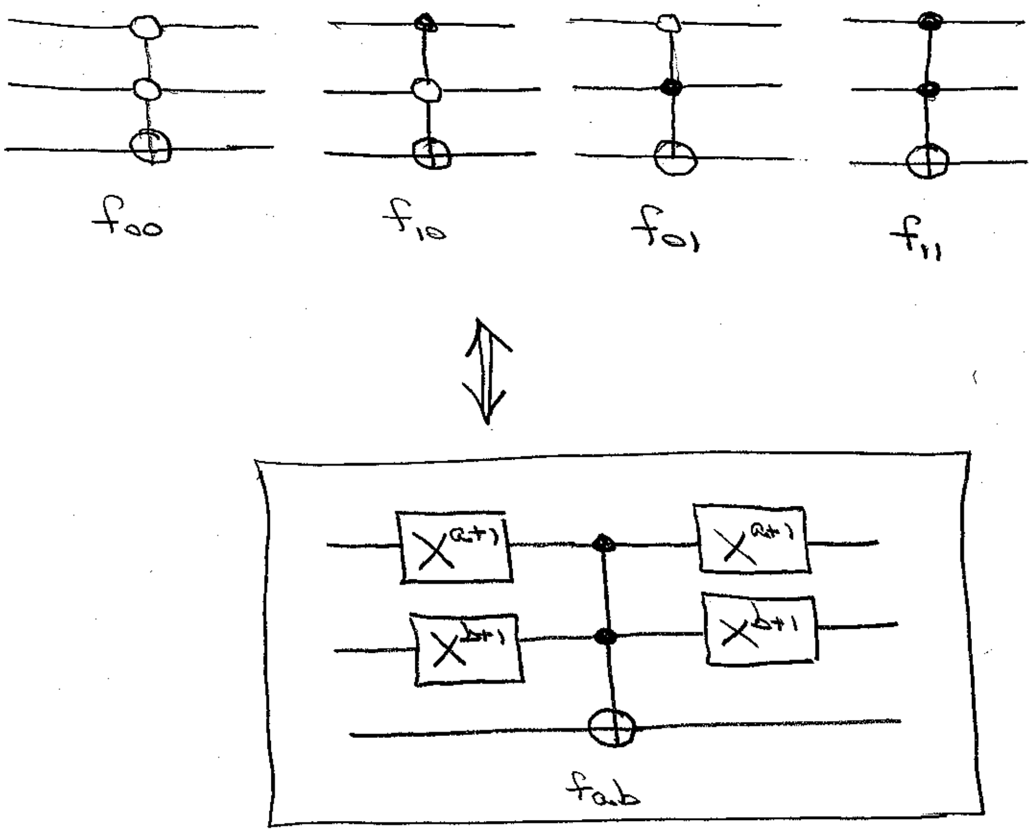
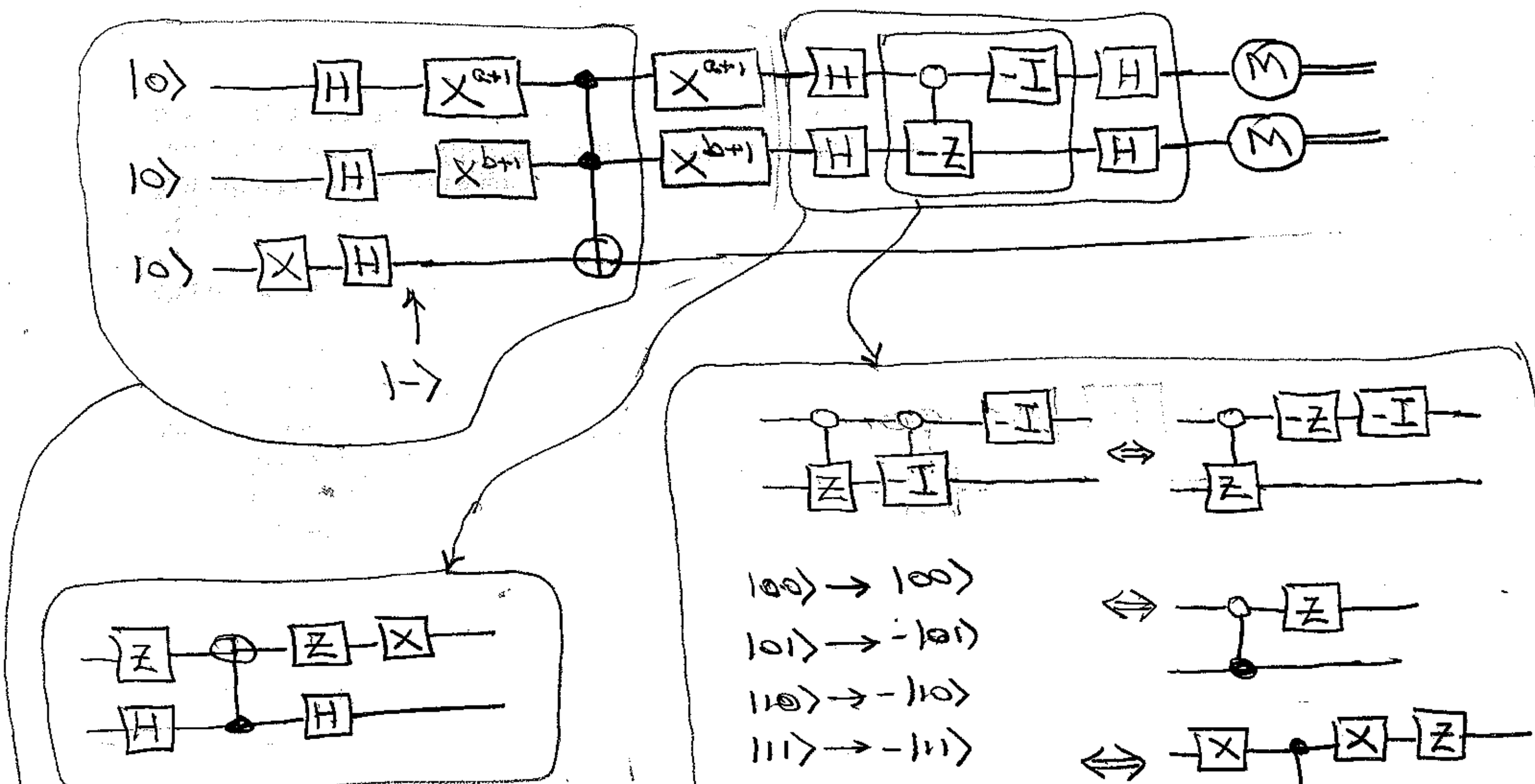
Solution 3.4.

