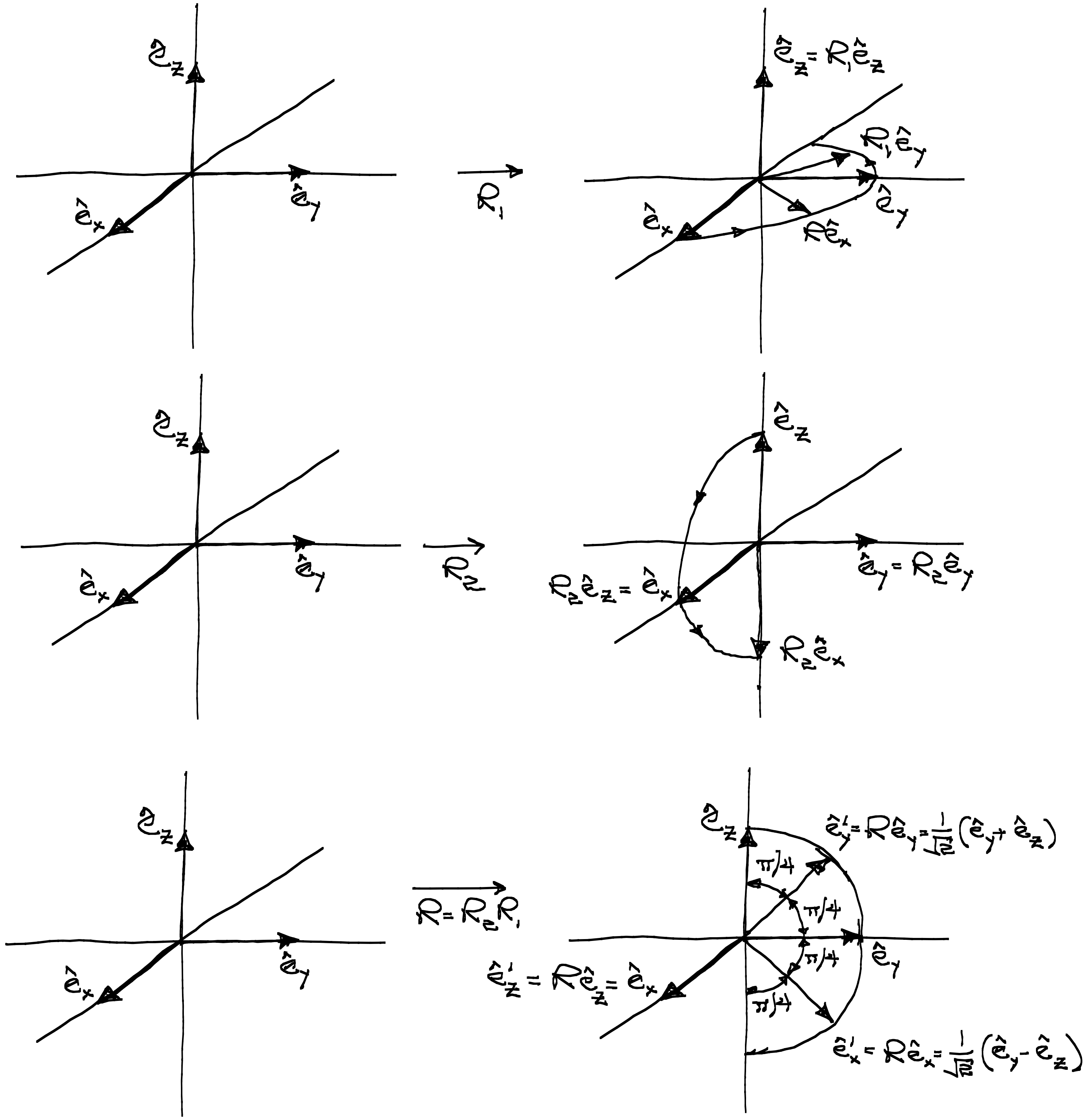
3.1



(a) $\hat{e}'_x = R\,\hat{e}_x = R_2 R_1\,\hat{e}_x = R_2 \frac{1}{\sqrt{2}}(\hat{e}_x + \hat{e}_y) = \frac{1}{\sqrt{2}}(-\hat{e}_z + \hat{e}_y)$

