

9.4. This is easy. All we have to do is to plug the proposed Fourier transform into the formula for reconstructing $f_L(x)$ from its Fourier transform and show that it works:

$$\int_{-\infty}^{\infty} \frac{dk}{2\pi} \tilde{f}_L(k) e^{ikx} = \frac{1}{\sqrt{L}} \sum_{n=-\infty}^{\infty} c_n \underbrace{\int_{-\infty}^{\infty} dk \delta(k - k_n) e^{ikx}}_{= e^{ik_n x}} = \frac{1}{\sqrt{L}} \sum_{n=-\infty}^{\infty} c_n e^{ik_n x} = f_L(x),$$

So it works. The Fourier transform of a periodic function consists of δ contributions at the wave numbers corresponding to those in the Fourier series.

Simple though this is, there is a lesson relative to problem 9.2. The Fourier transform of a periodic function has δ -function peaks at the wave numbers corresponding to the Fourier series. The Fourier transform of the same function restricted to the central domain $[-L/2, L/2]$ can be reconstructed from the Fourier-series coefficients of the corresponding periodic function, but δ peaks in the Fourier transform of the periodic function are fuzzed out into sinc functions in the Fourier transform of the restricted function. The fuzzing out is necessary to get the destructive interference that destroys all the parts of the periodic function that are outside the central domain.