Should we think of quantum probabilities as Bayesian probabilities?

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C. M. Caves, C. A. Fuchs, R. Schack, "Subjective probability and quantum certainty," Studies in History and Philosophy of Modern Physics **38**, 255--274 (2007)..

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Yes, because facts never determine probabilities or quantum states.

Solipsism? Waving the red flag

Is there something in nature even when there are no observers or agents about? At the practical level, it would seem hard to deny this, and neither of the authors wish to be viewed as doing so. The world persists without the observer---there is no doubt in either of our minds about that. But then, does that require that two of the most celebrated elements (namely, quantum states and operations) in quantum theory---our best, most all-encompassing scientific theory to date---must be viewed as objective, agent-independent constructs? There is no reason to do so, we say. In fact, *we think there is everything to be gained from carefully delineating which part of the structure of quantum theory is about the world and which part is about the agent's interface with the world*.

C. A. Fuchs and R. Schack, "Unknown quantum states and operations, a Bayesian view," in *Quantum State Estimation,* edited by M. Paris and J. Řeháček (Springer, Berlin, 2004), pp. 147–187.

Some mathematical objects in a scientific theory are our tools; others correspond to reality. Which is which?

Oljeto Wash
Southern Utah

Subjective Bayesian probabilities

Category distinction

Facts

Outcomes of *events*
Truth values of *propositions*

Objective

Probabilities

Agent's *degree of belief* in outcome of an event or truth of a proposition

Subjective

Facts never imply probabilities.

Two agents in possession of the same facts can assign different probabilities.

Subjective Bayesian probabilities

Probabilities

Agent's *degree of belief* in outcome of an event or truth of a proposition.

Consequence of ignorance

Agent's *betting odds*

Subjective

Rules for manipulating probabilities are *objective* consequences of consistent betting behavior (Dutch book).

Subjective Bayesian probabilities

Facts in the form of observed data *d* are used to update probabilities via Bayes's rule:

conditional (model, likelihood)

$$p(h|d) = \frac{p(d|h)\,p(h)}{p(d)}$$

posterior **prior**

The posterior always depends on the prior, *except* when *d* logically implies h_0:

$$\mathrm{Pr}(d|h) = 0 \text{ for } h \neq h_0 \implies \mathrm{Pr}(h_0|d) = 1\,.$$

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Objective probabilities

- Logical probabilities (objective Bayesian): symmetry implies probability
  - Symmetries are applied to judgments, not to facts.
- Probabilities as frequencies: probability as verifiable fact
  - Frequencies are facts, not probabilities.
  - Bigger sample space: exchangeability.

QM: Derivation of quantum probability rule from infinite frequencies?

C. M. Caves, R. Schack, ``Properties of the frequency operator do not imply the quantum probability postulate," Annals of Physics **315**, 123-146 (2005) [Corrigendum: **321**, 504--505 (2006)].

- Objective chance (propensity): probability as specified fact
  - Some probabilities are ignorance probabilities, but others are specified by the facts of a "chance situation."
  - Specification of "chance situation": same, but different.

objective *chance*

QM: Probabilities from physical law. Salvation of objective chance?

Bungle Bungle Range
Western Australia

	Classical (realistic, deterministic) world		Quantum world	
State space	Simplex of probabilities for microstates		Convex set of density operators	
State	Extreme point Microstate	Ensemble	Extreme point Pure state State vector	Ensemble Mixed state Density operator
	x_j	$p(x_j)$	$\vert\psi\rangle$	$\rho = \sum_j p_j \vert\psi_j\rangle\langle\psi_j\vert$

Scorecard:

1. Predictions for fine-grained measurements
2. Verification (state determination)
3. State change on measurement
4. Uniqueness of ensembles
5. Nonlocal state change (steering)
6. Specification (state preparation)

Objective	Subjective	Objective	Subjective

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Fine-grained measurement	Certainty	Probabilities	Certainty or Probabilities	Probabilities

x_j $\qquad p(x_j)$ $\qquad |\psi\rangle$ $\qquad \rho = \sum_j p_j |\psi_j\rangle\langle\psi_j|$

Certainty:
Orthonormal measurement basis that contains $|\psi\rangle$.

Objective	**Subjective**	Objective	**Subjective**

	Classical (realistic, deterministic) world		Quantum world	
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Verification: state determination	Yes	No	No	No

Whom do you ask for the system state? The system or an agent?

