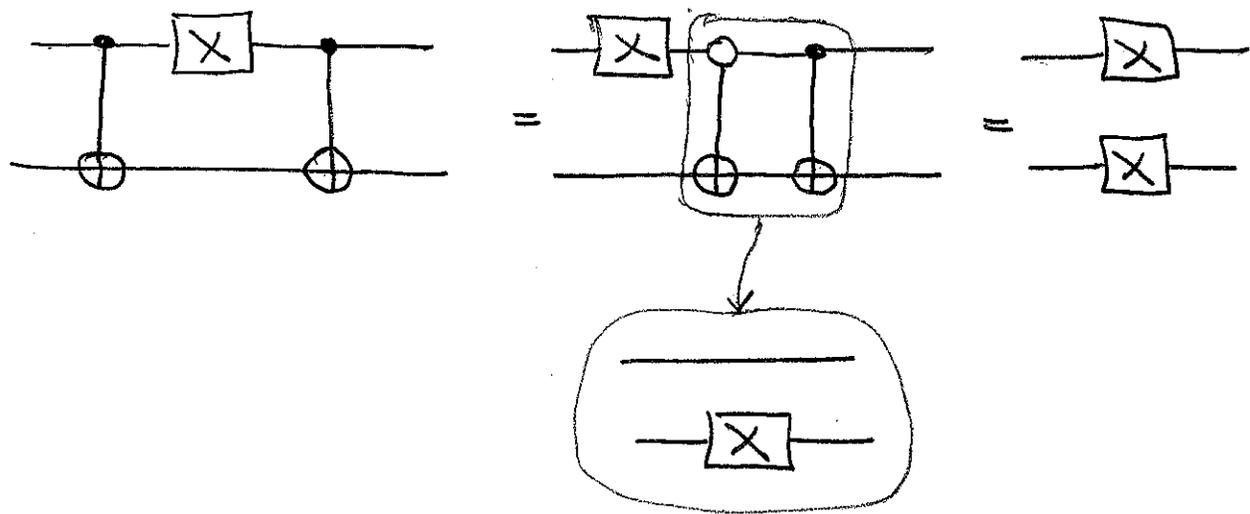
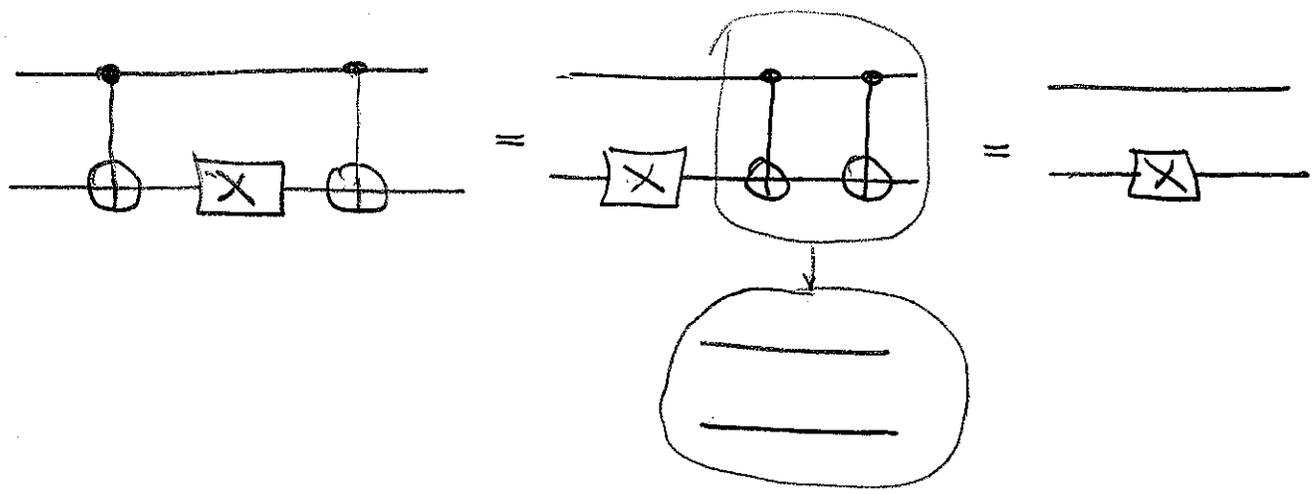


