9.3. To do this problem, you just have to remember the conclusion of problem 9.2: *The Fourier transform of a function that is restricted to a domain of finite size can be reconstructed, using the sinc function, from the values of the Fourier transform at a series of uniformly spaced points.* Since a Fourier transform and its inverse look exactly the same, we can change the wording to the following: *A function whose Fourier transform is restricted to a domain of finite size can be reconstructed, using the sinc function, from the values of the function at a series of uniformly spaced points.* The rest is commentary.

Here we have a function of time whose temporal Fourier transform is restricted to the interval $[-\Omega, \Omega]$, so the function is given by

$$f(t) = \int_{-\Omega}^{\Omega} \frac{d\omega}{2\pi} \, \tilde{f}(\omega) e^{-i\omega t} \, .$$

with Fourier transform

$$\tilde{f}(\omega) = \int_{\infty}^{\infty} dt \, f(t) e^{i\omega t} \, .$$

In problem 9.2, we had a function $f(x)$ restricted to an interval $[-L/2, L/2]$; its Fourier transform is given by

$$\tilde{f}(k) = \int_{-L/2}^{L/2} dx \, f(x) e^{-ikx} \, ,$$

with the function given by

$$f(x) = \int_{-\infty}^{\infty} \frac{dk}{2\pi} \, \tilde{f}(k) e^{ikx} \, .$$

We showed that the Fourier transform could be constructed as

$$\tilde{f}(k) = \sum_{n=-\infty}^{\infty} \sqrt{L} c_n \frac{\sin[(k - k_n)L/2]}{(k - k_n)L/2} = \sum_{n=-\infty}^{\infty} \tilde{f}(k_n) \frac{\sin[(k - k_n)L/2]}{(k - k_n)L/2} \, .$$

All we have to is to realize that the new problem is the same as the old one with the roles of spatial domain and Fourier domain reversed and with the constants identified correctly:

$$x \to \omega \, , \quad k \to t \, , \quad L \to 2\Omega$$
$$\tilde{f}(k) \to f(t) \, , \qquad f(x) \to \tilde{f}(\omega)/2\pi \, ,$$
$$n \to j \, , \quad k_n = 2\pi n/L \to t_j = \pi j/\Omega \, .$$

With these identifications, we have

$$f(t) = \sum_{j=-\infty}^{\infty} f(t_j) \frac{\sin[\Omega(t - t_j)]}{\Omega(t - t_j)} \, .$$

This is the Nyquist sampling result for a band-limited function.