	Classical (realistic, deterministic) world		Quantum world	
State space	**Simplex of probabilities for microstates**		**Convex set of density operators**	
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Can you reliably distinguish *two nonidentical* states?

iff orthogonal Always	iff orthogonal	iff orthogonal	iff orthogonal

	Classical (realistic, deterministic) world		Quantum world	
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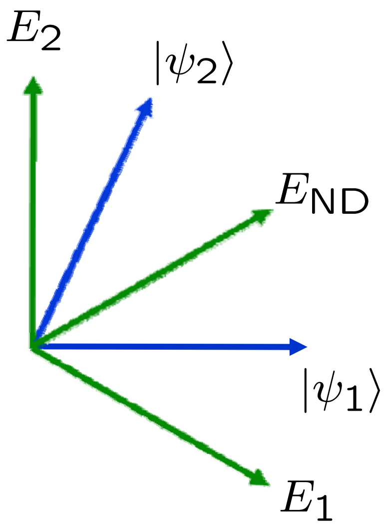

Can you unambiguously distinguish *two nonidentical* states?

Always $p_{\mathrm{ND}} = 0$	Sometimes (iff supports not identical) $p_{\mathrm{ND}} = \begin{pmatrix} \text{average} \\ \text{probability} \\ \text{in support} \\ \text{overlap} \end{pmatrix}$	Always (supports are not identical) $p_{\mathrm{ND}} = \lvert\langle\psi_1\vert\psi_2\rangle\rvert$	Sometimes (iff supports not identical) $p_{\mathrm{ND}} = ?$

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Objective	**Subjective**	Ubjective	**Subjective**

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State change on measurement	No	Yes	Yes	Yes

State-vector reduction or wave-function collapse

Real physical disturbance?

Objective	**Subjective**	Ubjective	**Subjective**

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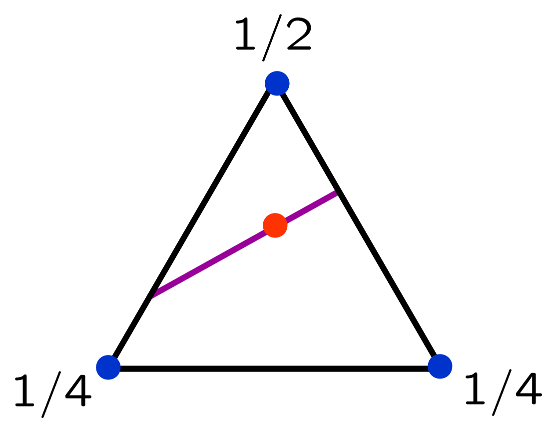
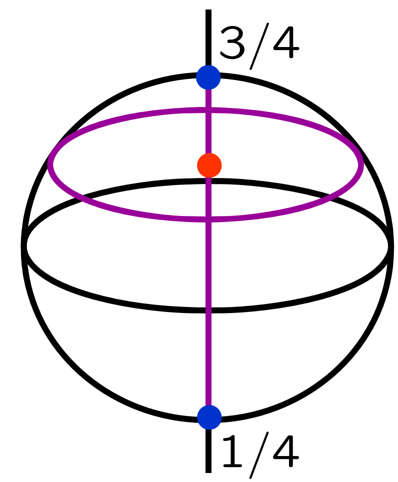
Uniqueness of ensembles	Yes	No	No	No





Objective	**Subjective**	Ubjective	**Subjective**

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State space	Simplex of probabilities for microstates		Convex set of density operators	
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Nonlocal state change (steering)	No	Yes	Yes	Yes

$$p_0 = 1/2 \qquad p_1 = 1/2$$

$$p_{0|0} = 3/4 \qquad p_{0|1} = 1/4$$
$$p_{1|0} = 1/4 \qquad p_{1|1} = 3/4$$

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$
$$= \frac{1}{\sqrt{2}}(|\mathbf{n}, -\mathbf{n}\rangle - |-\mathbf{n}, \mathbf{n}\rangle)$$

Real nonlocal physical disturbance?

Objective	**Subjective**	Subjective	**Subjective**

Truchas from East Pecos Baldy
Sangre de Cristo Range
Northern New Mexico

	Classical (realistic, deterministic) world		**Quantum world**	
State space	**Simplex of probabilities for microstates**		**Convex set of density operators**	
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Specification: state preparation	Yes	No	Copenhagen: Yes	Copenhagen: Yes

Copenhagen interpretation: Classical facts specifying the properties of the preparation device determine a pure state.

Copenhagen (objective preparations view) becomes the home of objective chance, with nonlocal physical disturbances.

