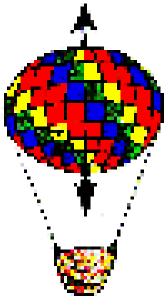
GHZ correlations are just a bit nonlocal

Carlton M. Caves
University of New Mexico
http://info.phys.unm.edu/~caves



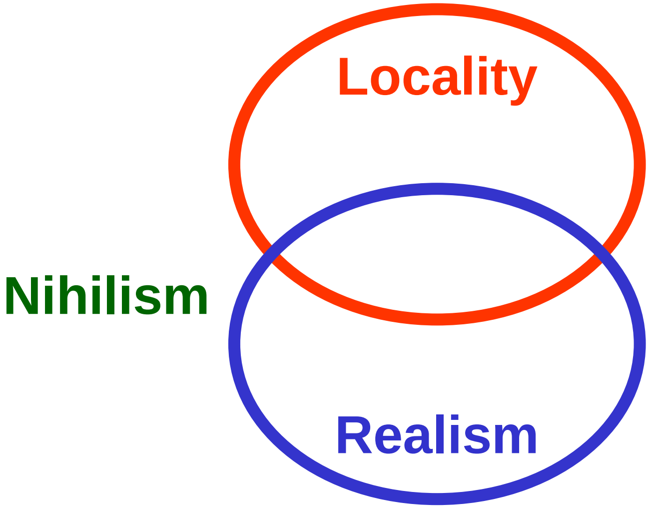
Seminar
date

Please join the
APS Topical Group on Quantum Information, Concepts, and Computation

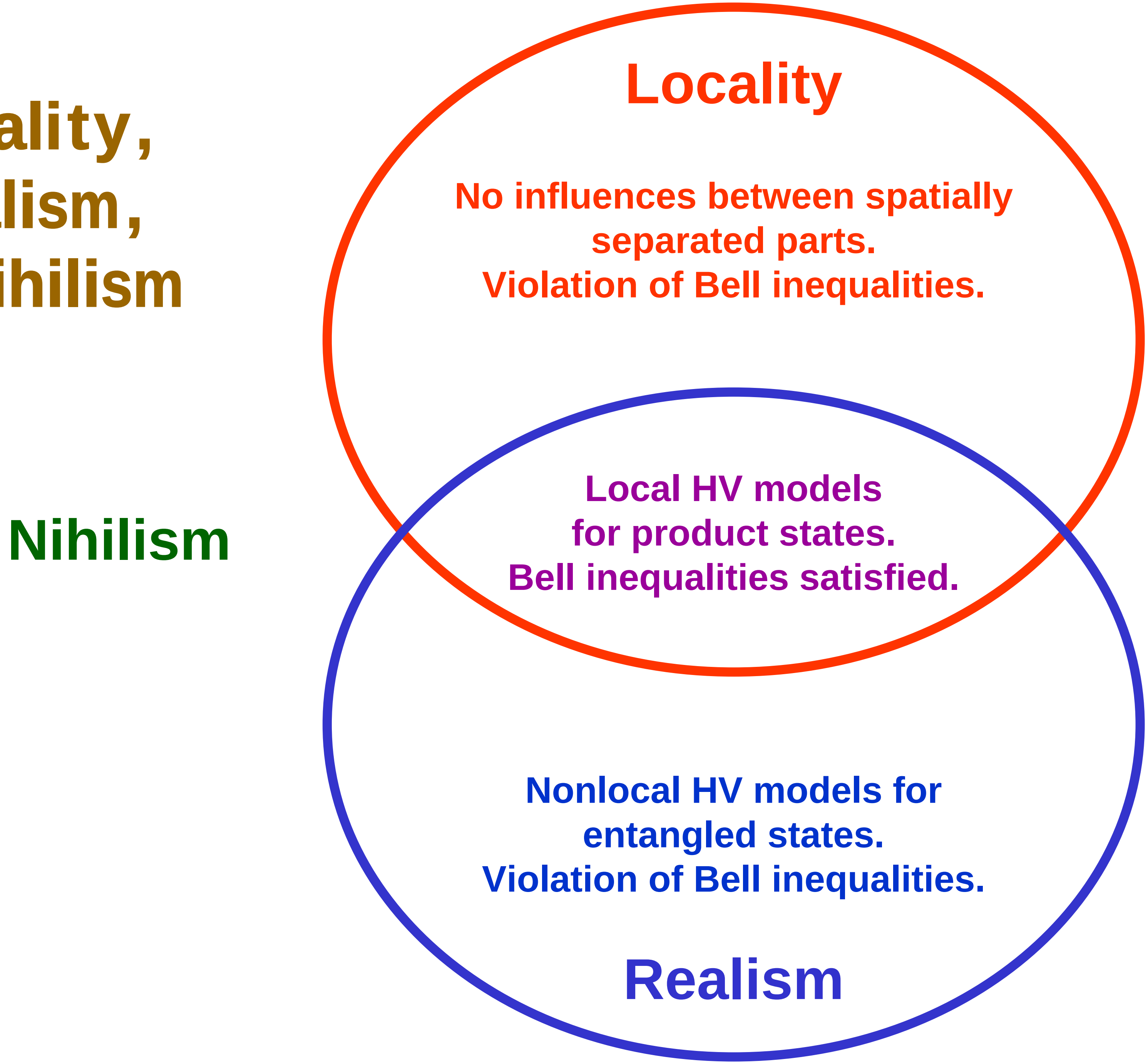
Locality, realism, or nihilism

We consider the consequences of the observed violations of Bell's inequalities. Two common responses are (i) the rejection of realism and the retention of locality and (ii) the rejection of locality and the retention of realism. Here we critique response (i). We argue that locality contains an implicit form of realism, since in a worldview that embraces locality, spacetime, with its usual, fixed topology, has properties independent of measurement. Hence we argue that response (i) is incomplete, in that its rejection of realism is only partial.

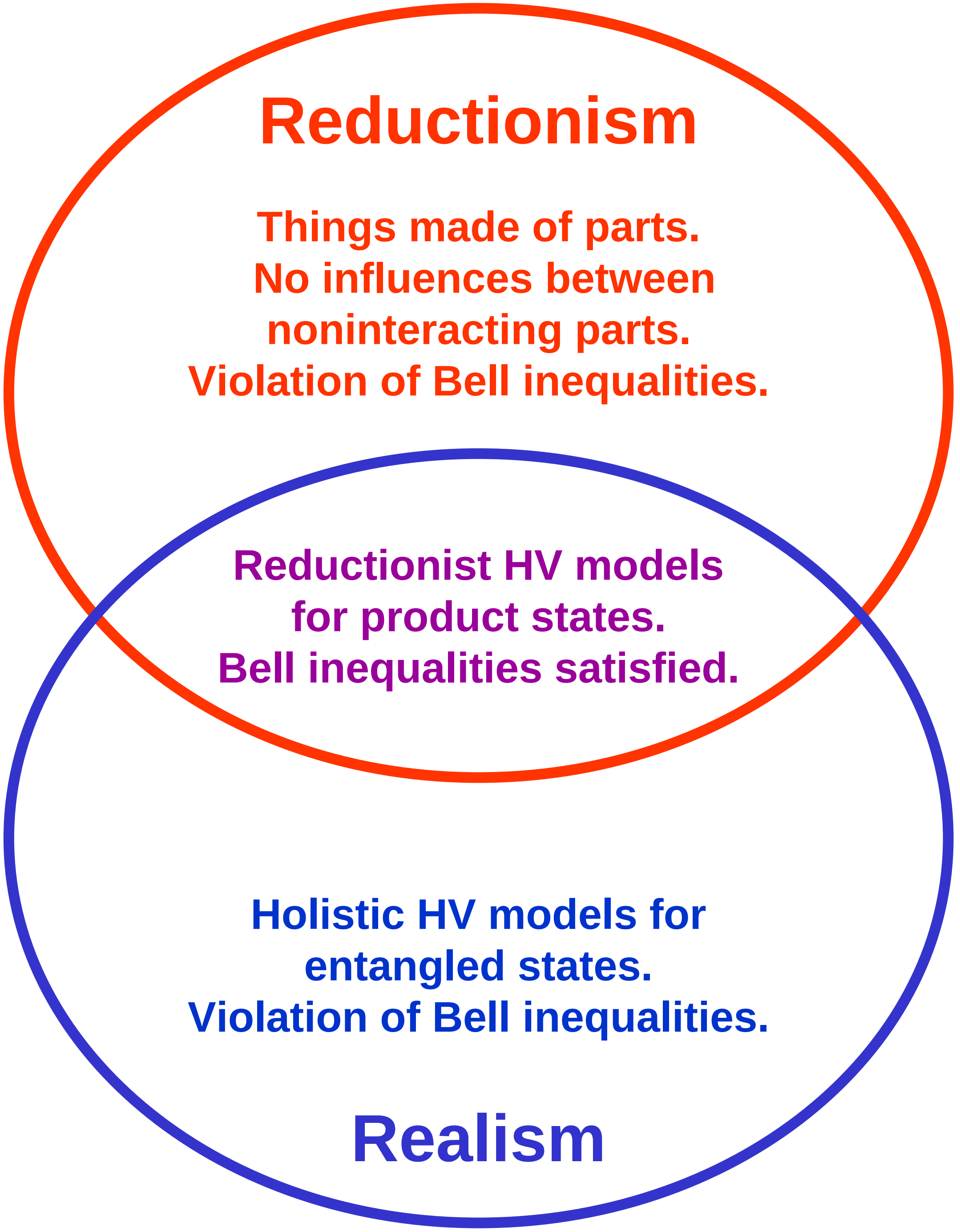
R. Y. Chiao and J. C. Garrison
"Realism or Locality: Which Should We Abandon?"
Foundations of Physics 29, 553-560 (1999).



