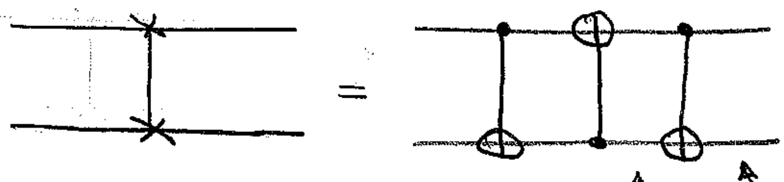
Solution 1.6.

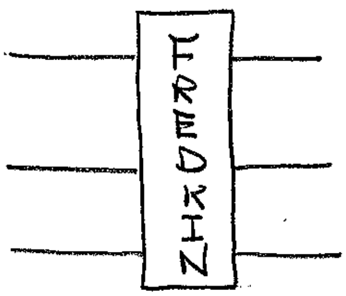
SWAP

(a)

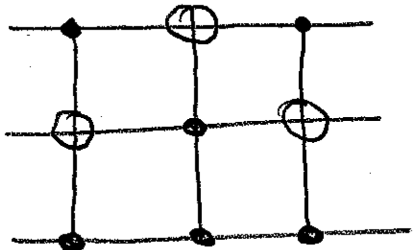


$|a,b\rangle$ $\quad$ $|a, b\oplus a\rangle$ $\quad$ $|a\oplus b\oplus a, b\oplus a\rangle = |b, b\oplus a\rangle$ $\quad$ $|b, b\oplus a\oplus b\rangle = |b,a\rangle$

$|a\rangle$, $|b\rangle$, $|c\rangle$



$|cb\oplus \bar{c}a\rangle$, $|ca\oplus \bar{c}b\rangle$, $|c\rangle$ =



$|a,b,c\rangle$

$|a, b\oplus ac, c\rangle$

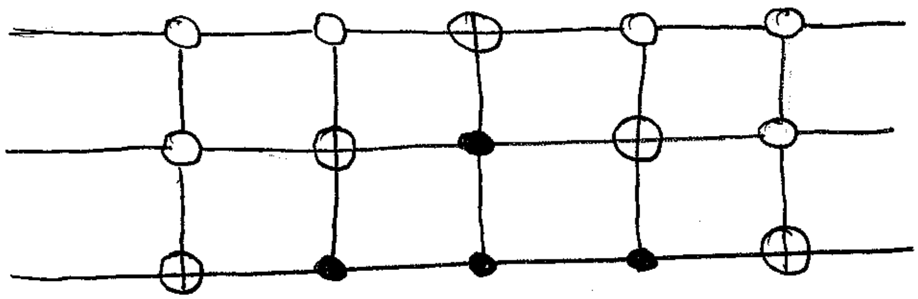
$$|a\oplus (b\oplus ac)c,\ b\oplus ac, c\rangle$$
$$= |a\oplus bc\oplus ac,\ b\oplus ac, c\rangle$$
$$= |\bar{c}a\oplus cb,\ b\oplus ac, c\rangle$$

$$|\bar{c}a\oplus cb,\ \underbrace{b\oplus ac + c(\bar{c}a\oplus cb)}_{b\oplus ca\oplus cb = \bar{c}b\oplus ca}, c\rangle$$