(a) 2-bit Boolean functions



(c) Quantum search circuit



to p. 3

(b) Direct check: on original circuit

$$|000\rangle \xrightarrow[H\otimes H\otimes HX]{} |\bar{0}\bar{0}\bar{1}\rangle$$

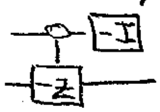
$$\xrightarrow[X^{a+1}\otimes X^{b+1}\otimes I]{} |\bar{0}\bar{0}\bar{1}\rangle = \frac{1}{2}\sum_{c,d} |c,d\rangle \otimes |\bar{1}\rangle$$

$$\xrightarrow[\text{Toffoli}]{} \frac{1}{2}\sum_{c,d} (-1)^{cd} |c,d\rangle \otimes |\bar{1}\rangle$$

$|\bar{1}\rangle \longrightarrow$ omit for rest of calculation

$$\xrightarrow[X^{a+1}\otimes X^{b+1}]{} \frac{1}{2}\sum_{c,d} (-1)^{cd} |c\oplus a\oplus 1, d\oplus b\oplus 1\rangle$$

$$\xrightarrow[H\otimes H]{} \frac{1}{4}\sum_{c,d,e,f} (-1)^{cd} (-1)^{(c+a+1)e} (-1)^{(d+b+1)f} |e,f\rangle$$



