3.4 $$\epsilon_{mnp} \det M = \epsilon_{jkl} M_{mj} M_{nk} M_{pl} \qquad (2)$$

(a) Multiply (2) by ϵ_{mnp} and sum on repeated indices

$$\underbrace{\epsilon_{mnp}\epsilon_{mnp}} \det M = \epsilon_{mnp}\epsilon_{jkl} M_{mj} M_{nk} M_{pl}$$

This sum runs over all 27 values of mnp, but only $3! = 6$ are nonzero, and all those values are equal to 1, so the sum is $3! = 6$.

$$\Longrightarrow \det M = \frac{1}{3!} \epsilon_{mnp}\epsilon_{jkl} M_{mj} M_{nk} M_{pl}$$

In this formula the determinant is calculated 6 times, and then the six are summed, so we have to divide by 6.

(b) Multiply (2) by $(M^{-1})_{qp}$, and sum over p:

$$(M^{-1})_{qp}\, \epsilon_{mnp} \det M = \epsilon_{jkl} M_{mj} M_{nk} \underbrace{M_{pl} (M^{-1})_{qp}}_{(M^{-1}M)_{ql} = \delta_{lq}}$$

$$\epsilon_{mnp} (M^{-1})_{qp} = \frac{1}{\det M} \epsilon_{jkq} M_{mj} M_{nk}$$

Multiply by ϵ_{mnl}, and sum over repeated indices:

$$\underbrace{\epsilon_{mnl}\epsilon_{mnp}}_{2\delta_{lp}} (M^{-1})_{qp} = \frac{1}{\det M} \epsilon_{mnl}\epsilon_{jkq} M_{mj} M_{nk}$$

$$(M^{-1})_{ql} = \frac{1}{\det M} \frac{1}{2} \epsilon_{mnl}\, \epsilon_{jkq} M_{mj} M_{nk}$$

Relabel indices, $q \to j$, $l \to k$, $j \to l$, $k \to m$, $m \to n$, $n \to p$, and cyclically permute:

$$(M^{-1})_{jk} = \frac{1}{\det M}\,\frac{1}{2}\,\epsilon_{jlm}\,\epsilon_{knp}\,M_{nl}\,M_{pm}$$

(c) Check:

$$(M^{-1})_{jk}\,M_{kq} = \frac{1}{\det M}\,\frac{1}{2}\,\epsilon_{jlm}\underbrace{\epsilon_{knp}\,M_{kq}\,M_{nl}\,M_{pm}}_{\epsilon_{qlm}\det M \qquad \text{Eq. (2)}}$$

$$= \frac{1}{2}\,\epsilon_{jlm}\,\epsilon_{qlm}$$

$$= \delta_{jq} \quad \checkmark$$