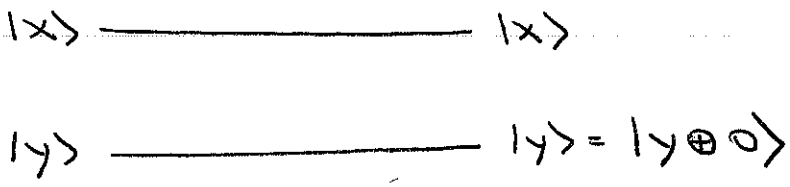
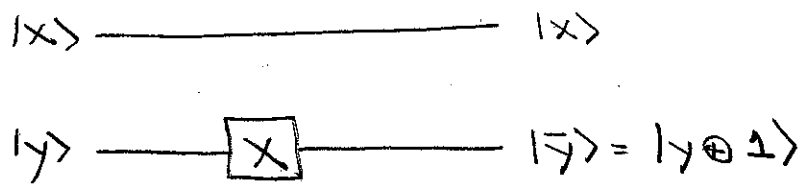
Solution 1.4

(a)

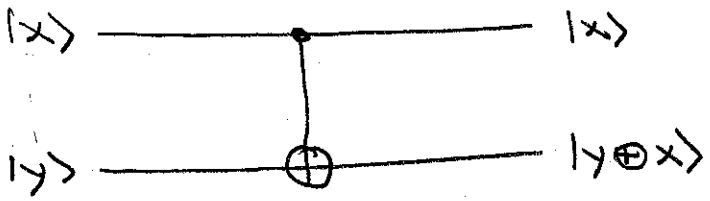
$f(x) = 0$



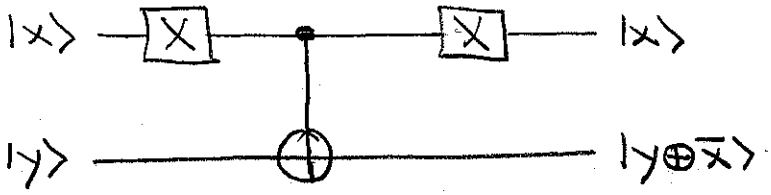