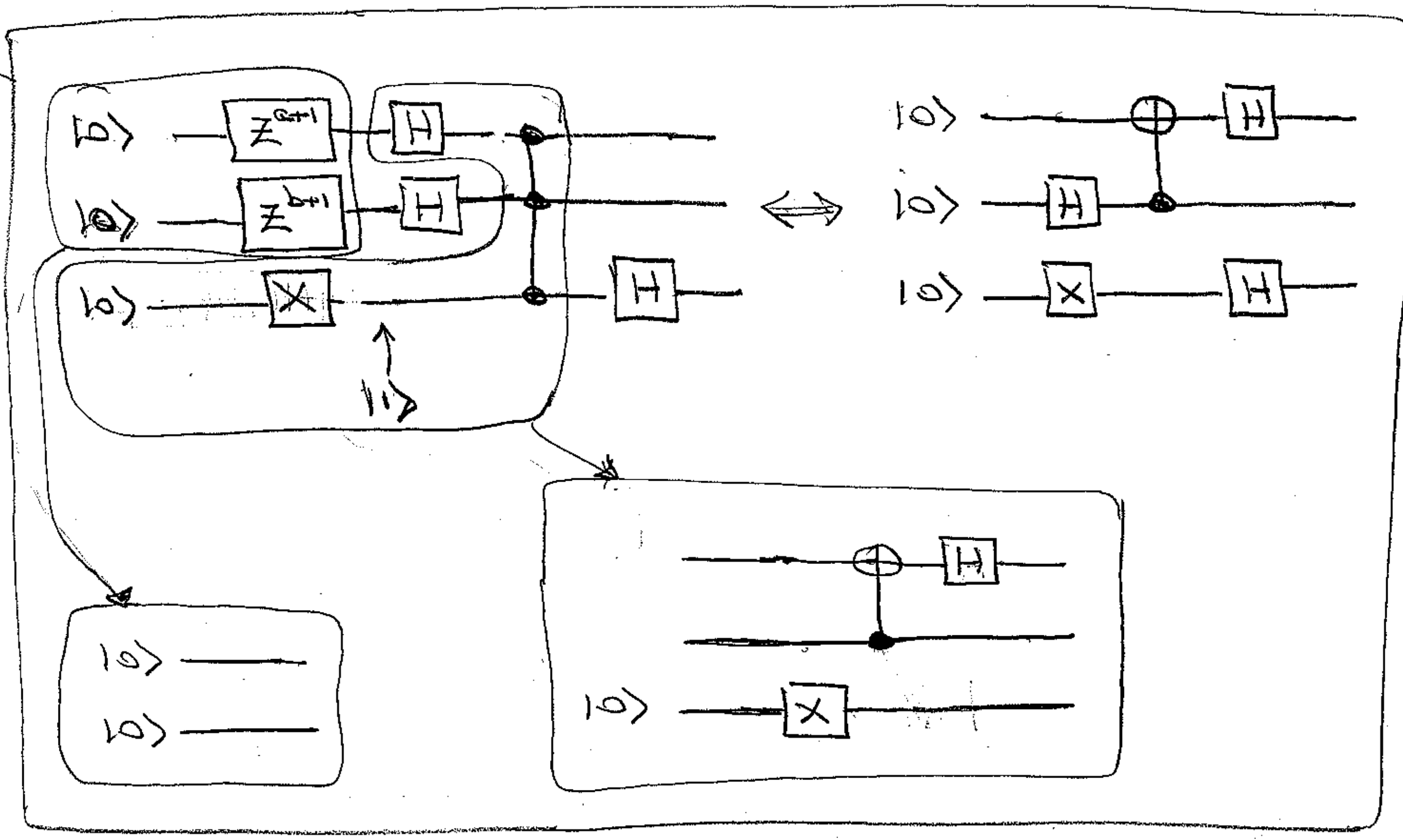
$$\longrightarrow -\frac{1}{4}\sum_{c,d,e,f} (-1)^{cd} (-1)^{(c+a+1)e} (-1)^{(d+b+1)f} (-1)^{(e+1)(f+1)} |e,f\rangle$$

$$= -\frac{1}{2}\sum_{e,f} |e,f\rangle (-1)^{(e+1)(f+1)} (-1)^{e(a+1)} (-1)^{f(b+1)} \times \underbrace{\sum_{d} (-1)^{df} \underbrace{\frac{1}{2}\sum_{c} (-1)^{c(d+e)}}_{\delta_{d\oplus e,0} = \delta_{de}}}_{(-1)^{ef}}$$

