Phys 521 Quantum Mechanics I **Fall 2011**

Homework Assignment #2 **Due Tuesday, September 13**
(50 points) **(at lecture)**

2.5 (10 points) Challenge problem. **The δ-function potential in momentum space.** Consider a particle moving in one spatial dimension, whose Hamiltonian is

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - \alpha\delta(x) \; ,$$

where α is a positive constant with dimensions of energy-length.

(a) *Write* the eigenvalue equation of H and the Fourier transform of this equation. *Solve* for the momentum-space eigenfunctions $\overline{\varphi}(p)$.

(b) Assuming that $E < 0$ (bound states), *calculate* $\varphi(x)$ by performing an inverse Fourier transform of $\overline{\varphi}(p)$, and *find* the one allowed energy eigenvalue.

(c) The average kinetic energy of the particle can be written as

$$E_k = \frac{1}{2m}\int_{-\infty}^{\infty} dp\, p^2 |\overline{\varphi}(p)|^2 \; .$$

Show formally that the average kinetic energy can also be written as

$$E_k = -\frac{\hbar^2}{2m}\int_{\infty}^{\infty} dx\, \varphi^*(x)\frac{d^2\varphi(x)}{dx^2} \; .$$

For the bound state found in part (b), *show* that the two formulas give the same result.

The following part is for extra credit only.

(d) (10 extra points) Assuming that $E > 0$ (continuum states), *find* the energy eigenfunction that describes a particle incident from the left by performing an inverse Fourier transform of $\overline{\varphi}(p)$. (Hint: This is hard and requires careful consideration of how to move the poles, which now lie on the real axis.)