Locality, realism, or nihilism



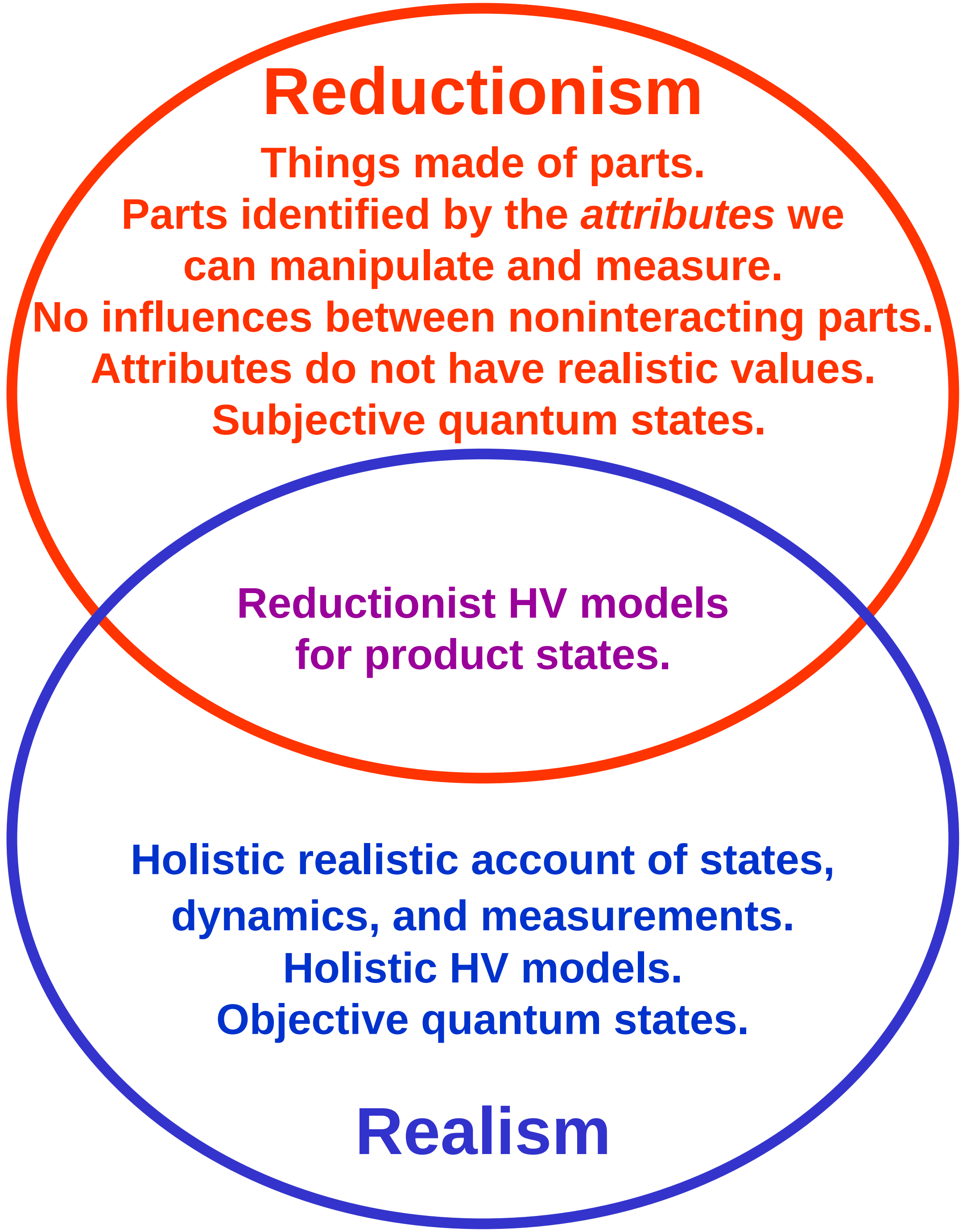
Reductionism or realism



Quantum mechanics

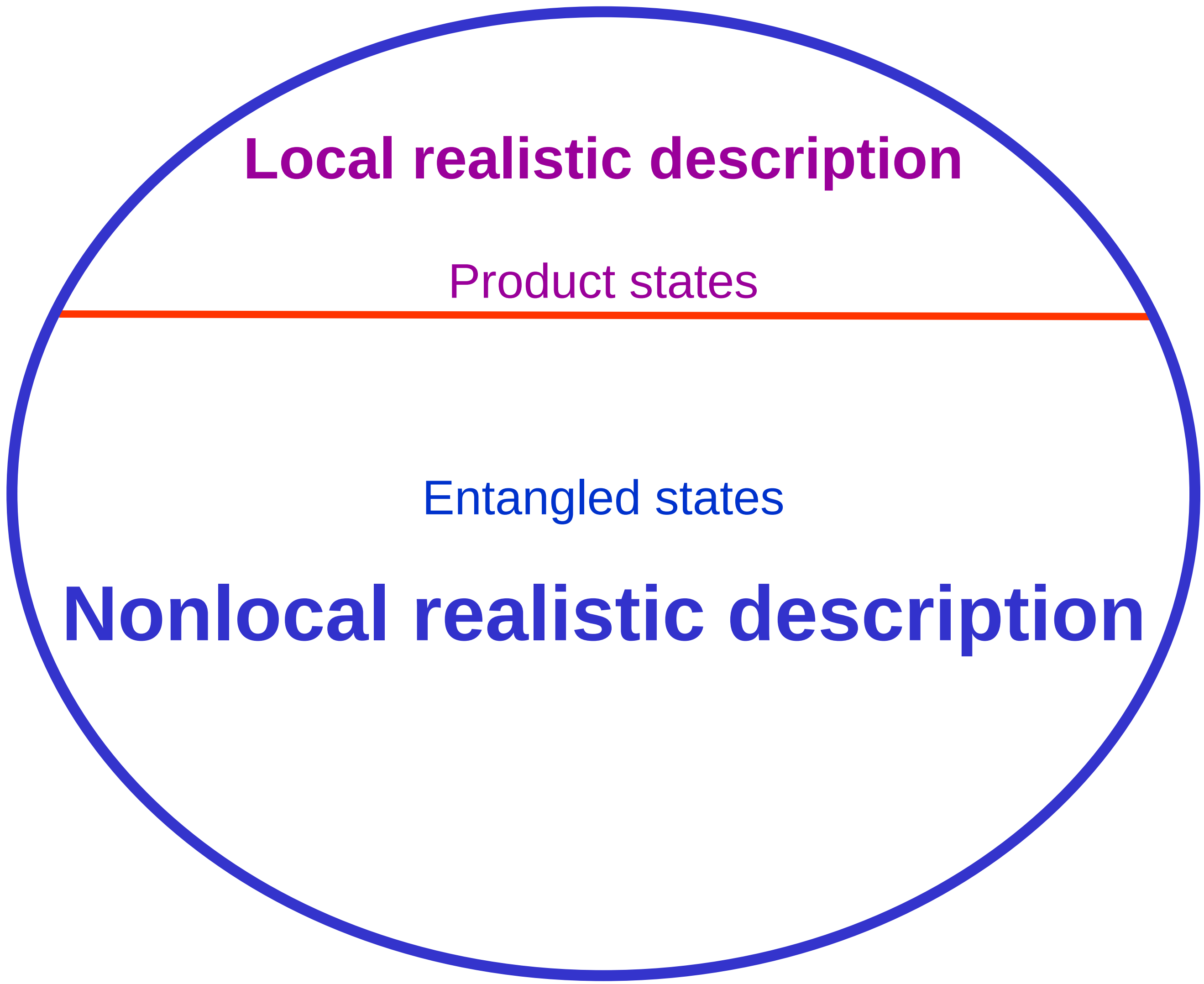
or

Stories about a reality beneath quantum mechanics

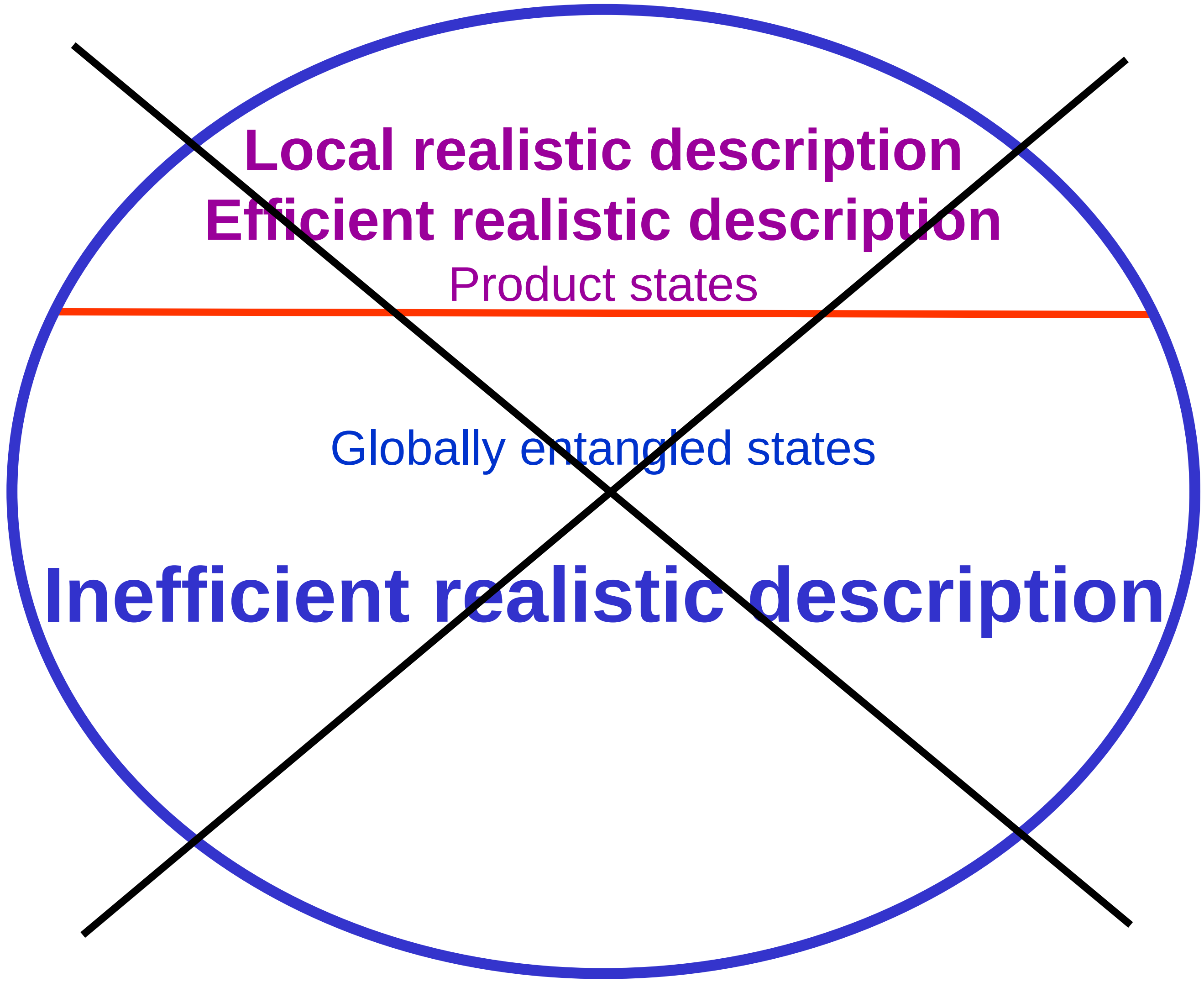


Why not a different story, one that comes from quantum information science?

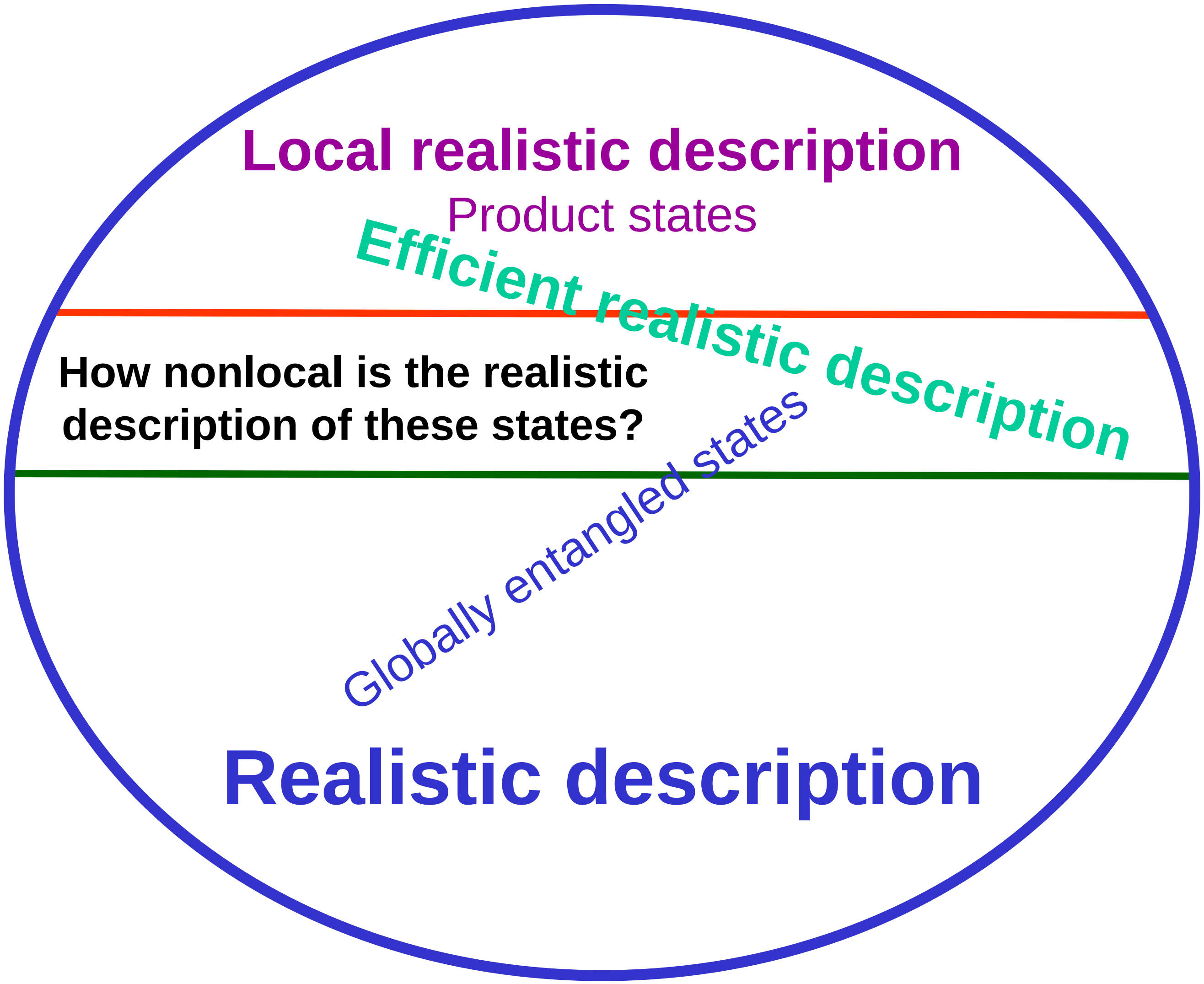
The old story



A new story from quantum information?



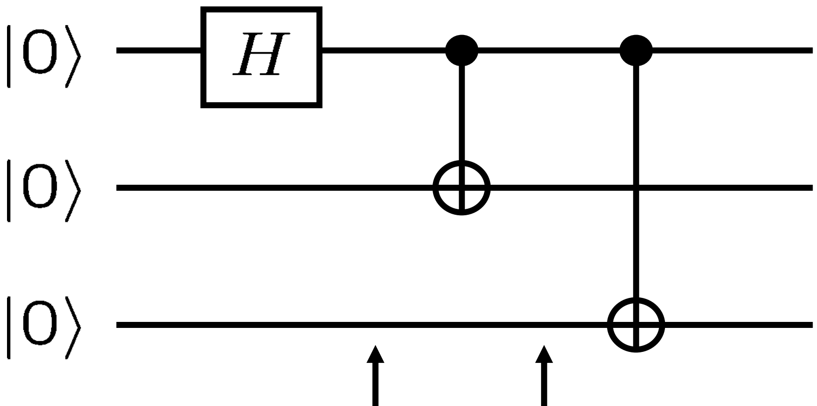
A new story from quantum information



Modeling GHZ (cat) correlations

Measure *XYY, YXY,* and *YYX*: All yield result -1.
Local* realism implies *XXX* = -1, *but
quantum mechanics says *XXX* = +1.

$$ZII \quad IZI \quad IIZ$$



$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|00\rangle$$

