[redacted] 3.2

Trajectory $\vec{r}(t) = \underset{\uparrow \atop x(t)}{b\hat{x}} + \underset{\uparrow \atop y(t)}{vt\,\hat{y}}$

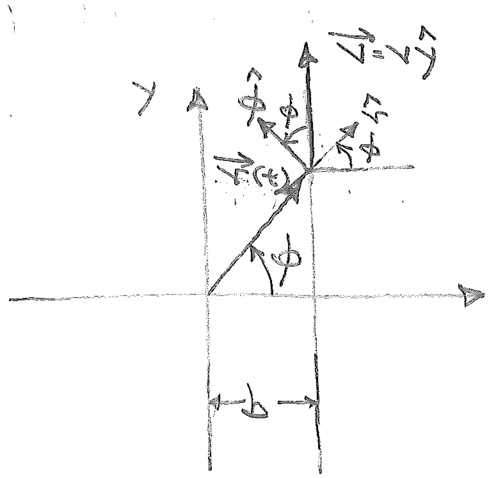
(a) $\vec{v}(t) = \dfrac{d\vec{r}}{dt} = \boxed{v\hat{y} = \vec{v}}$

Constant velocity

The magnitude of $\vec{v}$ is v, and the direction is the y-direction

No acceleration

(b) **Method 1:**



$\vec{v} = v\sin\phi\,\hat{r} + v\cos\phi\,\hat{\phi}$

$x(t) = b = r\cos\phi$

$y(t) = vt = r\sin\phi$

$r = \sqrt{x^2+y^2} = \sqrt{b^2+v^2t^2}$

$\tan\phi = \dfrac{y}{x} = \dfrac{vt}{b}$

$\cos\phi = \dfrac{x}{r} = \dfrac{b}{\sqrt{b^2+v^2t^2}}$

$\sin\phi = \dfrac{y}{r} = \dfrac{vt}{\sqrt{b^2+v^2t^2}}$

$$\boxed{\vec{v} = \frac{v^2t}{\sqrt{b^2+v^2t^2}}\,\hat{r} + \frac{vb}{\sqrt{b^2+v^2t^2}}\,\hat{\phi}}$$

2.) Method 2: Use the formula for $\vec{v}(t)$ in cylindrical coördinates

$$\vec{v} = \dot{r}\,\hat{r} + r\dot{\phi}\,\hat{\phi} + \underset{\overset{\|}{0}}{\dot{z}}\,\hat{z}$$

$$r = \sqrt{x^2+y^2} = \sqrt{b^2+v^2t^2} \Longrightarrow \dot{r} = \frac{1}{2}\frac{2v^2t}{\sqrt{b^2+v^2t^2}} = \frac{v^2t}{\sqrt{b^2+v^2t^2}}$$

$$\cos\phi = \frac{x}{r} = \frac{b}{\sqrt{b^2+v^2t^2}} \Longrightarrow -\dot{\phi}\sin\phi = -\frac{1}{2}\frac{b\,2v^2t}{(b^2+v^2t^2)^{3/2}}$$

$$\Longrightarrow \dot{\phi}\underbrace{\sin\phi}_{\frac{y}{r} = \frac{vt}{\sqrt{b^2+v^2t^2}}} = +\frac{bv^2t}{(b^2+v^2t^2)^{3/2}}$$

$$\Longrightarrow \dot{\phi} = \frac{bv}{b^2+v^2t^2}$$

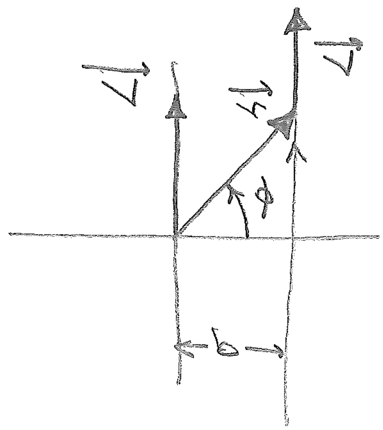
$$\Longrightarrow r\dot{\phi} = \frac{bv}{\sqrt{b^2+v^2t^2}}$$

Plugging these results into $\vec{v}$, we get

$$\boxed{\vec{v} = \frac{v^2t}{\sqrt{b^2+v^2t^2}}\,\hat{r} + \frac{bv}{\sqrt{b^2+v^2t^2}}\,\hat{\phi}}$$

(c) $\vec{L} = m\vec{r}\times\vec{v}$

Method 1: Abstract



$$\vec{L} = m\vec{r}\times\vec{v} = m\underset{r}{|\vec{r}|}\,\underset{v}{|\vec{v}|}\,\underbrace{\sin(\pi/2-\phi)}_{\cos\phi}\,\underset{\text{rh rule}}{\hat{z}}$$

$$= mv\underbrace{r\cos\phi}_{x=b}\,\hat{z}$$

$$\boxed{\vec{L} = mbv\,\hat{z}}$$

Method 2: Cartesian coördinates

$$\vec{L} = m\vec{r}\times\vec{v} = m(b\hat{x}+vt\hat{y})\times v\hat{y} = mbv\underbrace{\hat{x}\times\hat{y}}_{\hat{z}}$$

$$\boxed{\vec{L} = mbv\,\hat{z}}$$

Method 3: Cylindrical coördinates

$$\vec{L} = m\vec{r}\times\vec{v} = mr\hat{r}\times\left(\frac{v^2t}{r}\hat{r} + \frac{bv}{r}\hat{\phi}\right) = mbv\underbrace{\hat{r}\times\hat{\phi}}_{\hat{z}}$$

$$\boxed{\vec{L} = mbv\,\hat{z}}$$