$f(x) = 1$



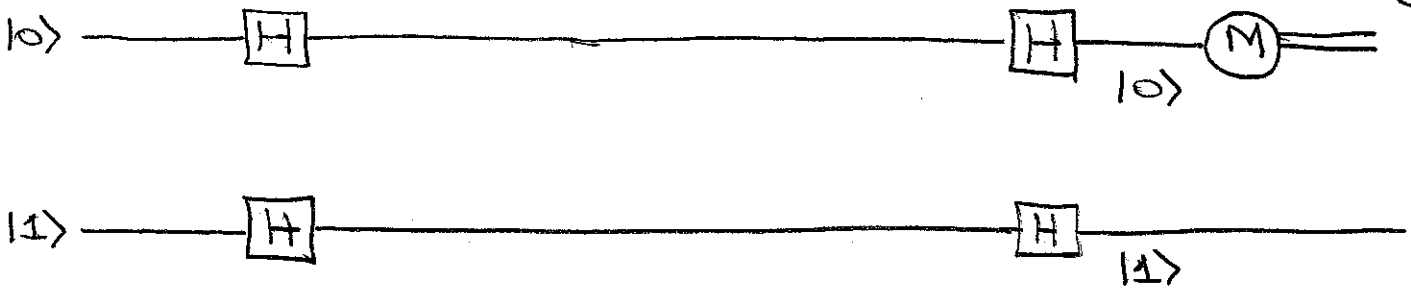
$f(x) = x$



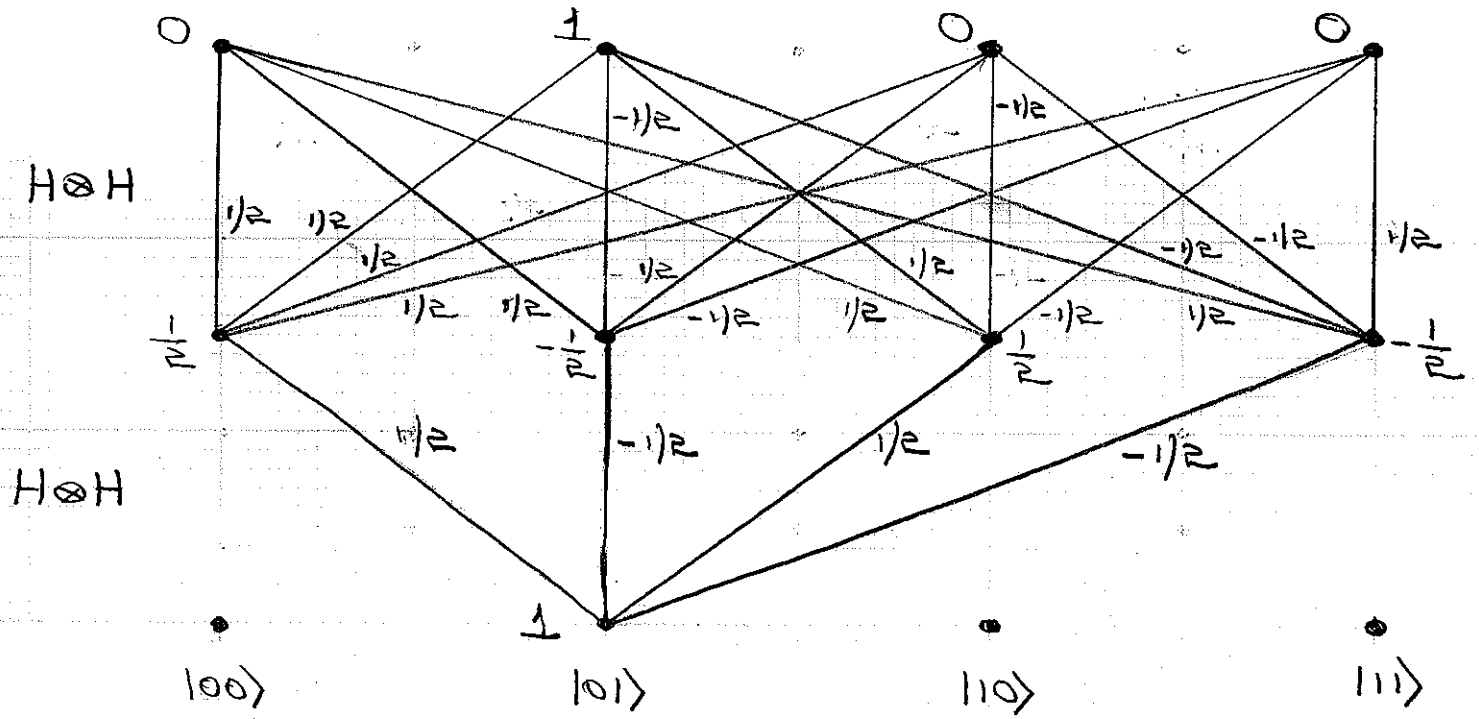
$f(x) = \bar{x} = 1 \oplus x$

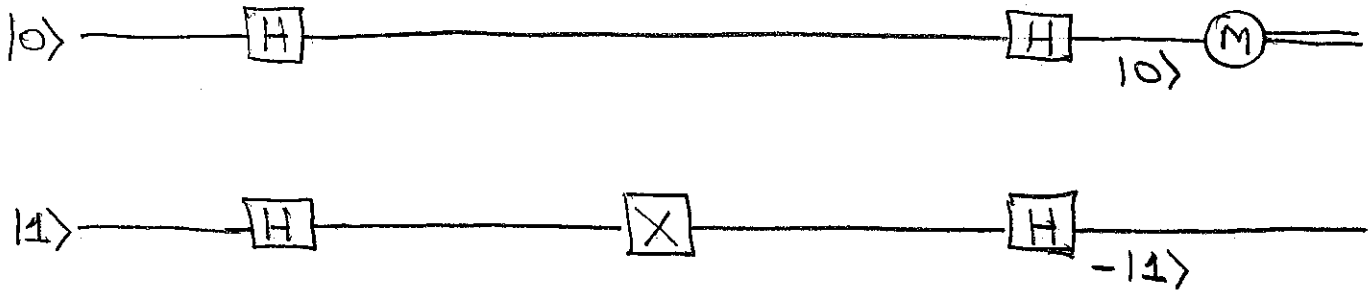


(b)