$$XII, IZI, IIZ$$

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

$$XXI, ZZI, IIZ$$

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

GHZ (cat) entangled state

$$XXX, ZZI, ZIZ$$

$$S = \left\{ \begin{array}{c} III, ZZI, ZIZ, IZZ, \\ XXX, -XYY, -YXY, -YYX \end{array} \right\}$$

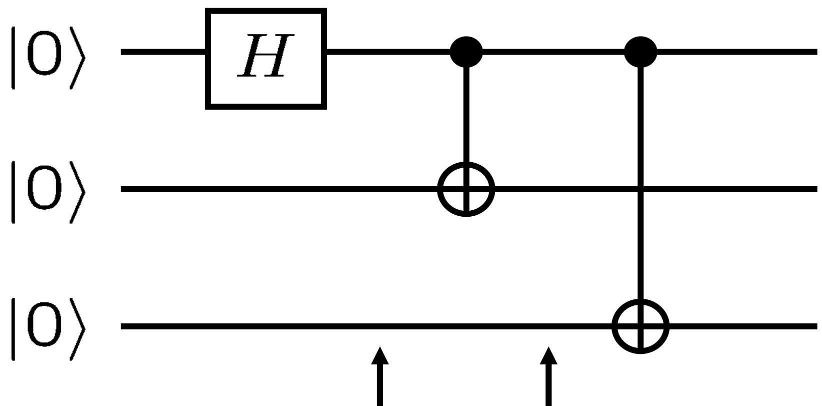
Stabilizer formalism

Efficient* (nonlocal) realistic description of *states, dynamics,* and *measurements

Modeling GHZ (cat) correlations

$ZZI = ZIZ = IZZ = XXX = +1$; $XYY = YXY = YYX = -1$. To get correlations right requires 1 bit of classical communication: party 2 tells party 1 whether Y is measured on qubit 2; party 1 flips her result if Y is measured on either 1 or 2.