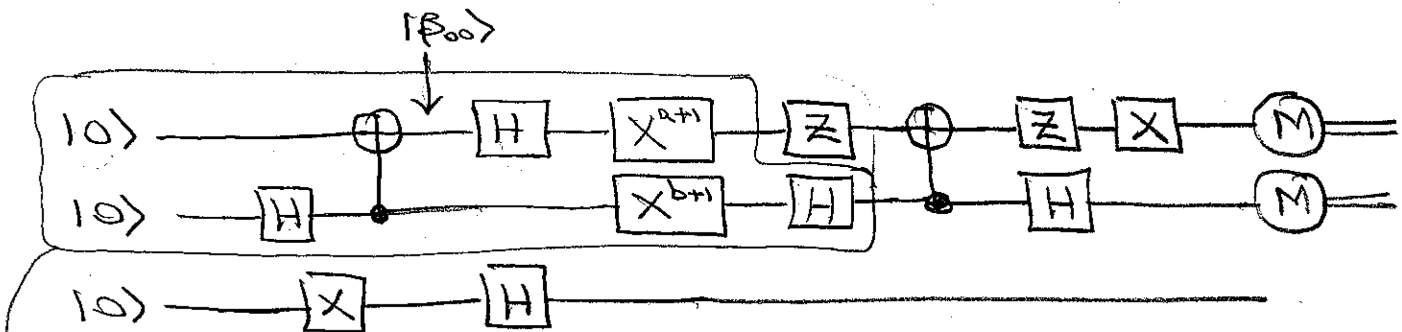
$$= \frac{1}{2}\sum_{e,f} |e,f\rangle (-1)^{ae} (-1)^{bf} = |\bar{a},\bar{b}\rangle$$

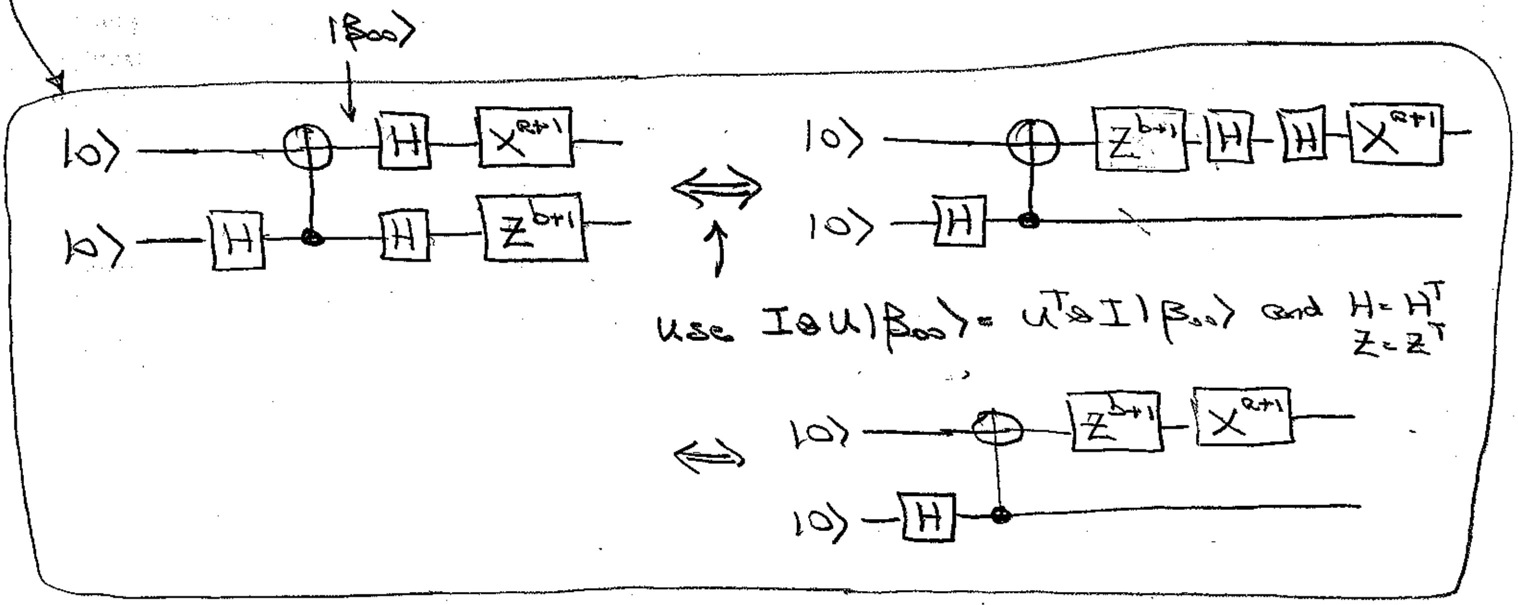
$$\xrightarrow[H\otimes H]{} |a,b\rangle$$