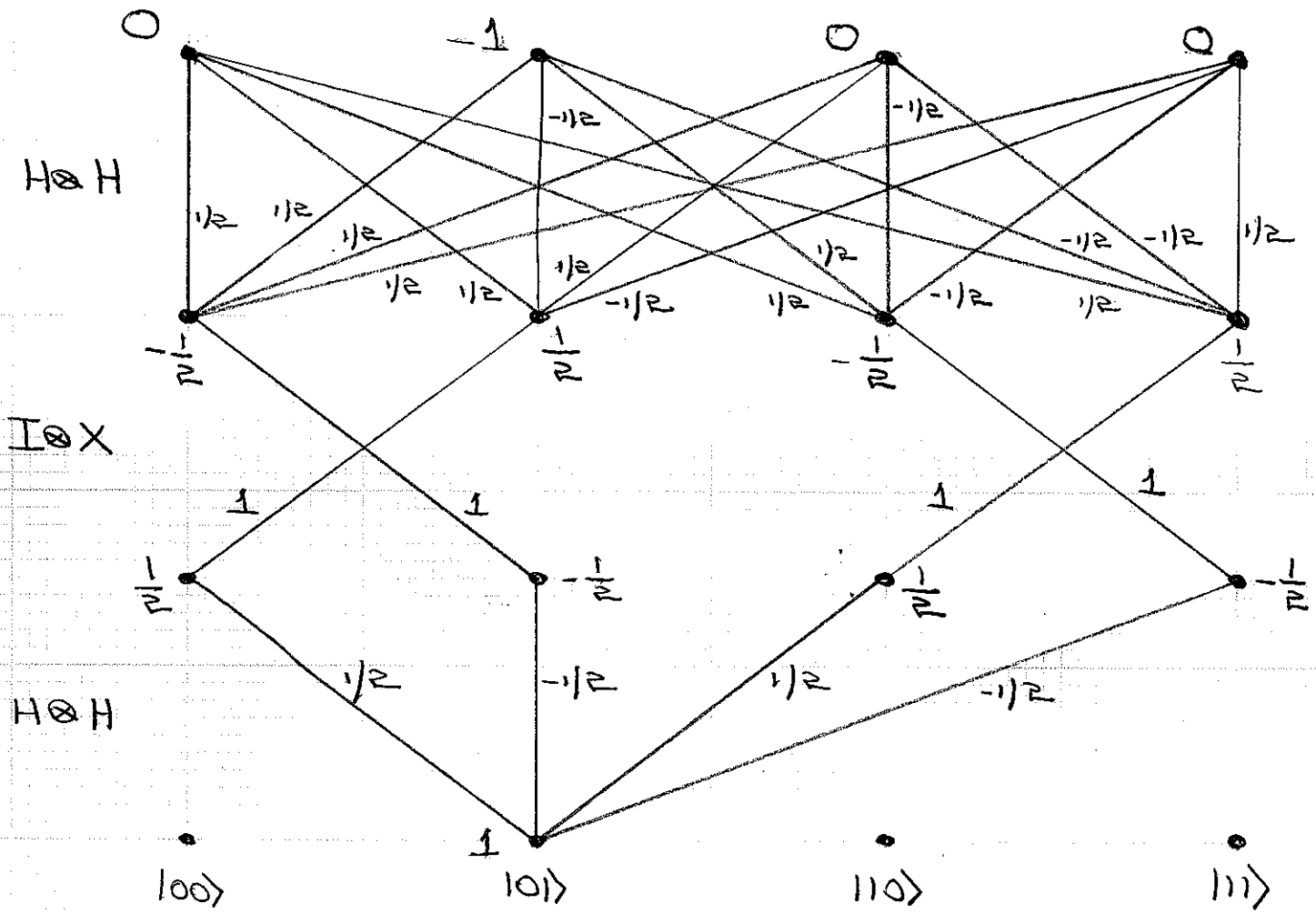

|0⟩
H
H
M
|0⟩
|1⟩
H
H
|1⟩

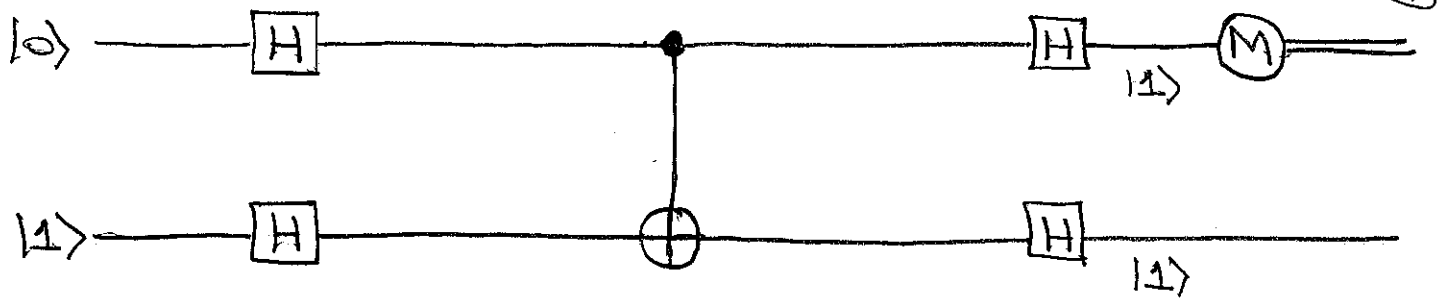
$f(x) = 0$


H⊗H
H⊗H
0
1
0
0
-1/2
-1/2
1/2
1/2
1/2
1/2
1/2
1/2
1/2
-1/2
-1/2
-1/2
-1/2
1/2
1/2
1/2
1/2
1/2
-1/2
-1/2
-1/2
1/2
-1/2
1/2
-1/2
1
|00⟩
|01⟩
|10⟩
|11⟩

