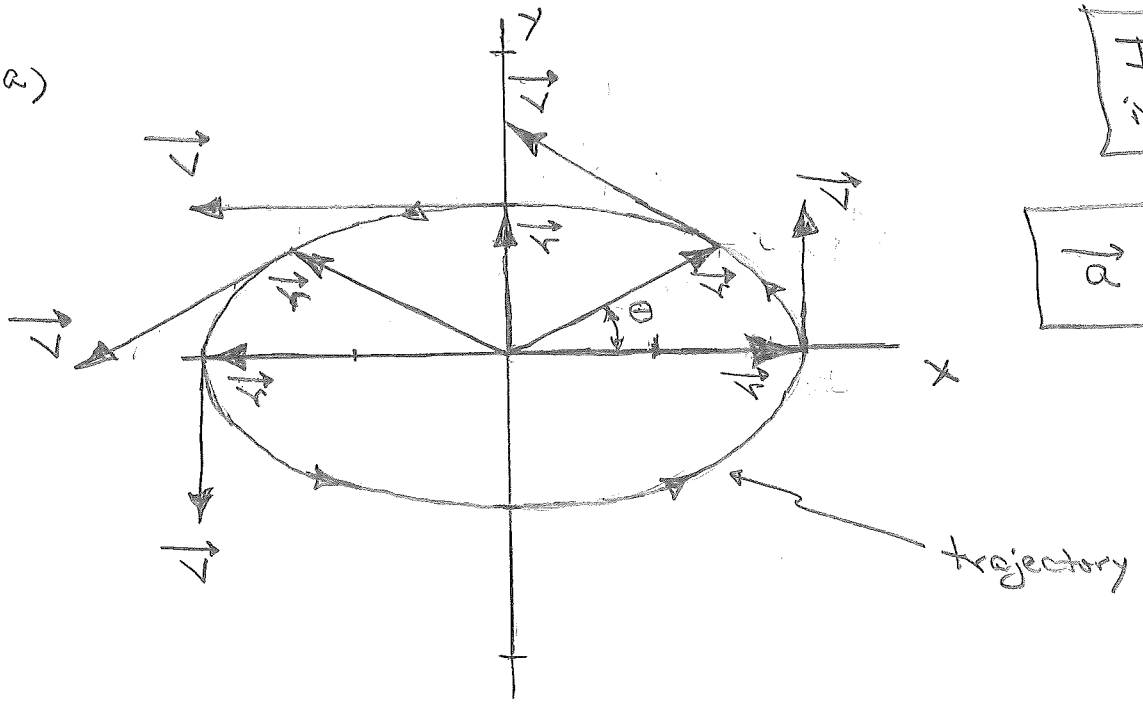
3.3

$\vec{r}(t) = 2b\cos\omega t\,\hat{x} + b\sin\omega t\,\hat{y} = b(2\cos\omega t\,\hat{x} + \sin\omega t\,\hat{y})$ (1)

(a)



I used $\omega = 1$ in drawing

$\vec{a}$ is opposite $\vec{r}$

$t = 0$: $\cos\omega t = 1$, $\sin\omega t = 0$

$\vec{r} = 2b\hat{x}$

$t = \pi/4\omega$: $\cos\omega t = 1/\sqrt{2}$, $\sin\omega t = 1/\sqrt{2}$

$\vec{r} = b(\sqrt{2}\hat{x} + \hat{y}/\sqrt{2}) = \sqrt{2}b(\hat{x} + \hat{y}/2)$

$t = \pi/2\omega$: $\cos\omega t = 0$, $\sin\omega t = 1$

$\vec{r} = b\hat{y}$

$\tan\theta = 1/2$

$\Rightarrow \theta = 0.46\ \text{rad} = 26.6^\circ$

$t = 3\pi/4\omega$: $\cos\omega t = -1/\sqrt{2}$, $\sin\omega t = 1/\sqrt{2}$

$\vec{r} = b(-\sqrt{2}\hat{x} + \hat{y}/\sqrt{2}) = \sqrt{2}b(-\hat{x} + \hat{y}/2)$

$t = \pi/\omega$: $\cos\omega t = -1$, $\sin\omega t = 0$

$\vec{r} = -2b\hat{x}$

(b) $\vec{v} = \dot{\vec{r}} = b\omega(-2\sin\omega t\,\hat{x} + \cos\omega t\,\hat{y})$

$(\text{speed}) = |\vec{v}| = b\omega(4\sin^2\omega t + \cos^2\omega t)^{1/2}$

$= b\omega(1 + 3\sin^2\omega t)^{1/2}$

$t = 0: \quad \vec{v} = b\omega\hat{y}$

$t = \pi/4\omega: \quad \vec{v} = b\omega(-\sqrt{2}\hat{x} + \hat{y})/\sqrt{2}) = \sqrt{2}\,b\omega(-\hat{x} + \hat{y})/2)$

$t = \pi/2\omega: \quad \vec{v} = -2b\omega\hat{x}$

$t = 3\pi/4\omega: \quad \vec{v} = b\omega(-\sqrt{2}\hat{x} - \hat{y})/\sqrt{2}) = -\sqrt{2}\,b\omega(\hat{x} + \hat{y})/2)$

$t = \pi/\omega: \quad \vec{v} = -b\omega\hat{y}$

(c) $\vec{a} = \dot{\vec{v}} = b\omega^2(-2\cos\omega t\,\hat{x} - \sin\omega t\,\hat{y}) = -\omega^2\vec{r}$

$\vec{a}$ is opposite $\vec{r}$

(d) $t = 0: \quad \pi/2 = 90^\circ$

$t = \pi/4\omega: \quad 2\theta = 0.93\ \text{rad} = 53.1^\circ$

$t = \pi/2\omega: \quad \pi/2 = 90^\circ$

$t = 3\pi/4\omega: \quad \pi - 2\theta = 2.2\ \text{rad} = 127^\circ$

$t = \pi/\omega: \quad \pi/2 = 90^\circ$