x	y	z
r_1	$-r_1$	1
r_2	r_2	1
r_3	r_3	1



$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|00\rangle$$

x	y	z
1	r_1	r_1
r_2	r_2	1
r_3	r_3	1

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

x	y	z
r_2	$r_1 r_2$	r_1
r_2	$r_1 r_2$	r_1
r_3	r_3	1

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

GHZ (cat) entangled state

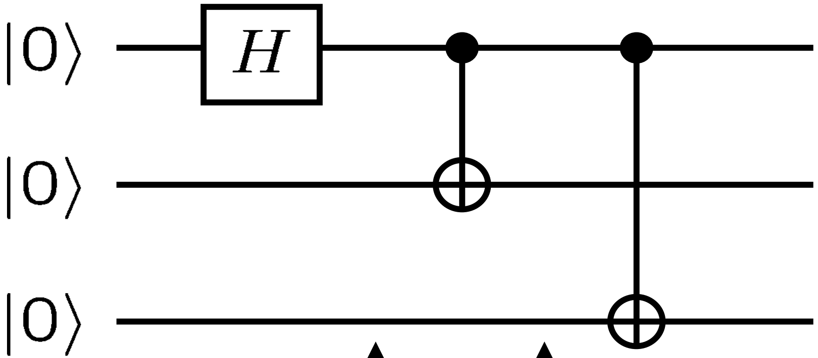
x	y	z
$r_2 r_3$	$r_1 r_2 r_3$	r_1
r_2	$r_1 r_2$	r_1
r_3	$r_1 r_3$	r_1

When party 1 flips her result, this can be thought of as a nonlocal disturbance that passes from qubit 2 to qubit 1. The communication protocol quantifies the required amount of nonlocality.

Modeling GHZ (cat) correlations

ZZI = *ZIZ* = *IZZ* = *XXX* = +1; *XYY* = *YXY* = *YYX* = -1. To get correlations right requires 1 bit of classical communication: party 2 tells party 1 whether *Y* is measured on qubit 2; party 1 flips her result if *Y* is measured on either 1 or 2.

x	y	z
r_1	$-r_1$	1
r_2	r_2	1
r_3	r_3	1



$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

GHZ (cat) entangled state

x	y	z
$r_2 r_3$	$r_1 r_2 r_3$	r_1
r_2	$r_1 r_2$	r_1
r_3	$r_1 r_3$	r_1

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|00\rangle$$

x	y	z
1	r_1	r_1
r_2	r_2	1
r_3	r_3	1

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

x	y	z
r_2	$r_1 r_2$	r_1
r_2	$r_1 r_2$	r_1
r_3	r_3	1

T. E. Tessier, C. M. Caves, I. H. Deutsch, B. Eastin, and D. Bacon, ``Optimal classical-communication-assisted local model of *n*-qubit Greenberger-Horne-Zeilinger correlations,'' Phys. Rev. A **72**, 032305 (2005).

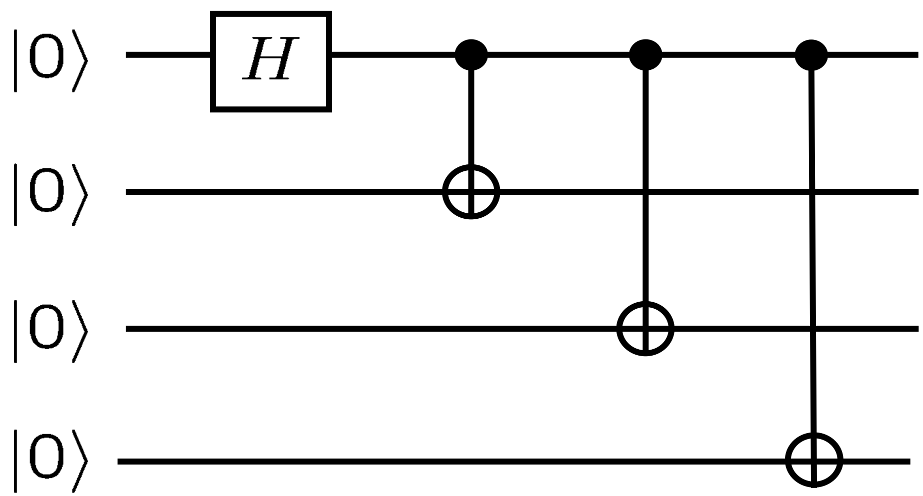
For *N*-qubit GHZ states, this same procedure gives a *local realistic* description, aided by *N-2* bits of *classical communication* (provably minimal), of *states, dynamics,* and *measurements* (of Pauli products).

Communication-assisted LHV model

Modeling GHZ (cat) correlations

Assume 1 bit of communication between qubits 1 and 2. Let *S=XXII* and *T=XYII* be Pauli products for qubits 1 and 2; then we have *SYY=TXY=TYX* = -1. *Local* realism implies *SXX* = -1, *but* quantum mechanics says *SXX* = +1.

$$ZIII \quad IZII \quad IIZI \quad IIIZ$$



$$\frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle)$$

4-qubit GHZ entangled state

$$XXXX, ZZII, ZIZI, ZIIZ$$

$$S = \left\{ \begin{array}{c} IIII, ZZII, ZIZI, ZIIZ, IZZI, IZIZ, IIZZ \\ XXXX, -XXYY, -XYXY, -XYYX, \\ -YXXY, -YXYX, -YYXX, YYYY \end{array} \right\}$$

For *N*-qubit GHZ states, a simple extension of this argument shows that *N-2* bits of *classical communication* is the minimum required to mimic the predictions of quantum mechanics for measurements of Pauli products.

Clifford circuits: Gottesman-Knill theorem

- N qubits in an initial product state in Z basis
- Allowed gates: Pauli operators X, Y, and Z, plus H, S, and C-NOT
- Allowed measurements: Products of Pauli operators

Global entanglement

but

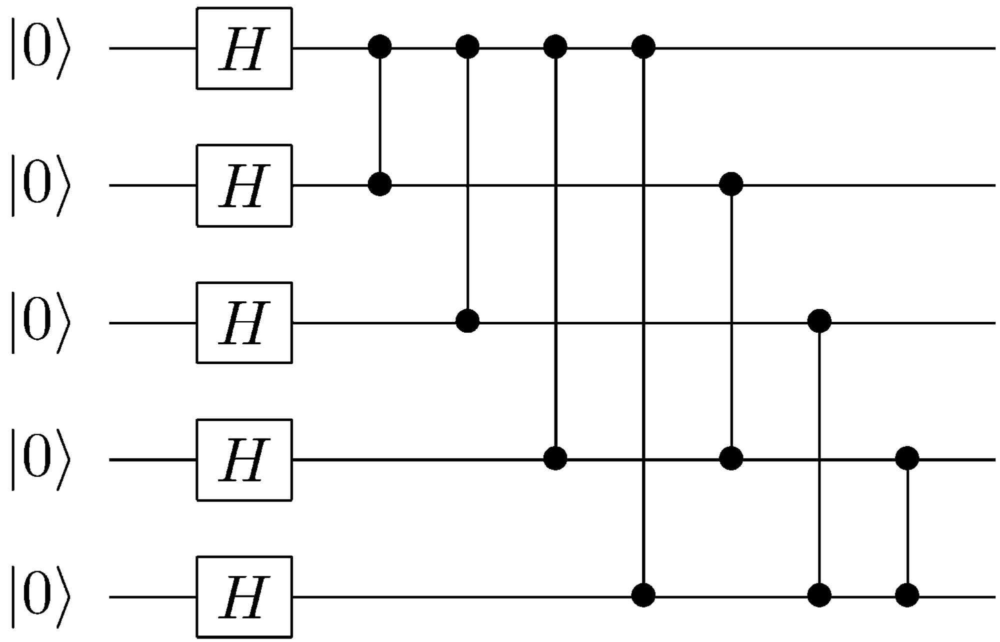
Efficient (nonlocal) realistic description of states, dynamics, and measurements (in terms of stabilizer generators)

This kind of global entanglement, when measurements are restricted to the Pauli group, can be simulated efficiently and thus does not provide an exponential speedup for quantum computation.

Graph states

All stabilizer (Clifford) states are related to graph states by *Z*, Hadamard, and S gates.

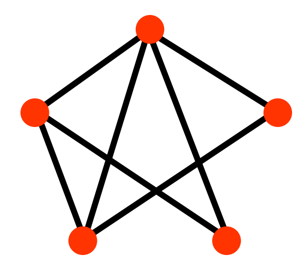
$$\begin{matrix} ZIIII \\ IZIII \\ IIZII \\ IIIZI \\ IIIIZ \end{matrix}$$



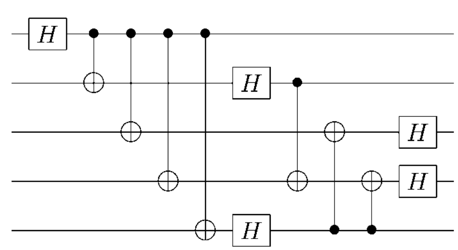
$$\begin{matrix} XZZZZ \\ ZXIZI \\ ZIXIZ \\ ZZIXZ \\ ZIZZX \end{matrix}$$

$$|\psi\rangle = \frac{1}{2\sqrt{2}}\big(|00\bar{0}\bar{0}0\rangle + |01\bar{0}\bar{1}0\rangle + |00\bar{1}\bar{1}1\rangle + |01\bar{1}\bar{0}1\rangle$$

$$+ |10\bar{1}\bar{1}0\rangle - |11\bar{1}\bar{0}0\rangle - |10\bar{0}\bar{0}1\rangle + |11\bar{0}\bar{1}0\rangle\big)$$