(a) C-C-NOT

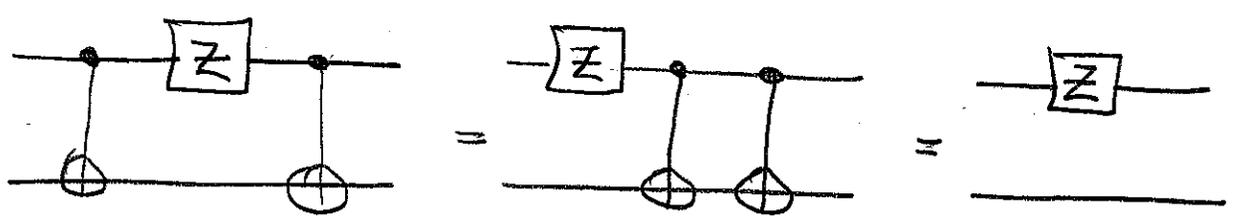
$$C(X \otimes I)C^\dagger = C(X \otimes I)C = X \otimes X$$



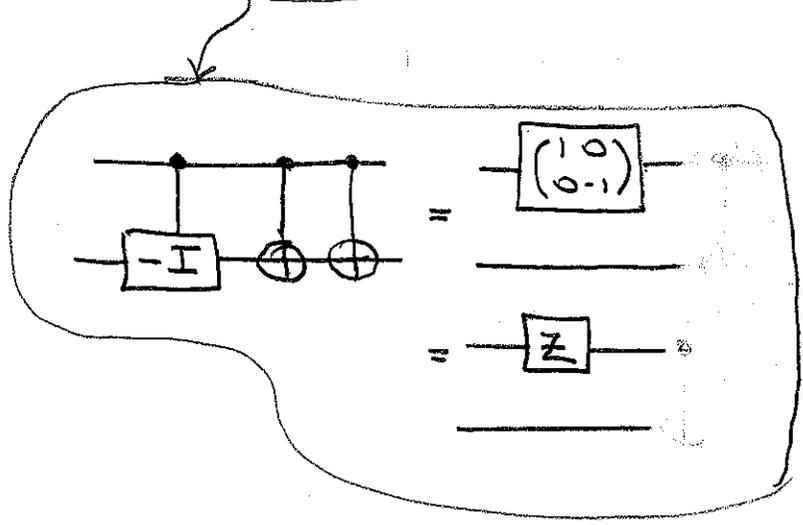
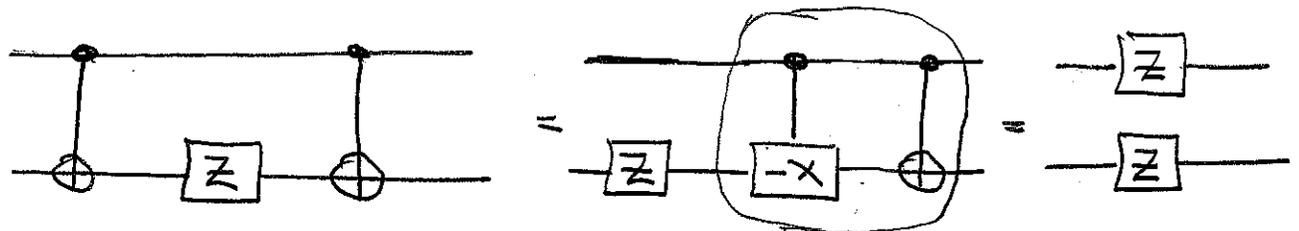
$$C(I \otimes X)C^\dagger = C(I \otimes X)C = I \otimes X$$



$$C(Z \otimes I)C^\dagger = C(Z \otimes I)C = Z \otimes I$$



$$C(I \otimes Z)C^\dagger = C(I \otimes Z)C = Z \otimes Z$$



$$C(X \otimes I)C^\dagger = X \otimes X$$

$$C(I \otimes X)C^\dagger = I \otimes X$$

$$C(Z \otimes I)C^\dagger = Z \otimes I$$

$$C(I \otimes Z)C^\dagger = Z \otimes Z$$

(b) The four Pauli products generate all the others, so we can get the rest by doing things like

$$\begin{aligned} C(X \otimes X)C^\dagger &= C(X \otimes I)C^\dagger C(I \otimes X)C^\dagger \\ &= (X \otimes X)(I \otimes X) \\ &= X \otimes I \end{aligned}$$