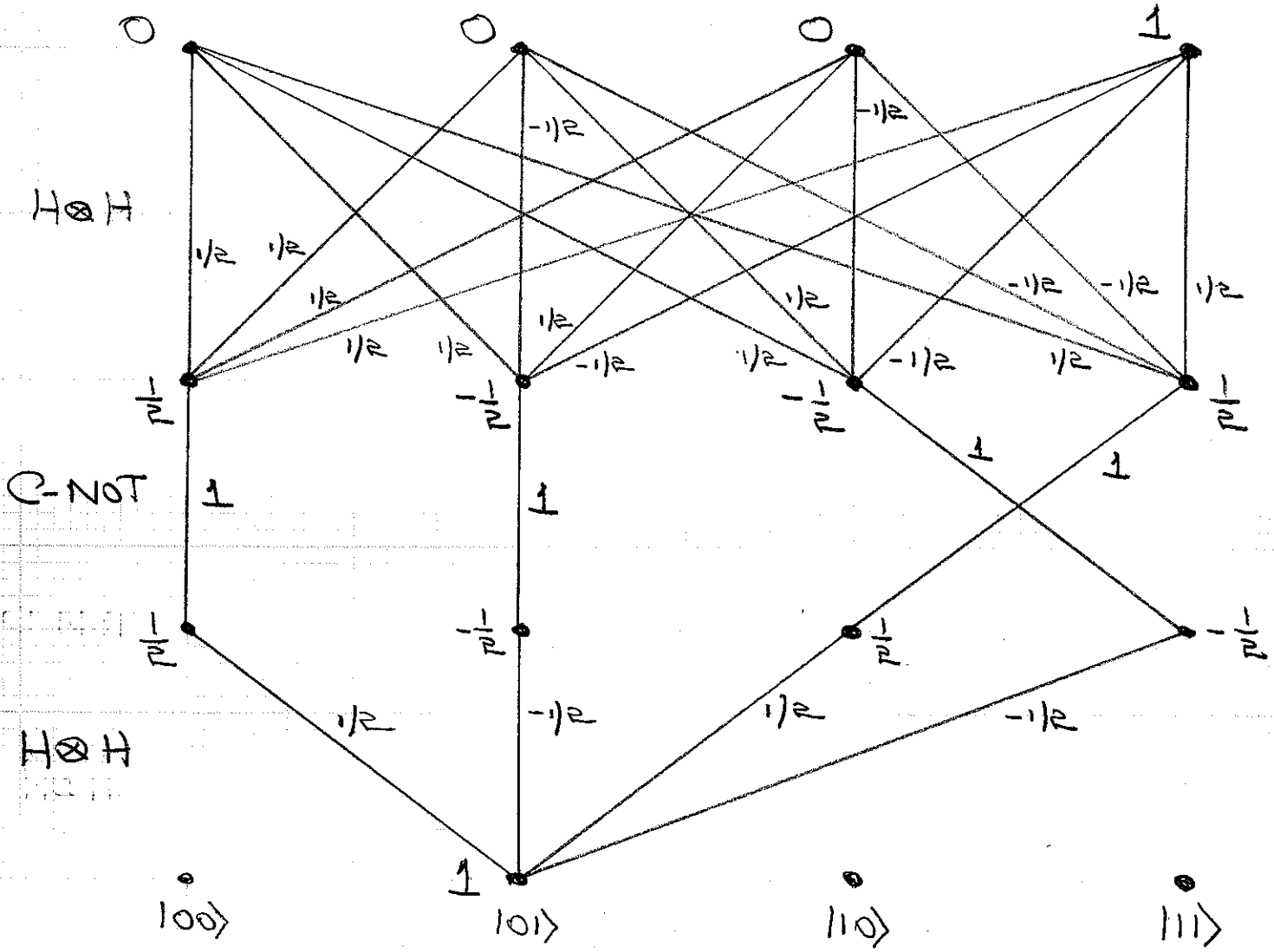


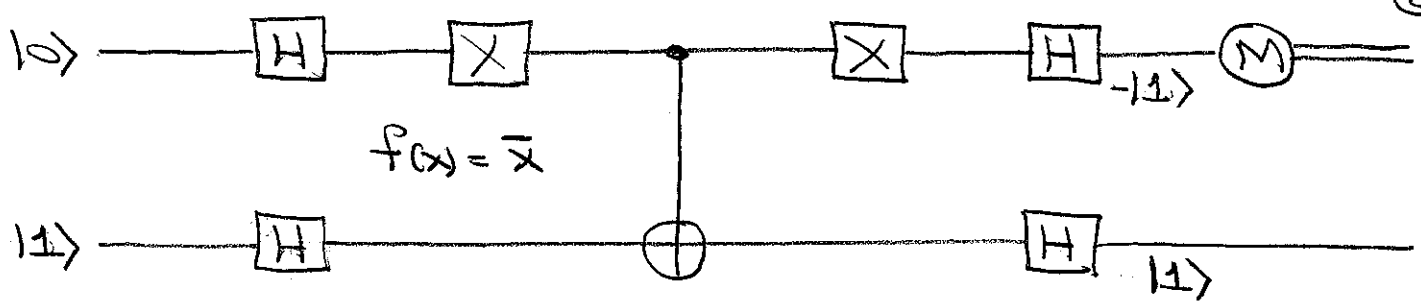
$f(x) = 1$

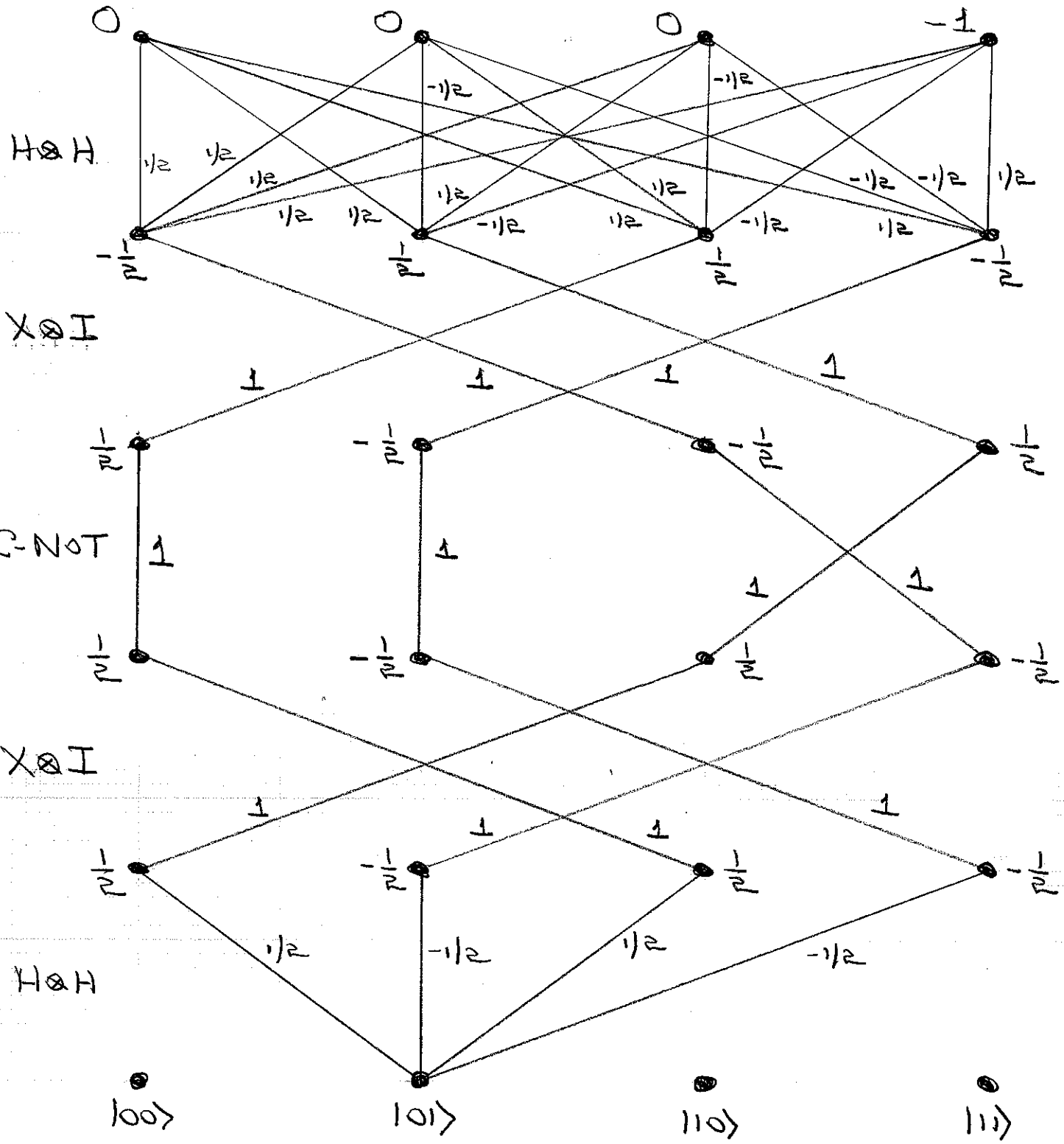




$f(x) = x$