	Objective	**Subjective**	**Objective**	**Objective**

Copenhagen	Classical (realistic, deterministic) world		Quantum world	
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Verification: state determination	Yes	No	No	No
State change on measurement	No	Yes	Yes	Yes
Uniqueness of ensembles	Yes	No	No	No
Nonlocal state change (steering)	No	Yes	Yes	Yes
Specification: state preparation	Yes	No	Yes	Yes
	Objective	**Subjective**	**Objective**	**Objective**

Classical and quantum updating

Facts in the form of observed data *d* are used to update probabilities via Bayes's rule:

conditional (model, likelihood)

$$p(h|d) = \frac{p(d|h)\,p(h)}{p(d)}$$

prior

posterior

The posterior always depends on the prior, *except* when *d* logically implies h_0:

$$\mathrm{Pr}(d|h) = 0 \text{ for } h \neq h_0$$
$$\Longrightarrow \quad \mathrm{Pr}(h_0|d) = 1\ .$$

Facts in the form of observed data *d* are used to update quantum states:

quantum operation (model)

$$\rho_d = \frac{\mathcal{A}_d(\rho)}{p(d)}$$

prior

posterior

Quantum state preparation:

ρ_d does not depend on ρ.

The posterior state *always* depends on prior beliefs, *even* for quantum state preparation, because there is a judgment involved in choosing the quantum operation.

Facts never determine probabilities or quantum states.

Where does Copenhagen go wrong?

The Copenhagen interpretation forgets that the preparation device is quantum mechanical. A detailed description of the operation of a preparation device (provably) involves prior judgments in the form of quantum state assignments.

It is possible to show that neither deterministic nor stochastic preparation devices can prepare the same system state independent of system and device initial states.

Subjective Bayesian	Classical (realistic, deterministic) world		Quantum world	
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Fine-grained measurement	Certainty	Probabilities	Certainty or Probabilities	Probabilities
Verification: state determination	Yes	No	No	No
State change on measurement	No	Yes	Yes	Yes
Uniqueness of ensembles	Yes	No	No	No
Nonlocal state change (steering)	No	Yes	Yes	Yes
Specification: state preparation	Yes	No	No	No
	Objective	**Subjective**	**Subjective**	**Subjective**

Echidna Gorge
Bungle Bungle Range
Western Australia

Quantum states vs. probabilities

Are quantum states the same as probabilities? No, though both are subjective, there are differences, but these differences can be stated in Bayesian terms.

A quantum state is a catalogue of probabilities, but the rules for manipulating quantum states are different than for manipulating probabilities.

The rules for manipulating quantum states are *objective* consequences of restrictions on how agents interface with the real world.

Is a quantum coin toss more random than a classical one?
Why trust a quantum random generator over a classical one?

$$|\psi\rangle = |\uparrow\rangle = (|\rightarrow\rangle + |\leftarrow\rangle)/\sqrt{2}$$

Measure spin along *z* axis: $p_\uparrow = 1$ $p_\downarrow = 0$

Measure spin along *x* axis: $p_\rightarrow = 1/2$ $p_\leftarrow = 1/2$

C. M. Caves, R. Schack, "Quantum randomness," in preparation.

quantum coin toss

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Measure spin along *x* axis: $p_{\rightarrow} = 1/2$ $p_{\leftarrow} = 1/2$

quantum coin toss

Standard answer: The quantum coin toss is objective, with probabilities guaranteed by physical law.

Subjective Bayesian answer? No inside information.

Pure states and inside information

Party *B* has *inside information* about event *E,* relative to party *A*, if *A* is willing to agree to a bet on *E* that *B* believes to be a sure win. *B* has *one-way inside information* if *B* has inside information relative to *A*, but *A* does not have any inside information relative to *A*.

The unique situation in which *no other party can have one-way inside information* relative to a party *Z* is when *Z* assigns a pure state. *Z* is said to have a *maximal belief structure.*

Subjective Bayesian answer

We trust quantum over classical coin tossing because an agent who believes the coin is fair cannot rule out an insider attack, whereas the beliefs that lead to a pure-state assignment are inconsistent with any other party's being able to launch an insider attack.

Cape Hauy
Tasman Peninsula

Taking a stab at ontology

CMC only

Quantum systems are defined by *attributes*, such as position, momentum, angular momentum, and energy or Hamiltonian. These attributes—and thus the numerical particulars of their eigenvalues and eigenfunctions and their inner products—are *objective* properties of the system.

The *value* assumed by an attribute is not an objective property, and the *quantum state* that we use to describe the system is purely subjective.

Taking a stab at ontology

1. The attributes orient and give structure to a system's Hilbert space. Without them we are clueless as to how to manipulate and interact with a system.
2. The attributes are unchanging properties of a system, which can be determined from facts. The attributes determine the structure of the world.
3. The system Hamiltonian is one of the attributes, playing the special role of orienting a system's Hilbert space now with the same space later.
4. Convex combinations of Hamiltonian evolutions are essentially unique (up to degeneracies).

Why should you care?
If you do care, how can this be made convincing?
Status of quantum operations?
Effective attributes and effective Hamiltonians? "Effective reality"?