Of course, this one we can also get from

$$C(X \otimes I)C^\dagger \cdot X \otimes X \Rightarrow \underbrace{C^\dagger(X \otimes X)C}_{C(X \otimes X)C^\dagger} = X \otimes I$$

since  $C = C^\dagger$ .

The full list is

$$C(I \otimes I)C^\dagger = I \otimes I$$

$$C(I \otimes X)C^\dagger = I \otimes X$$

$$C(I \otimes Y)C^\dagger = Z \otimes Y$$

$$C(I \otimes Z)C^\dagger = Z \otimes Z$$

$$C(Z \otimes I)C^\dagger = Z \otimes I$$

$$C(Z \otimes X)C^\dagger = Z \otimes X$$

$$C(Z \otimes Y)C^\dagger = I \otimes Y$$

$$C(Z \otimes Z)C^\dagger = I \otimes Z$$

$$C(X \otimes I)C^\dagger = X \otimes X$$

$$C(X \otimes X)C^\dagger = X \otimes I$$

$$C(X \otimes Y)C^\dagger = Y \otimes Z$$

$$C(X \otimes Z)C^\dagger = -Y \otimes Y$$

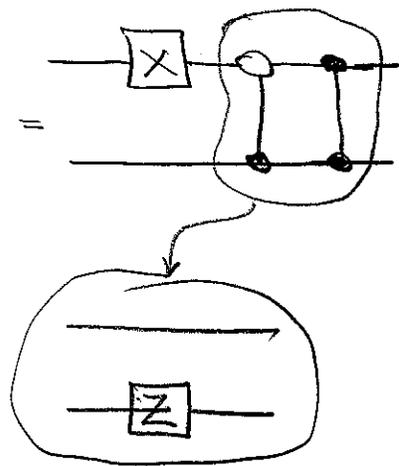
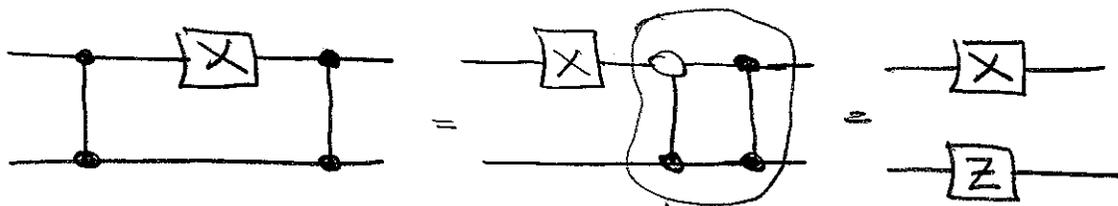
$$C(Y \otimes I)C^\dagger = Y \otimes X$$

$$C(Y \otimes X)C^\dagger = Y \otimes I$$

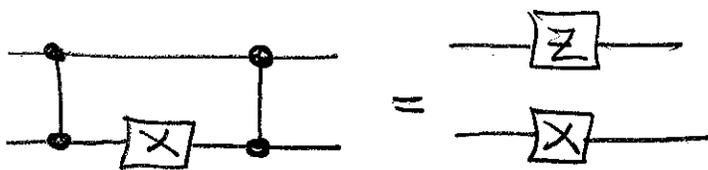
$$C(Y \otimes Y)C^\dagger = -X \otimes Z$$