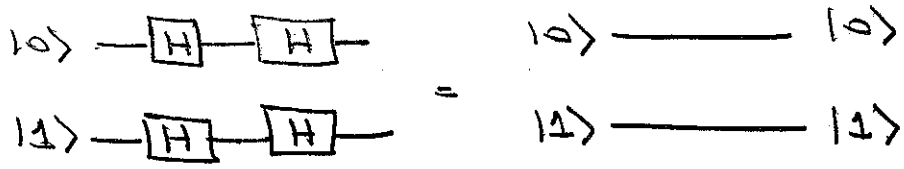

|0⟩
H
X
X
H
M
-|1⟩
f(x) = x̄
|1⟩
H
H
|1⟩


H⊗H
X⊗I
C-NOT
X⊗I
H⊗H
|00⟩
|01⟩
|10⟩
|11⟩

(c)

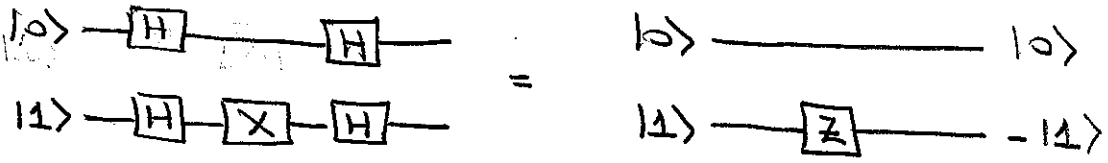