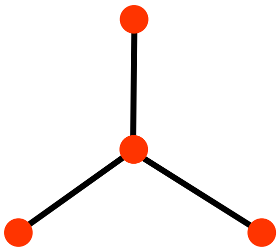


$$|\bar{a}\rangle \equiv H|a\rangle = (|0\rangle + (-1)^a|1\rangle)/\sqrt{2}$$

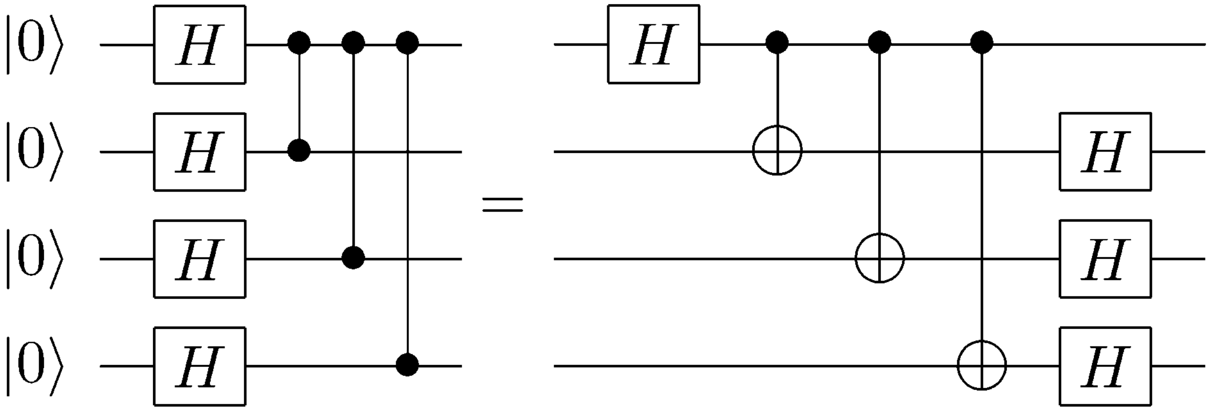


Graph states

4-qubit GHZ graph state



$$\begin{matrix} ZIII \\ IZII \\ IIZI \\ IIIZ \end{matrix}$$

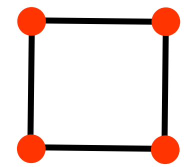


$$\begin{matrix} XZZZ \\ ZXII \\ ZIXI \\ ZIIX \end{matrix}$$

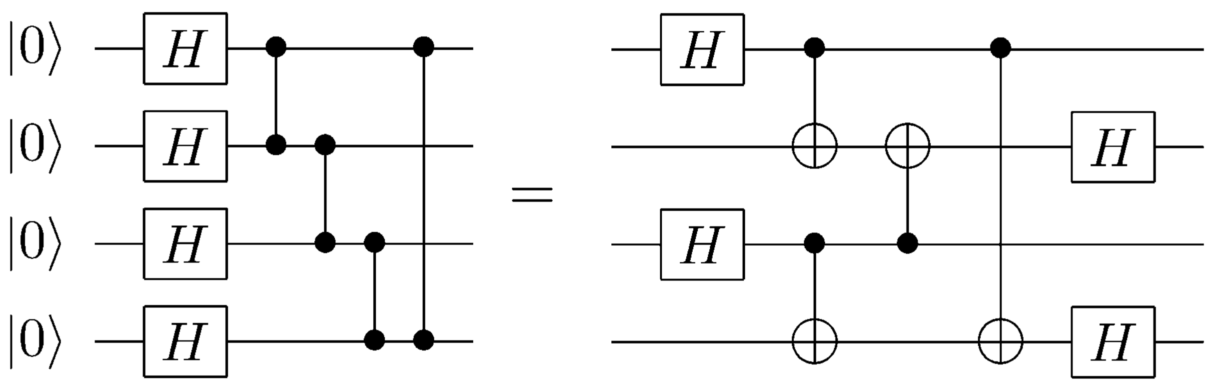
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\bar{0}\bar{0}\bar{0}\rangle + |1\bar{1}\bar{1}\bar{1}\rangle)$$

Graph states

2 x 2 cluster state



$$\begin{matrix} ZIII \\ IZII \\ IIZI \\ IIIZ \end{matrix}$$



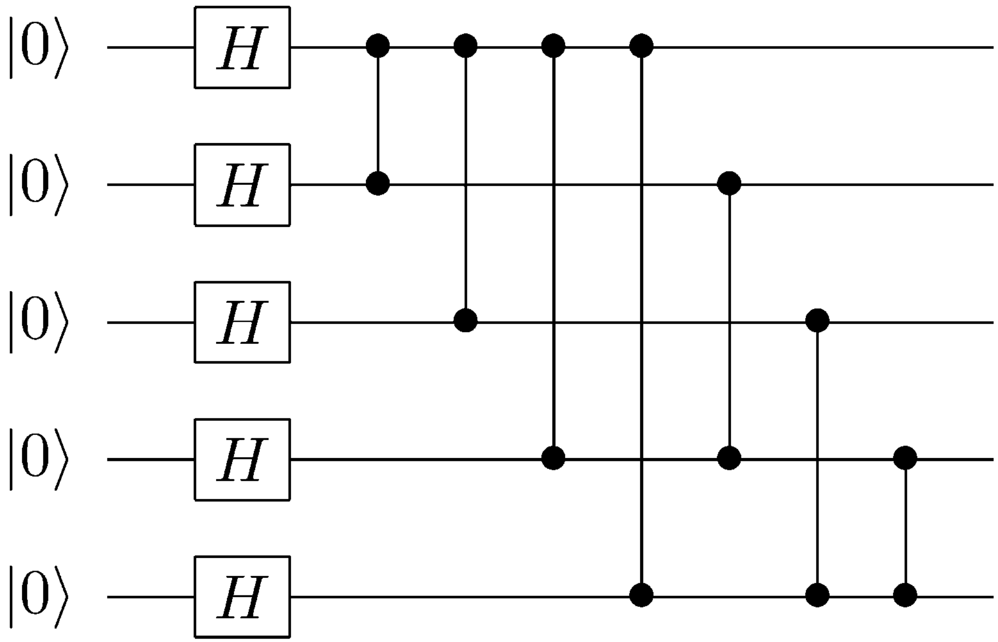
$$\begin{matrix} XZIZ \\ ZXZI \\ IZXZ \\ ZIZX \end{matrix}$$

$$|\psi\rangle = \frac{1}{2}\Big(|0\bar{0}0\bar{0}\rangle + |1\bar{1}0\bar{1}\rangle + |0\bar{1}1\bar{1}\rangle + |1\bar{0}1\bar{0}\rangle\Big)$$

Graph states: LHV model

J. Barrett, C. M. Caves, B. Eastin, M. B. Elliott, and S. Pironio, "Modeling Pauli measurements on graph states with nearest-neighbor classical communication," submitted to PRA.

$ZIIII$
$IZIII$
$IIZII$
$IIIZI$
$IIIIZ$



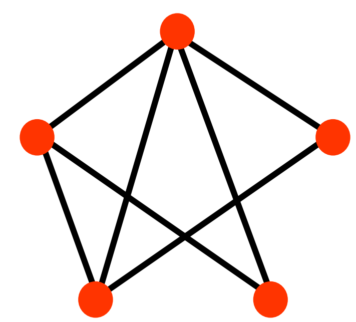
$$g_1 = XZZZZ$$
$$g_2 = ZXIZI$$
$$g_3 = ZIXIZ$$
$$g_4 = ZZIXZ$$
$$g_5 = ZIZZX$$

$$g_j = X_j \bigotimes_{k\in\mathcal{N}(j)} Z_k$$

$$x_j = \prod_{k\in\mathcal{N}(j)} z_k\ ,$$

$$x_j y_j z_j = \pm 1$$

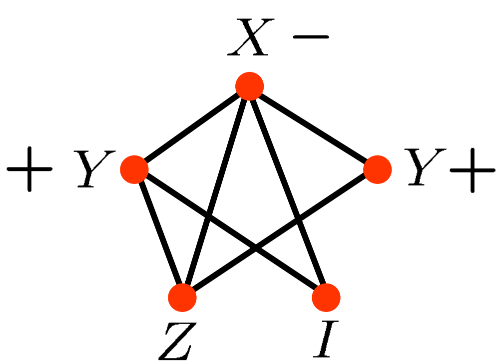
x	y	z
$z_2z_3z_4z_5$	$z_1z_2z_3z_4z_5$	z_1
z_1z_4	$z_1z_2z_4$	z_2
z_1z_5	$z_1z_3z_5$	z_3
$z_1z_2z_5$	$z_1z_2z_4z_5$	z_4
$z_1z_3z_4$	$z_1z_3z_4z_5$	z_5



Graph states: Nearest-neighbor (single-round) communication protocol

For qubit j, let n_j be the number of neighbors that measure X or Y. Certainty (stabilizer element) requires

$$n_j = \begin{cases} 0 \bmod 2\,, & \text{if qubit } j \text{ measures } I \text{ or } X, \\ 1 \bmod 2\,, & \text{if qubit } j \text{ measures } Z \text{ or } Y. \end{cases}$$



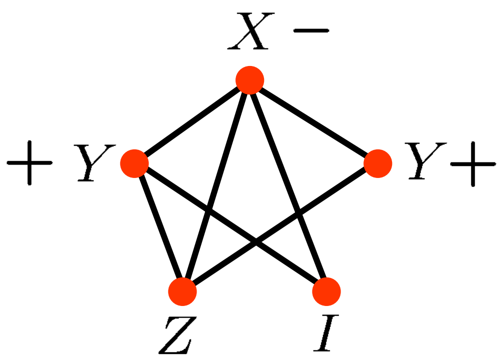
$$\begin{aligned} g_1 &= XZZZZ \\ g_2 &= ZXIZI \\ g_3 &= ZIXIZ \\ g_4 &= ZZIXZ \\ g_5 &= ZIZZX \end{aligned}$$

$$g_1 g_2 g_5 = -XYIZY$$

$$\begin{pmatrix} \text{overall} \\ \text{sign} \end{pmatrix} = (-1)^{(\#\text{ of } X \text{ qubits with } n_j = 2 \bmod 4)} \times (-1)^{(\#\text{ of } Y \text{ qubits with } n_j = 3 \bmod 4)}\,.$$

All that matters is that a qubit measuring $X(Y)$ doesn't flip when $n_j = 0(1)\text{mod}4$ and does flip when $n_j = 2(3)\text{mod}4$.