Measurement reads out values of a and b.

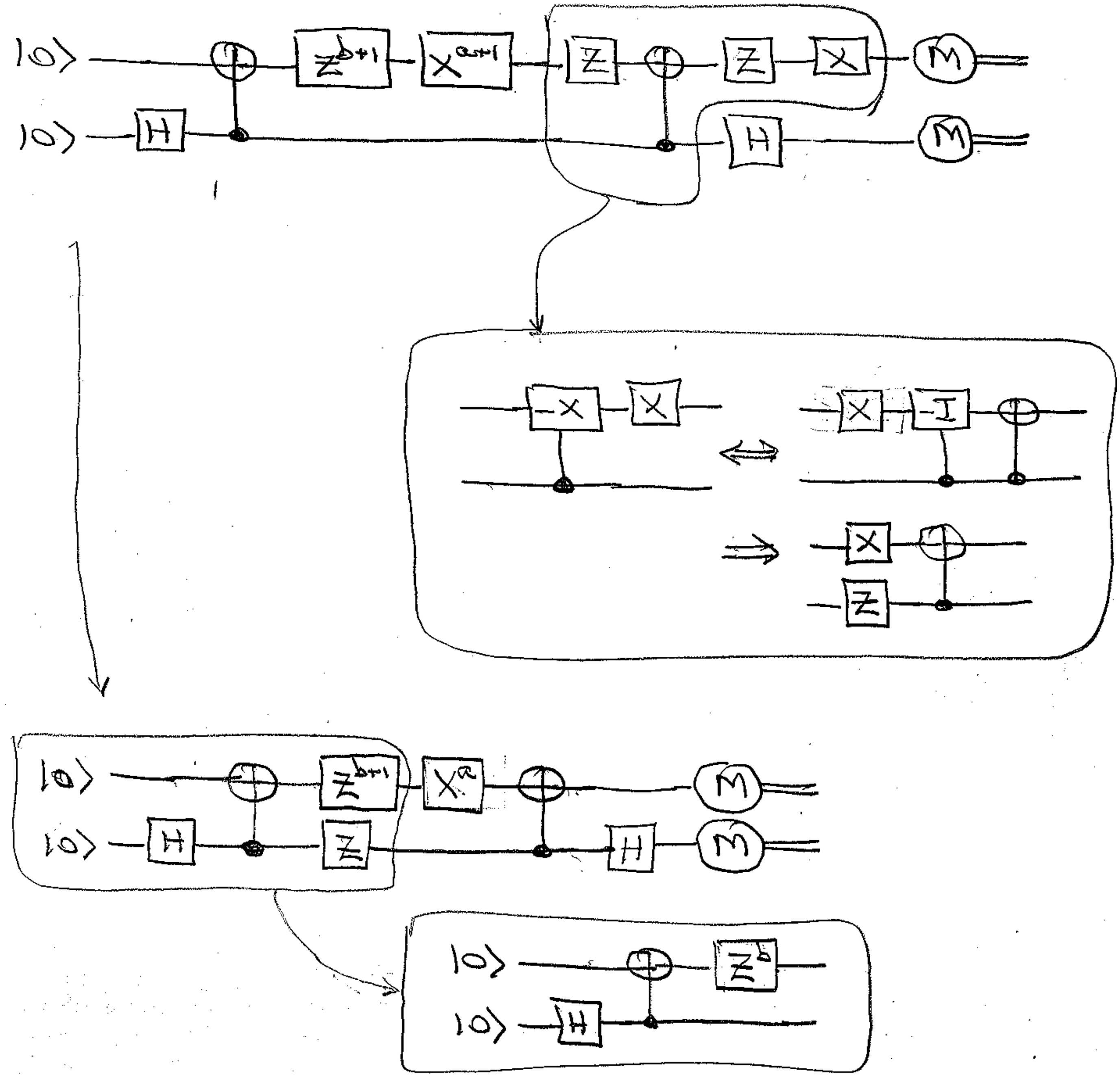


So the quantum search circuit is equivalent to

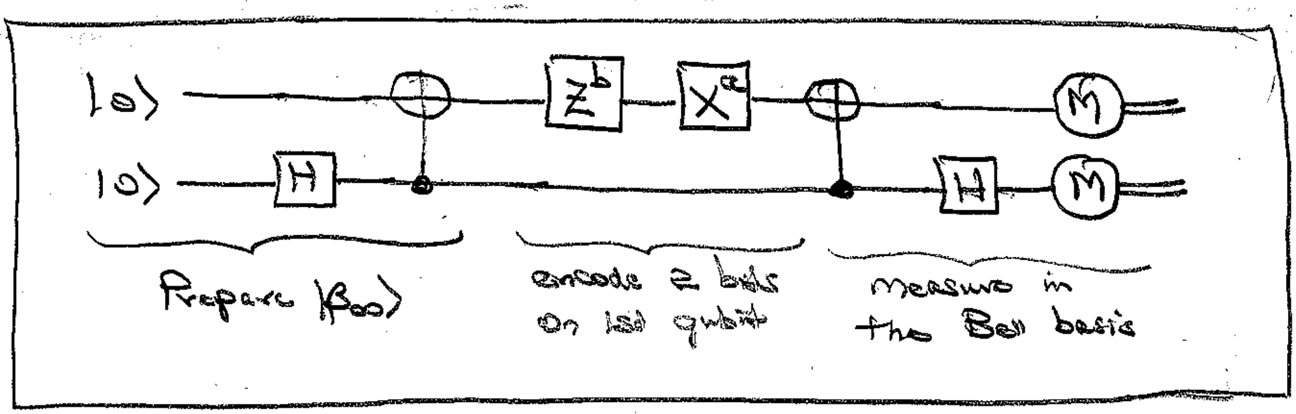