$\hat{e}'_y = R\,\hat{e}_y = R_2 R_1\,\hat{e}_y = R_2 \frac{1}{\sqrt{2}}(-\hat{e}_x + \hat{e}_y) = \frac{1}{\sqrt{2}}(\hat{e}_z + \hat{e}_y)$

$\hat{e}'_z = R\,\hat{e}_z = R_2 R_1\,\hat{e}_z = R_2\hat{e}_z \doteq \hat{e}_x$

(b) $\vec{A} = 2\hat{e}_x - \hat{e}_y$

$$\vec{A}' = R\vec{A} = 2R\hat{e}_x - R\hat{e}_y$$ notice that this is $2\hat{e}'_x - \hat{e}'_y$

$$= 2\frac{1}{\sqrt{2}}(-\hat{e}_z + \hat{e}_y) - \frac{1}{\sqrt{2}}(\hat{e}_z + \hat{e}_y)$$

$$= \underbrace{\left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)}_{\frac{1}{\sqrt{2}}}\hat{e}_y - \underbrace{\left(\sqrt{2} + \frac{1}{\sqrt{2}}\right)}_{\frac{3}{\sqrt{2}}}\hat{e}_y$$

$$\boxed{\vec{A}' = \frac{1}{\sqrt{2}}(\hat{e}_y - 3\hat{e}_z)}$$

(c) $R_{jk} = \hat{e}_j \cdot R\hat{e}_k$

$R_{xx} = \hat{e}_x \cdot R\hat{e}_x = 0 \qquad R_{xy} = \hat{e}_x \cdot R\hat{e}_y = 0 \qquad R_{xz} = \hat{e}_x \cdot R\hat{e}_z = 1$

$R_{yx} = \hat{e}_y \cdot R\hat{e}_x = \frac{1}{\sqrt{2}} \qquad R_{yy} = \hat{e}_y \cdot R\hat{e}_y = \frac{1}{\sqrt{2}} \qquad R_{yz} = \hat{e}_y \cdot R\hat{e}_z = 0$

$R_{zx} = \hat{e}_z \cdot R\hat{e}_x = -\frac{1}{\sqrt{2}} \qquad R_{zy} = \hat{e}_z \cdot R\hat{e}_y = \frac{1}{\sqrt{2}} \qquad R_{zz} = \hat{e}_y \cdot R\hat{e}_z = 0$

$$\boxed{\| R_{jk} \| = \begin{pmatrix} 0 & 0 & 1 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \end{pmatrix}}$$

Components of $\vec{A}'$ in $\hat{e}_j$ basis
$$A'_j = \hat{e}_j \cdot R\vec{A} = \sum_k \overbrace{\hat{e}_j \cdot R\hat{e}_k}^{R_{jk}} A_k = \sum_k R_{jk} A_k$$

$$\begin{pmatrix} A'_x \\ A'_y \\ A'_z \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \sqrt{2} - 1/\sqrt{2} \\ -\sqrt{2} - 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 1/\sqrt{2} \\ -3/\sqrt{2} \end{pmatrix}$$

M is the passive transformation matrix

(d)

Components of $\vec{A}$ in $\hat{e}'_j$ basis
$$A'_j = \hat{e}'_j \cdot \vec{A} = \sum_k R\hat{e}_j \cdot \hat{e}_k A_k = \sum_k \overbrace{\hat{e}_k \cdot R\hat{e}_j}^{R_{kj} = M_{jk}} A_k = \sum_k R_{kj} A_k$$

Same notation, but different components

$$\begin{pmatrix} A'_x \\ A'_y \\ A'_z \end{pmatrix} = \begin{pmatrix} 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \\ 2 \end{pmatrix}$$