Quiz #2, Problem 1

$$r(t) = a \qquad\qquad x(t) = a\cos\phi(t) = a\cos[\pi(1-e^{-\gamma t})]$$

$$\phi(t) = \pi(1-e^{-\gamma t}) \Longrightarrow y(t) = a\sin\phi(t) = a\sin[\pi(1-e^{-\gamma t})]$$

(a)

$$\boxed{\vec{r}(t) = a\underbrace{(\cos\phi(t)\,\hat{x} + \sin\phi(t)\,\hat{y})}_{\hat{r}} = a\hat{r}}$$

(b)

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = a\dot{\phi}\,(-\sin\phi(t)\,\hat{x} + \cos\phi(t)\,\hat{y})$$

$$= \boxed{a\pi\gamma\, e^{-\gamma t}\underbrace{(-\sin\phi(t)\,\hat{x} + \cos\phi(t)\,\hat{y})}_{\hat{\phi}} = \vec{v}(t)}$$

$$\boxed{v(t) = a\pi\gamma\, e^{-\gamma t}}$$

$$\dot{\phi} = \pi\gamma e^{-\gamma t}$$

$$\ddot{\phi} = -\pi\gamma^2 e^{-\gamma t}$$

(c)

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = a\ddot{\phi}\,(-\sin\phi(t)\,\hat{x} + \cos\phi(t)\,\hat{y})$$

$$+ a\dot{\phi}^2\,(-\cos\phi(t)\,\hat{x} - \sin\phi(t)\,\hat{y})$$

$$\boxed{\begin{aligned}\vec{a}(t) &= -a\pi\gamma^2 e^{-\gamma t}\underbrace{(-\sin\phi(t)\,\hat{x} + \cos\phi(t)\,\hat{y})}_{\hat{\phi}} \\ &\quad - a\pi^2\gamma^2 e^{-2\gamma t}\underbrace{(\cos\phi(t)\,\hat{x} + \sin\phi(t)\,\hat{y})}_{\hat{r}}\end{aligned}}$$

Slows particle down → component opposite to $\vec{v}$

Curves path into a circle → component opposite to $\vec{r}$ and $\perp$ to $\vec{v}$

Doing this in cylindrical coördinates:

$$\vec{r}(t) = a\hat{r}$$

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = a\frac{d\hat{r}}{dt} = a\dot{\phi}\,\hat{\phi} = a\pi\gamma e^{-\gamma t}\,\hat{\phi}$$

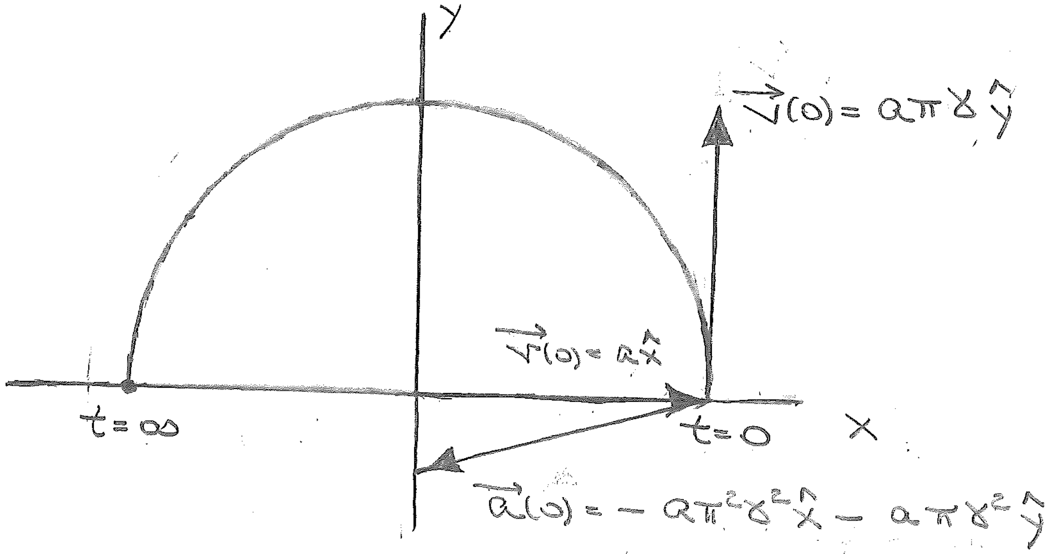
$$\vec{a}(t) = \frac{d\vec{v}}{dt} = a\ddot{\phi}\,\hat{\phi} + a\dot{\phi}\,\frac{d\hat{\phi}}{dt} = a\ddot{\phi}\,\hat{\phi} - a\dot{\phi}^2\,\hat{r}$$

$$\uparrow\ \dot{\phi}\hat{r}$$

$$= -a\pi\gamma^2 e^{-\gamma t}\,\hat{\phi} - a\pi^2\gamma^2 e^{-2\gamma t}\,\hat{r}$$

(a) $\phi(0) = 0, \quad \phi(\infty) = \pi, \quad 0 \le \phi(t) \le \pi$

The whole trajectory is a half circle



$$\frac{a_x(0)}{a_y(0)} = \pi$$

I drew this assuming $\gamma\pi = 1$.