Use $H^2 = I$

$f(x) = 0$

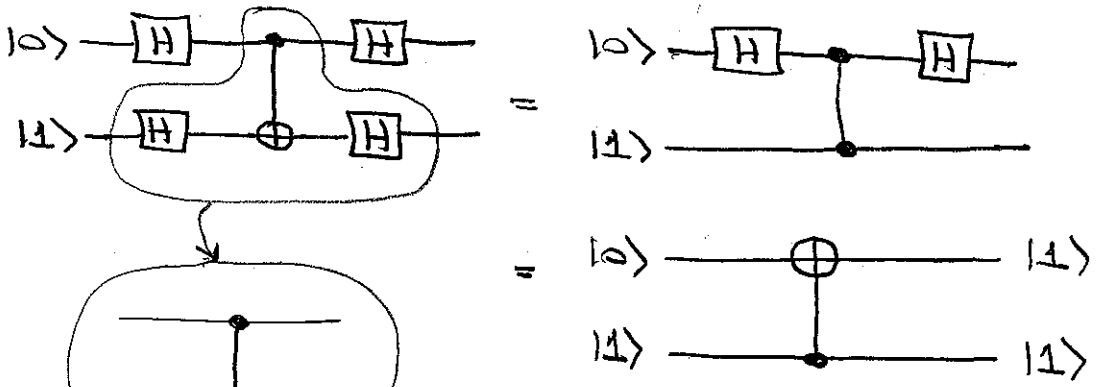


Use $H^2 = I$ and $HXH = Z$

$f(x) = 1$



$f(x) = x$



$f(x) = \bar{x}$