Graph states: Nearest-neighbor communication protocol



$$\boxed{\begin{aligned} g_1 &= XZZZZ \\ g_2 &= ZXIZI \\ g_3 &= ZIXIZ \\ g_4 &= ZZIXZ \\ g_5 &= ZIZZX \end{aligned}}$$

$$g_1 g_2 g_5 = -XYIZY$$

Each qubit tells its neighbors if it measures X or Y. A qubit flips its table entry if it measures X or Y and the number of neighbors measuring X or Y is $2, 3 \bmod 4$.

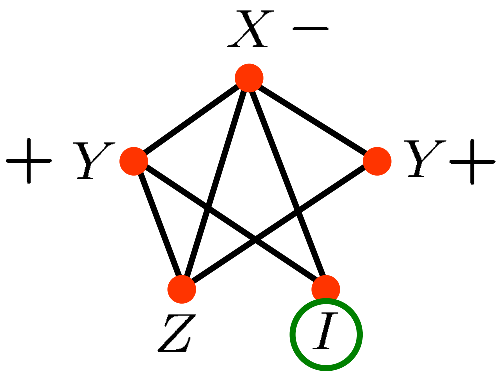
OR

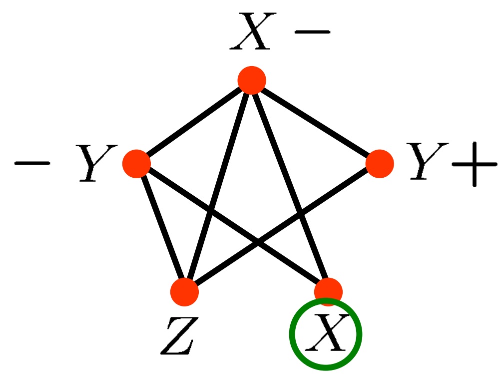
Each qubit tells its neighboring qubits if it measures X or Y. A qubit flips its table entry if it measures X (Y) and the number of neighbors measuring X or Y is $2, 3 \bmod 4$ ($0, 3 \bmod 4$)

Site-invariant nearest-neighbor communication protocols

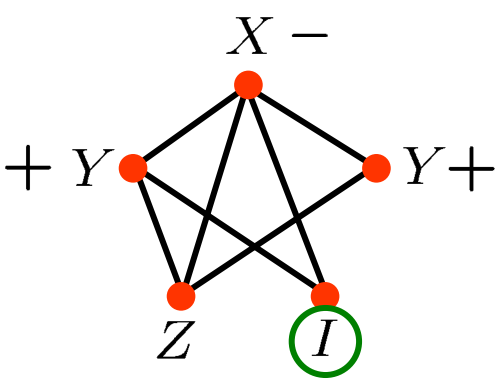
Graph states: Subcorrelations

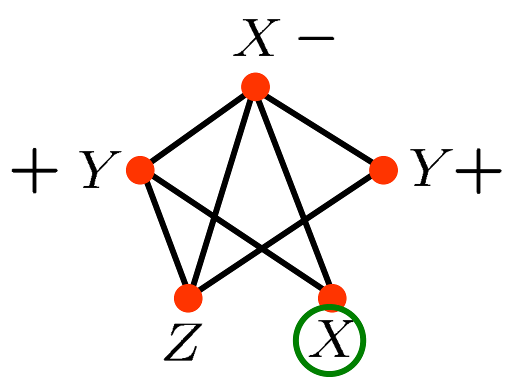
Each qubit tells its neighbors if it measures X or Y. A qubit flips its table entry if it measures X or Y and the number of neighbors measuring X or Y is $2, 3 \bmod 4$.





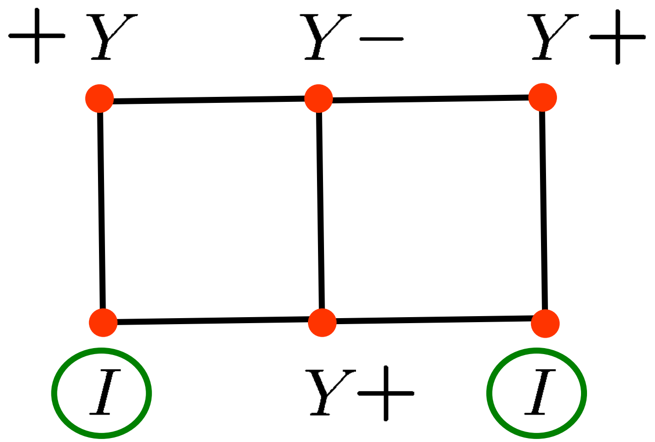
Each qubit tells its neighboring qubits if it measures X or Y. A qubit flips its table entry if it measures X (Y) and the number of neighbors measuring X or Y is $2, 3 \bmod 4$ $(0, 3 \bmod 4)$.



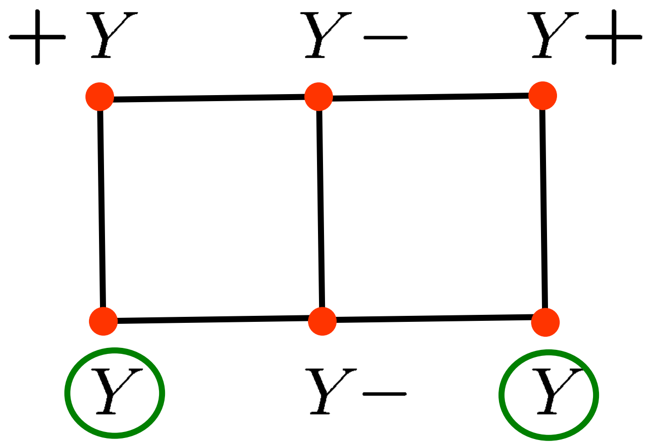


Graph states: Subcorrelations

Each qubit tells its neighboring qubits if it measures X or Y. A qubit flips its table entry if it measures X (Y) and the number of neighbors measuring X or Y is $2,3 \bmod 4$ $(0,3 \bmod 4)$



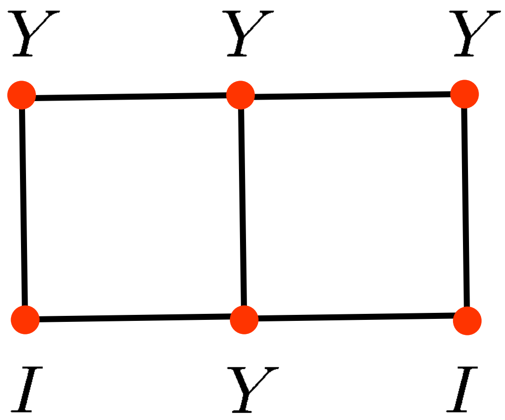
Certain result -1

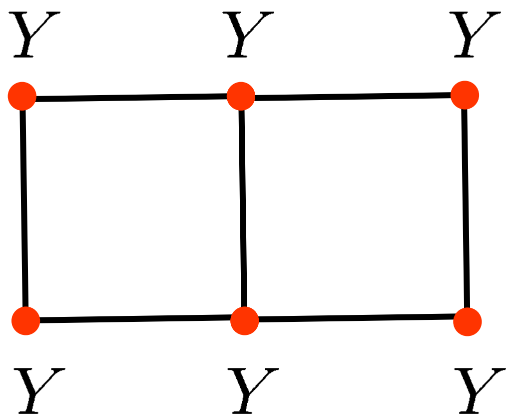


Overall random result

Protocol gets submeasurement wrong.

Graph states: Site invariance and communication distance





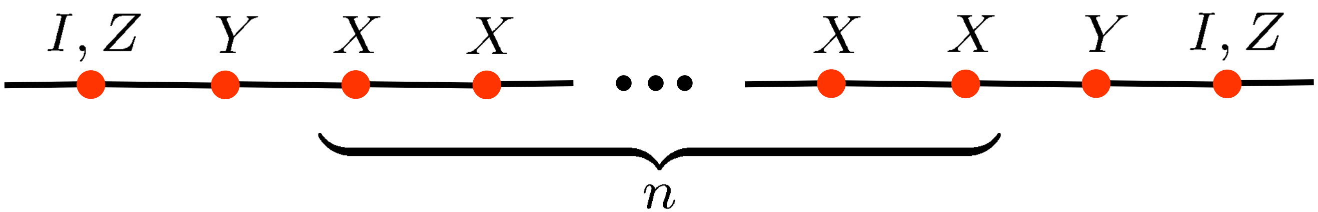
Certain result -1

A site-invariant protocol cannot introduce an overall sign flip when this measurement is viewed as a submeasurement of the one on the left.

Overall random result

Site-invariant protocols can get all correlations right, but even with unlimited-distance communication, such protocols fail on some subcorrelations for some graphs.