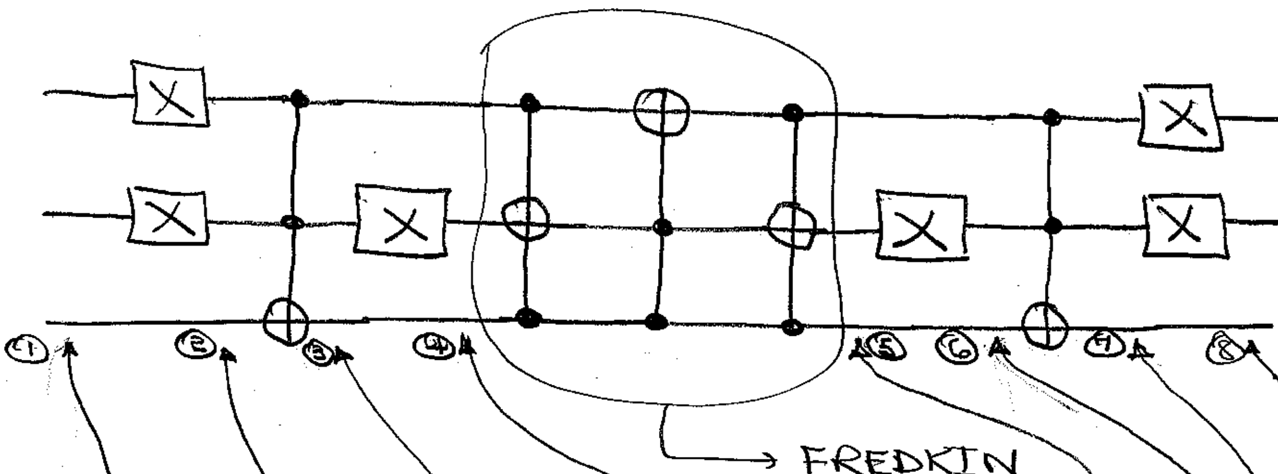
$$= |\bar{c}a\oplus cb,\ \bar{c}b\oplus ca, c\rangle$$

Circuit for 2-level transition $|000\rangle \longrightarrow |111\rangle$

$|111\rangle \longrightarrow |000\rangle$



(b) We convert a control-on-0 to a control-on-1 by putting an X gate on each side of the control. On the top line all these X's annihilate except the outer two, so the circuit becomes



(c) ① $|a,b,c\rangle$ ② $|\bar{a},\bar{b},c\rangle$

③ $|\bar{a},\bar{b},c+\bar{a}\bar{b}\rangle$

④ $|\bar{a},b,c+\bar{a}\bar{b}\rangle$

$$\Big|\underbrace{(c+\bar a\bar b)b+(\bar c+\bar a\bar b)\bar a}\,,\ \underbrace{(c+\bar a\bar b)\bar a+(\overline{c+\bar a\bar b})b}\,,\ \underbrace{c+\bar a\bar b}\Big\rangle$$

$$bc+\bar a\bar c+\bar a\bar b \qquad \bar a c+\bar a\bar b+b\bar c \qquad c+1+a+b+ab$$

$$= bc+\cancel{1}+\cancel{a}+c+ac+\cancel{1}+\cancel{a}+b+ab \qquad = c+ac+1+a+\cancel{b}+ab+\cancel{b}+bc \qquad = 1+a+b+c+ab$$

$$= b+c+g \qquad = 1+a+c+g$$

5

$$= \big|b+c+g,\ 1+a+c+g,\ 1+a+b+c+ab\big\rangle$$

6

$$\big|b+c+g,\ a+c+g,\ 1+a+b+c+ab\big\rangle$$

$$\big|b+c+g,\ a+c+g,\ 1+a+b+c+ab+\underbrace{(b+c+g)(a+c+g)}\big\rangle$$

$$(a+c)(b+c)+(a+b)g+g$$

$$= c+\cancel{g}+(a+b)g+\cancel{g}$$

$$= c+(a+b)g$$

$$= c+\cancel{ab}+ac+\cancel{abc}+\cancel{ab}+bc+\cancel{abc}$$

$$= c+ac+bc$$

$$1+a+b+g = a+b+\bar g$$

7

$$= \big|b+c+g,\ a+c+g,\ a+b+\bar g\big\rangle$$

8

$$\big|b\oplus c\oplus \bar g,\ a\oplus c\oplus \bar g,\ a\oplus b\oplus \bar g\big\rangle$$

It is easy to verify that the 3-bit function

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \longrightarrow \begin{pmatrix} b \oplus c \oplus \overline{\delta} \\ a \oplus c \oplus \overline{\delta} \\ a \oplus b \oplus \overline{\delta} \end{pmatrix}$$

exchanges 000 and 111 and leaves all other inputs unchanged.