$$C(Y \otimes Z)C^\dagger = X \otimes Y$$

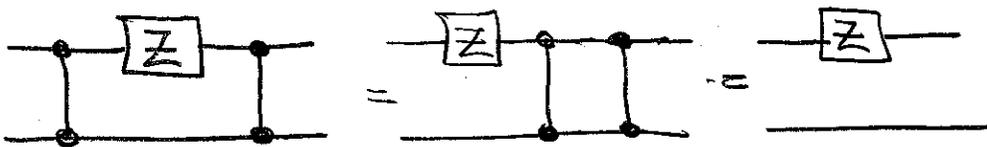
(c)  $C = C$ -SIGN



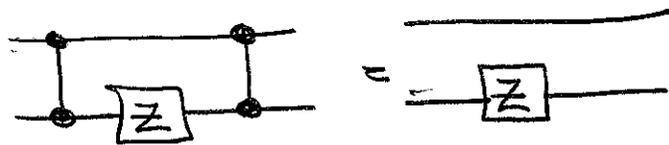
$$C(X \otimes I)C^\dagger = X \otimes Z$$



$$C(I \otimes X)C^\dagger = Z \otimes X$$



$$C(Z \otimes I)C^\dagger = Z \otimes I$$



$$C(I \otimes Z)C^\dagger = I \otimes Z$$

The full list is

$$C(I \otimes I) C^{\dagger} = I \otimes I$$

$$C(I \otimes X) C^{\dagger} = Z \otimes X$$

$$C(I \otimes Y) C^{\dagger} = Z \otimes Y$$

$$C(I \otimes Z) C^{\dagger} = I \otimes Z$$

$$C(Z \otimes I) C^{\dagger} = Z \otimes I$$

$$C(Z \otimes X) C^{\dagger} = I \otimes X$$

$$C(Z \otimes Y) C^{\dagger} = I \otimes Y$$

$$C(Z \otimes Z) C^{\dagger} = Z \otimes Z$$

$$C(X \otimes I) C^{\dagger} = X \otimes Z$$

$$C(X \otimes X) C^{\dagger} = Y \otimes Y$$

$$C(X \otimes Y) C^{\dagger} = -Y \otimes X$$

$$C(X \otimes Z) C^{\dagger} = X \otimes I$$

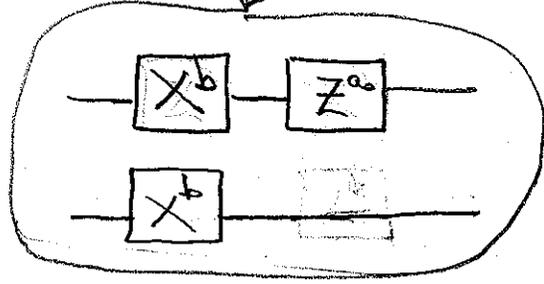
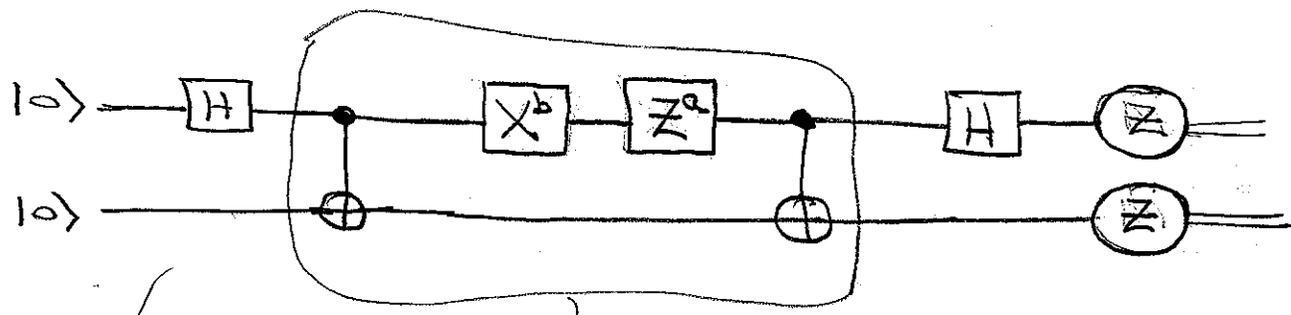
$$C(Y \otimes I) C^{\dagger} = Y \otimes Z$$

$$C(Y \otimes X) C^{\dagger} = -X \otimes Y$$

$$C(Y \otimes Y) C^{\dagger} = X \otimes X$$

$$C(Y \otimes Z) C^{\dagger} = Y \otimes I$$

(d) Superdense coding:



Using C-NOT transformations

