Phys 366 Mathematical Methods of Physics Fall 2016

Quiz 4 2016 November 4
(100 points)

Problem 1 (100 points) The inertia tensor

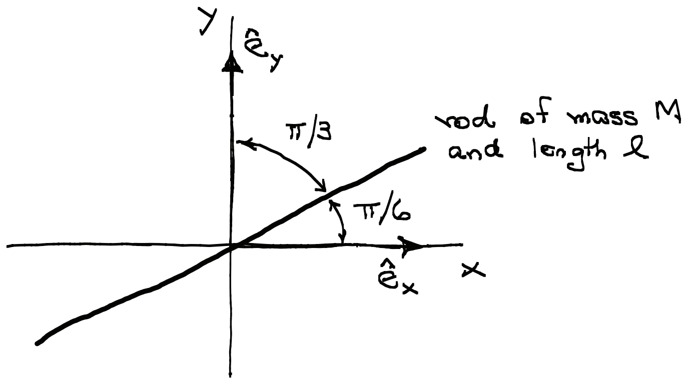
$$\mathbf{I} = \sum_{j,k} I_{jk} \hat{\mathbf{e}}_j \otimes \hat{\mathbf{e}}_k \ ,$$

has components given by

$$I_{jk} = \int dm \, (r^2 \delta_{jk} - x_j x_k) \ ,$$

where the integral is over the mass distribution.

This problem deals with a long, thin rod of length ℓ. As shown in the drawing, the rod lies in the x-y plane, its center is at the origin, and the rod makes an angle $\pi/6$ with the x axis. The mass in the rod is distributed uniformly along its length (i.e., uniform linear mass density), and the total mass of the rod is M.



(a) (35 points) *Find* the components I_{jk} of the inertia tensor.

(b) (30 points) *Give* the eigenvalues (principal moments of inertia) and (orthonormal) eigenvectors (principal axes) of the inertia tensor. [Hint: You can solve for these by diagonalizing the matrix of part (a), or you can guess (or argue from symmetry) the principal directions and verify your guess.]

(c) (35 points) *Find* the orthogonal matrix that transforms from the original Cartesian basis vectors $\hat{\mathbf{e}}_x$, $\hat{\mathbf{e}}_y$, $\hat{\mathbf{e}}_z$ to the basis vectors $\hat{\mathbf{e}}'_x$, $\hat{\mathbf{e}}'_y$, $\hat{\mathbf{e}}'_z$ that are the orthonormal eigenvectors (principal axes) of the inertia tensor. *Demonstrate* that the inertia tensor transforms from the components of part (a) to the diagonal form of part (b).