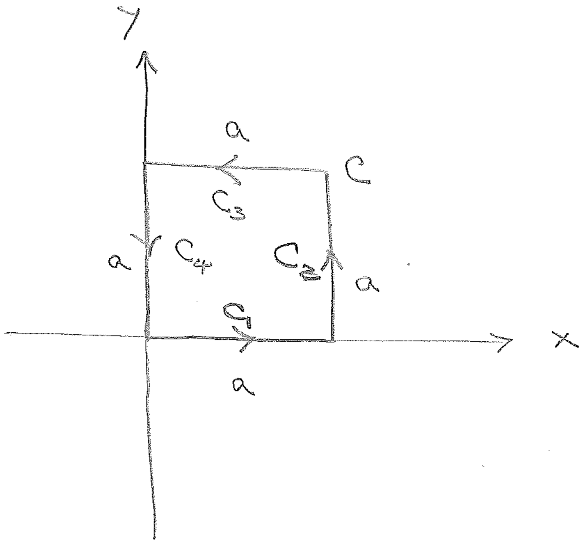
5.1. $\vec{A} = y^2\hat{e}_x - x^2\hat{e}_y$



(a)
$$\oint_C \vec{A}\cdot\vec{dl} = \underbrace{\int_{C_1}\vec{A}\cdot\underbrace{\vec{dl}}_{\hat{e}_x dx,\ (dx>0)}}_{\int_0^a \underbrace{A_x}_{=0} dx \,=\, 0} + \underbrace{\int_{C_2}\vec{A}\cdot\underbrace{\vec{dl}}_{\hat{e}_y dy\ (dy>0)}}_{\int_0^a \underbrace{A_y}_{-a^2} dy \,=\, -a^2\cdot a = -a^3} + \underbrace{\int_{C_3}\vec{A}\cdot\underbrace{\vec{dl}}_{-\hat{e}_x dx\ (dx>0)}}_{-\int_0^a \underbrace{A_x}_{a^2} dx \,=\, -a^2\cdot a = -a^3} + \underbrace{\int_{C_4}\vec{A}\cdot\underbrace{\vec{dl}}_{-\hat{e}_y dy,\ (dy>0)}}_{-\int_0^a \underbrace{A_y}_{0} dy \,=\, 0}$$

$$= 0 - a^3 - a^3 + 0$$

$$= -2a^3$$

(b)
$$\oint_C \vec{A}\cdot\vec{dl} = \int_S \underbrace{\nabla\times\vec{A}}\cdot\underbrace{\vec{da}}_{\hat{e}_z dx\,dy} = -2\int_0^a dx\int_0^a dy\,(x+y) = -2a^3$$

$$\hat{e}_z\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) = \hat{e}_z(-2x - 2y) = -2\hat{e}_z(x+y)$$