Graph states: Site invariance and communication range



For linear chains, the only measurements requiring a sign flip are those containing strings $(I \text{ or } Z)YX^{\otimes n}Y(I \text{ or } Z)$ with n odd. A site-invariant protocol with unlimited-distance communication can make appropriate sign flips. Since these measurements have no certain submeasurements, the protocol gets everything right.

Nonetheless, *any* protocol with limited-distance communication, site-invariant or not, fails for some graphs; thus for a protocol to be successful for all graphs, it (i) must not be site invariant and (ii) must have unlimited-distance communication.

Graph states: Getting it all right

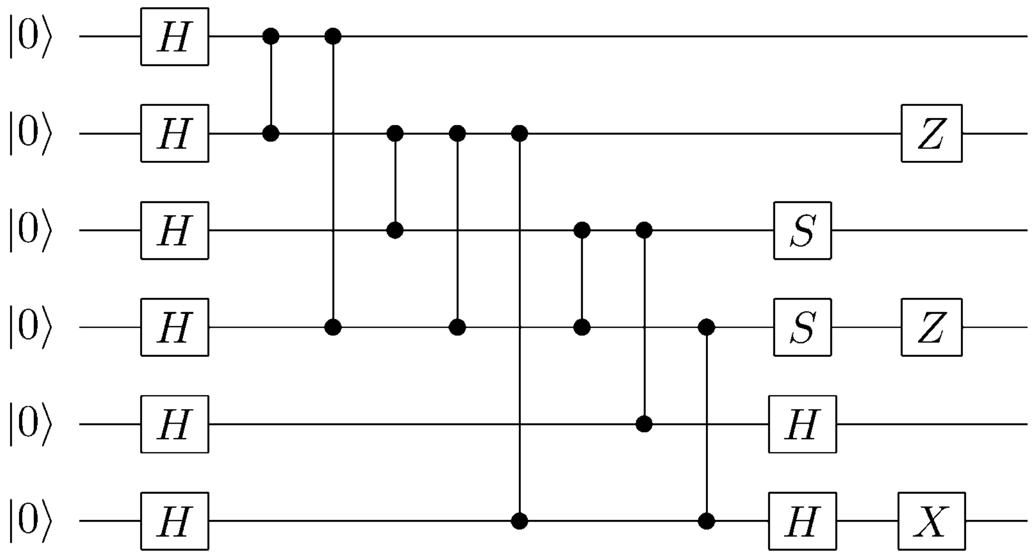
1. **Select a special qubit that knows the adjacency matrix of the graph.**
2. **Each qubit tells the special qubit if it measures *X* or *Y*.**
3. **From the adjacency matrix, the special qubit calculates a generating set of certain submeasurements (stabilizer elements), each of which has a representative qubit that participates in none of the other submeasurements. Since these submeasurements commute term by term, the overall sign for any certain submeasurement is a product of the signs for the participating submeasurements.**
4. **The special qubit tells each of the representative qubits whether to flip the sign of its table entry.**

Non-site-invariant, unlimited-distance communication protocol

M. B. Elliott, B. Eastin, and C. M. Caves, "Local-hidden-variables models assisted by classical communication for stabilizer states," in preparation.

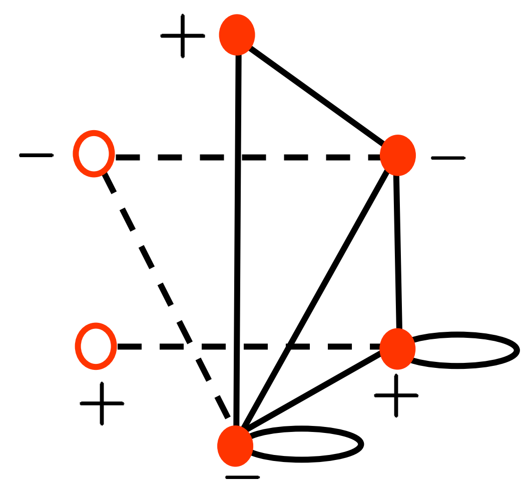
Stabilizer states

$ZIIIII$
$IZIIII$
$IIZIII$
$IIIZII$
$IIIIZI$
$IIIIIZ$



$$g_1 = XZIZII$$
$$g_2 = -ZXZZIX$$
$$g_3 = IZYZXI$$
$$g_4 = -ZZZYIX$$
$$g_5 = IIZIZI$$
$$g_6 = -IZIZIZ$$

x	y	z
z_2z_4	$z_1z_2z_4$	z_1
$-z_1z_3z_4z_6$	$-z_1z_2z_3z_4z_6$	z_2
$-z_2z_3z_4z_5$	$z_2z_4z_5$	z_3
$z_1z_2z_3z_4z_6$	$-z_1z_2z_3z_6$	z_4
z_5	$-z_3z_5$	z_3
z_6	$z_2z_4z_6$	$-z_2z_4$

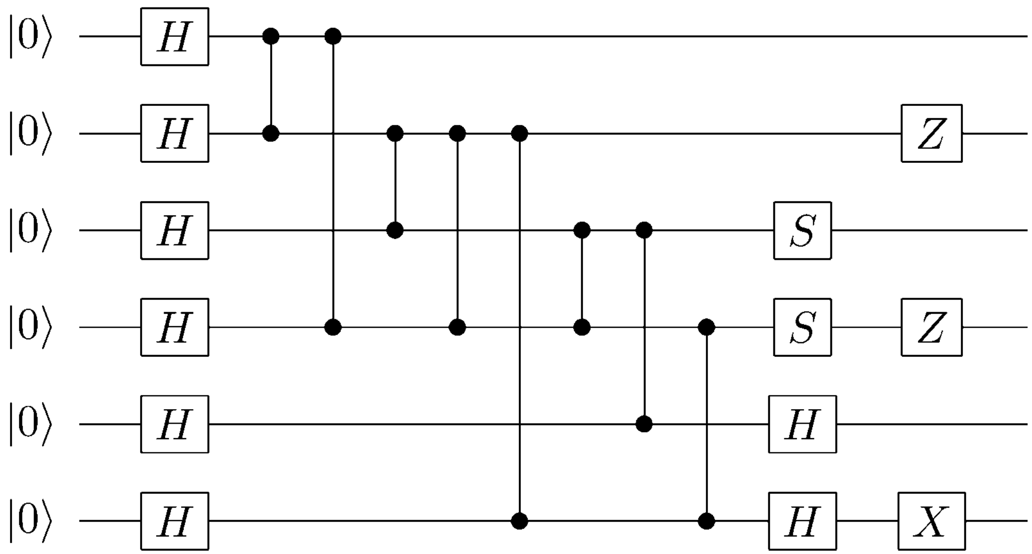


Generalized graph

A non-site-invariant, unlimited-distance protocol like that for graph states, based on the adjacency matrix of the qubits connected by solid lines, gets everything right.

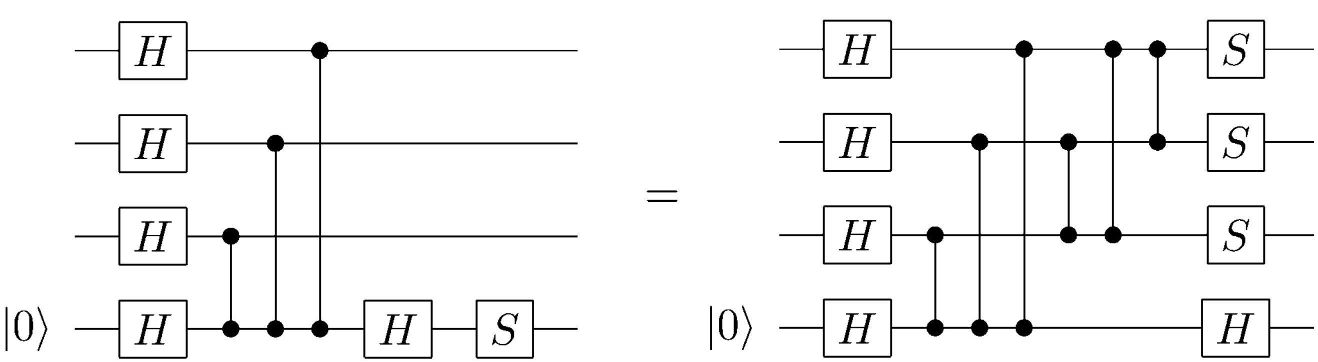
Stabilizer states

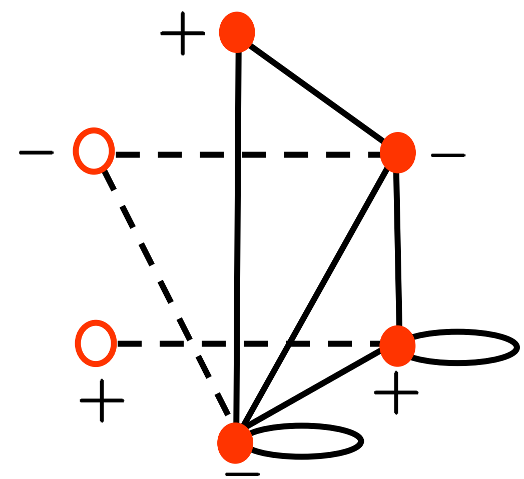
$$\begin{matrix} ZIIIII \\ IZIIII \\ IIZIII \\ IIIZII \\ IIIIZI \\ IIIIIZ \end{matrix}$$



$$\begin{aligned} g_1 &= X\,Z\,I\,Z\,I\,I \\ g_2 &= -Z\,X\,Z\,Z\,I\,X \\ g_3 &= I\,Z\,Y\,Z\,X\,I \\ g_4 &= -Z\,Z\,Z\,Y\,I\,X \\ g_5 &= I\,I\,Z\,I\,Z\,I \\ g_6 &= -I\,Z\,I\,Z\,I\,Z \end{aligned}$$

Simulation of Clifford circuits leads to local-complementation rules for generalized graphs subjected to Clifford gates, which can be expressed as powerful circuit identities.

