

Homework Problem 3.7

Discussion Friday, October 17

3.7 Schmidt-like decomposition for three qubits. Consider three qubits, A , B , and C , each of which has a basis labeled by $|0\rangle$ and $|1\rangle$.

(a) Show that an *arbitrary* pure state $|\Psi\rangle$ of the three qubits can be transformed to the following Schmidt-like form using local unitary operators on A , B , and C :

$$\begin{aligned} & \cos\theta|0\rangle \otimes \underbrace{(\cos\chi|0\rangle \otimes |0\rangle + \sin\chi|1\rangle \otimes |1\rangle)}_{=|\phi_0\rangle} \\ & + \sin\theta|1\rangle \otimes \underbrace{(\cos\xi(\sin\chi|0\rangle \otimes |0\rangle - \cos\chi|1\rangle \otimes |1\rangle) + e^{i\delta}\sin\xi(\cos\eta|0\rangle \otimes |1\rangle + \sin\eta|1\rangle \otimes |0\rangle))}_{=|\phi_1\rangle}. \end{aligned}$$

The states $|\phi_0\rangle$ and $|\phi_1\rangle$ are orthonormal states of BC . Five parameters, θ , χ , ξ , η , and δ , are necessary to specify an arbitrary three-qubit pure state; *determine* the range of these five parameters. [Hint: Schmidt decompose $|\Psi\rangle$ with respect to the division A vs. BC . Then Schmidt decompose one of the resulting Schmidt states of BC with respect to the division B vs. C , writing the other BC Schmidt state in the resulting Schmidt bases of B and C . Use the freedom to rephase states to reduce the number of parameters, and use the freedom to apply local unitaries to get into the standard basis.]

The presence of four terms in $|\phi_1\rangle$, instead of just the first two terms or the last two terms, prevents this from being a genuine three-qubit Schmidt decomposition. This illustrates why there is generally no three-particle Schmidt decomposition.

(b) Find the marginal density operators of the three qubits.