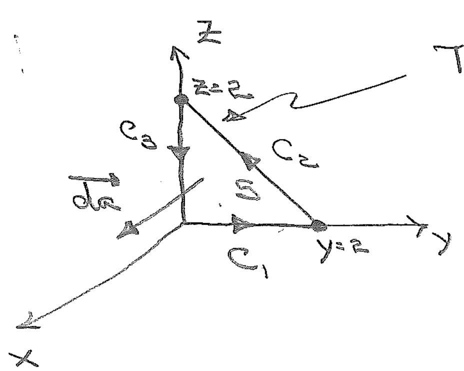
5.2.

$$\vec{v} = \underbrace{xy}_{v_x}\hat{x} + \underbrace{2yz}_{v_y}\hat{y} + \underbrace{3zx}_{v_z}\hat{z}$$

$$\nabla \times \vec{v} = \hat{x}\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) + \hat{y}\left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) + \hat{z}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)$$

$$= \hat{x}\Bigg(\underbrace{\frac{\partial(3zx)}{\partial y}}_{=0} - \underbrace{\frac{\partial(2yz)}{\partial z}}_{=2y}\Bigg) + \hat{y}\Bigg(\underbrace{\frac{\partial(xy)}{\partial z}}_{=0} - \underbrace{\frac{\partial(3zx)}{\partial x}}_{=3z}\Bigg) + \hat{z}\Bigg(\underbrace{\frac{\partial(2yz)}{\partial x}}_{=0} - \underbrace{\frac{\partial(xy)}{\partial y}}_{=x}\Bigg)$$

$$\nabla \times \vec{v} = -2y\hat{x} - 3z\hat{y} - x\hat{z}$$



This is the line $z = 2 - y$ and $x = 0$

First calculate

$$\int_S \nabla\times\vec{v}\cdot\vec{da} = \int \underbrace{\nabla\times\vec{v}\cdot\hat{x}}\,dy\,dz$$

$\uparrow$ $\hat{x}\,dy\,dz$ $\quad (\nabla\times\vec{v})_x = -2y$

If we'd been smart, we would have only calculated the x component of $\nabla\times\vec{v}$

We integrate y from y=0 to y=2, and for each value of y, we integrate z from z=0 to z=2-y.

$$\int_S \nabla\times\vec{v}\cdot\vec{da} = -2\int_0^2 y\,dy \underbrace{\int_0^{2-y} dz}_{2-y}$$

$$= -2\underbrace{\int_0^2 (2y-y^2)\,dy}$$

$$y^2\Big|_0^2 - \frac{1}{3}y^3\Big|_0^2 = 4 - \frac{1}{3}8 = \frac{4}{3}$$

$$\boxed{\int_S \nabla\times\vec{v}\cdot\vec{da} = -\frac{8}{3}}$$

Notice that we could have done this another way

$$\int_S \nabla\times\vec{v}\cdot\vec{da} = -2\int_0^2 dz \underbrace{\int_0^{2-z} y\,dy}$$

$$\frac{1}{2}y^2\Big|_0^{2-z} = \frac{1}{2}(2-z)^2$$

$$= -2 \int_0^2 dz\,(2-z)^2$$

$u = 2 - z$

$du = -dz$

$$= -\int_0^2 du\, u^2$$

$$\frac{1}{3}u^3\Big|_0^2 = \frac{8}{3}$$

$$\int_S \nabla \times \vec{v} \cdot \vec{da} = -\frac{8}{3}$$

Now let's do the line integral.

$$\oint_C \vec{v} \cdot \vec{dl} = \int_{C_1} \vec{v} \cdot \vec{dl} + \int_{C_2} \vec{v} \cdot \vec{dl} + \int_{C_3} \vec{v} \cdot \vec{dl}$$

For C_1: $\vec{dl} = dy\,\hat{y}$ $(dy > 0)$

$$= \int_0^2 v_y\, dy$$

$v_y = 2yz = 0$ because $z = 0$ along C_1

For C_3: $\vec{dl} = -dz\,\hat{z}$ $(dz > 0)$

$$-\int_0^2 v_z\, dz$$

$v_z = 3zx = 0$ because $x = 0$ along C_3

$$\therefore \oint_C \vec{v} \cdot \vec{dl} = \int_{C_2} \vec{v} \cdot \vec{dl}$$

$\vec{dl} = dy\,\hat{y} + dz\,\hat{z}$ $(dz > 0)$

$y = 2 - z$ along C_2

$dy = -dz$

$$= dz\,(-\hat{y} + \hat{z})$$

$$\int_{C_2} \vec{V} \cdot d\ell = \int_0^2 dz \left(-V_y + V_z \right)$$

V_z: $3zx = 0$ because $x = 0$ along C_2

V_y: $2yz = 2(2-z)z$ because $y = 2 - z$ along C_2

$$= -2\int_0^2 dz\,(2z - z^2)$$

$$= -2\left(z^2 \Big|_0^2 - \frac{1}{3} z^3 \Big|_0^2 \right)$$

$$= -2\left(4 - \frac{8}{3}\right)$$

$$\int_{C_2} \vec{V} \cdot \vec{d\ell} = -\frac{8}{3}$$

$$\therefore \boxed{\oint_C \vec{V} \cdot \vec{d\ell} = -\frac{8}{3}}$$

Checks