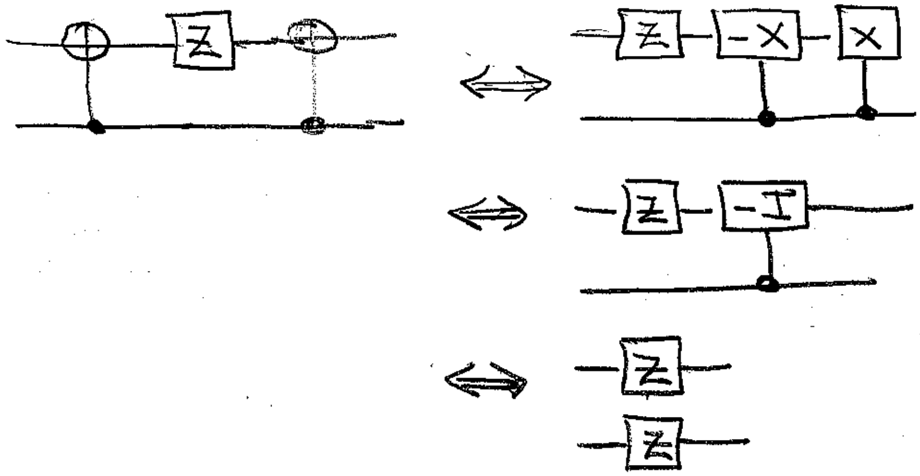
Leaving off the irrelevant 3rd qubit, we have



We end up with Superdense coding:



(d) Use



Dense coding circuit becomes