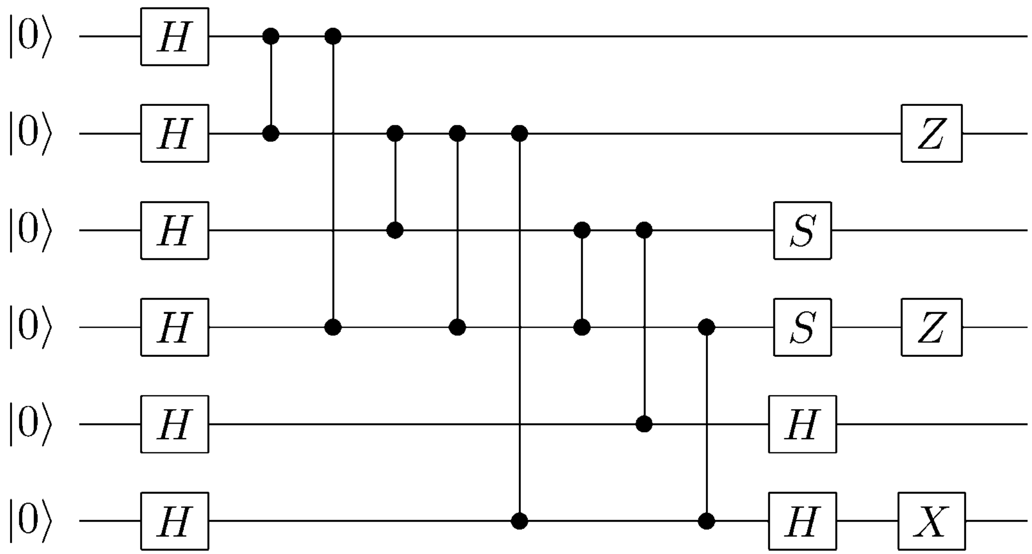




Generalized graph

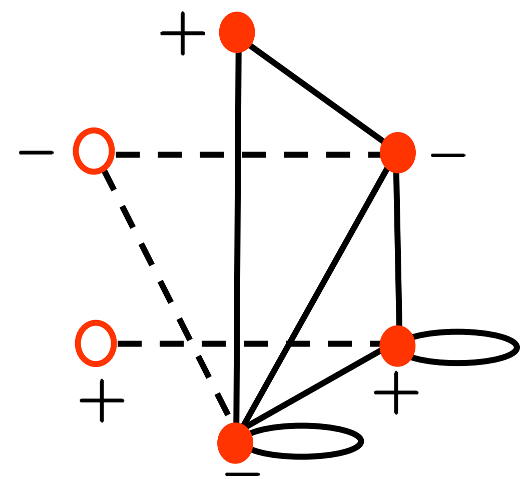
Stabilizer states

$$\begin{matrix} ZIIIII \\ IZIIII \\ IIZIII \\ IIIZII \\ IIIIZI \\ IIIIIZ \end{matrix}$$

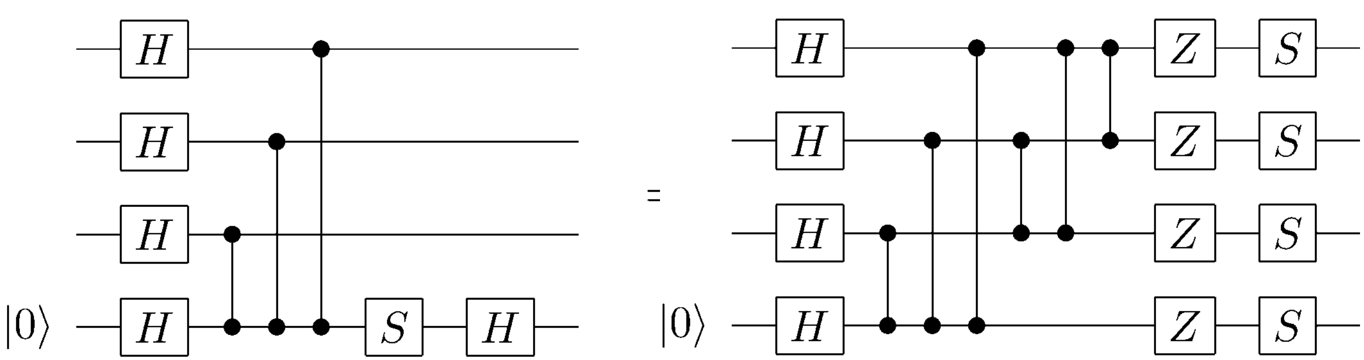


$$\begin{aligned} g_1 &= XZIZII \\ g_2 &= -ZXZZIX \\ g_3 &= IZYZXI \\ g_4 &= -ZZZYIX \\ g_5 &= IIZIZI \\ g_6 &= -IZIZIZ \end{aligned}$$

Simulation of Clifford circuits leads to local-complementation rules for generalized graphs subjected to Clifford gates, which can be expressed as powerful circuit identities.



Generalized graph



Clifford circuits: Gottesman-Knill theorem

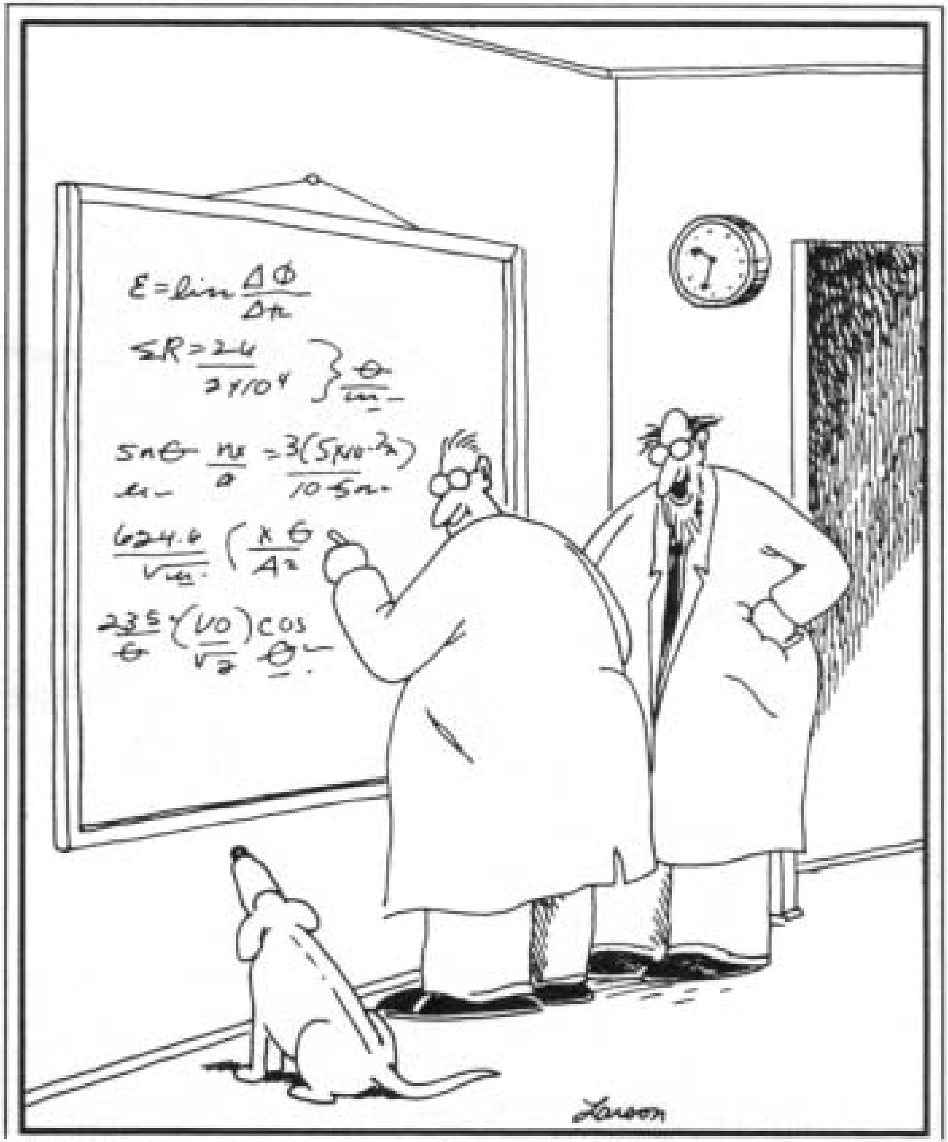
- N qubits in an initial product state in Z basis
- Allowed gates: Pauli operators X, Y, and Z, plus H, S, and C-NOT
- Allowed measurements: Products of Pauli operators

Global entanglement

but

Efficient (nonlocal) realistic description of states, dynamics, and measurements (in terms of stabilizer generators)

This kind of global entanglement, when measurements are restricted to the Pauli group, can be simulated efficiently because it can be described efficiently by local hidden variables assisted by classical communication.



"Ohhhhhh…Look at that, Schuster…Dogs are so cute when they try to comprehend quantum mechanics."

The problem is that it's not just dogs, so …

Quantum information science is the discipline that explores information processing within the quantum context where the mundane constraints of realism and determinism no longer apply. What better way could there be to explore the foundations of quantum mechanics?