Quantum control and feedback: A circuit-based perspective

I. Is there a problem?
II. Measurement-based and coherent control
III. ~~True quantum~~ Noncommutative control and feedback

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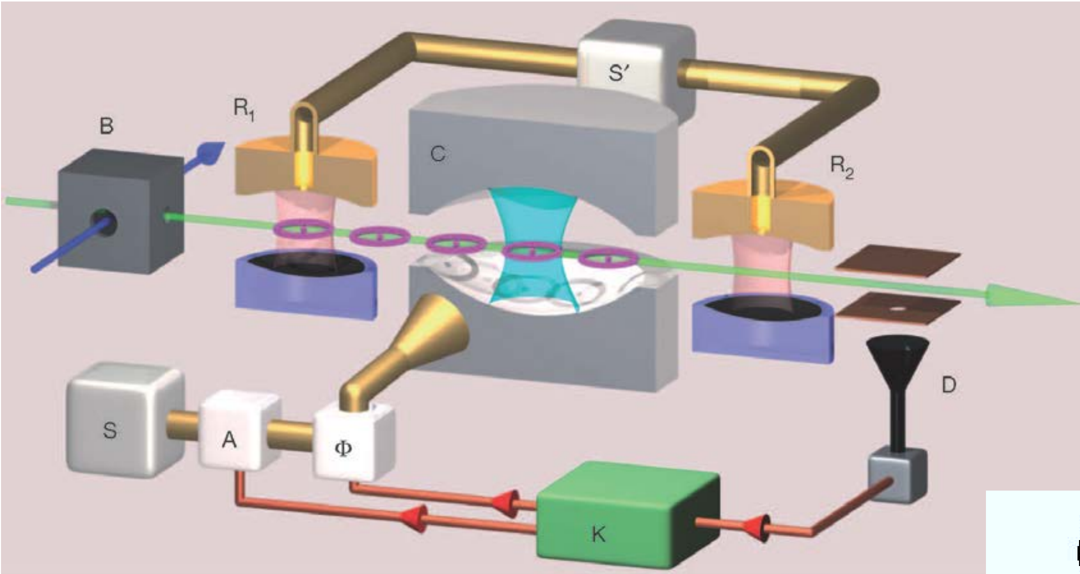
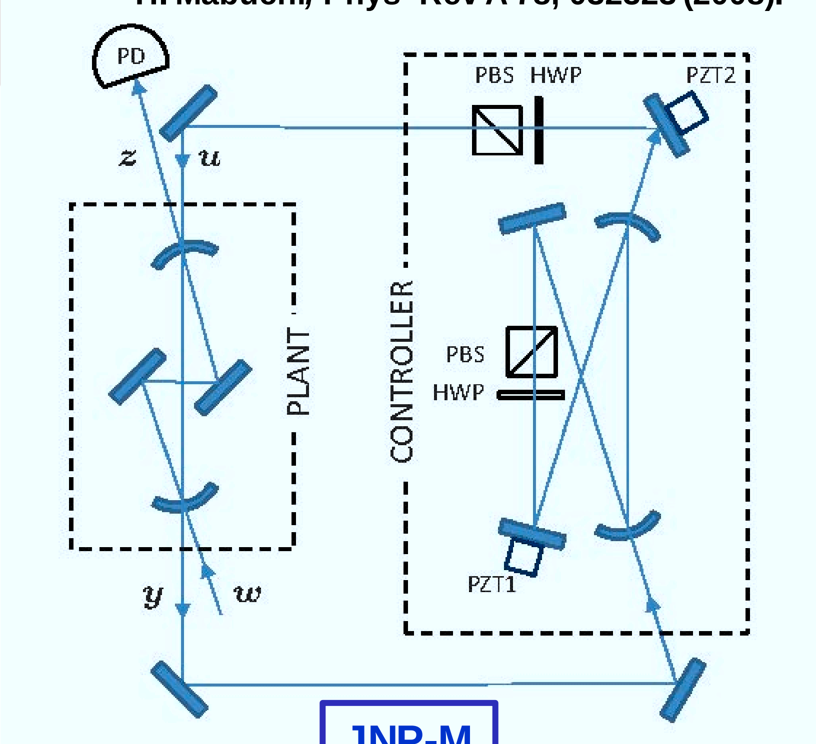
I. Is there a problem?

Examples and questions



Haroche lab: C. Sayrin *et al.,* Nature 477, 73 (2011).

M. R. James, H. I. Nurdin, and I. R.~Petersen, IEEE Trans. Auto. Control 53, 1787 (2008).
H. Mabuchi, Phys Rev A 78, 032323 (2008).



Feedforward? Feedback? Measurement-based control? Coherent control? Quantum control?

JNP-M

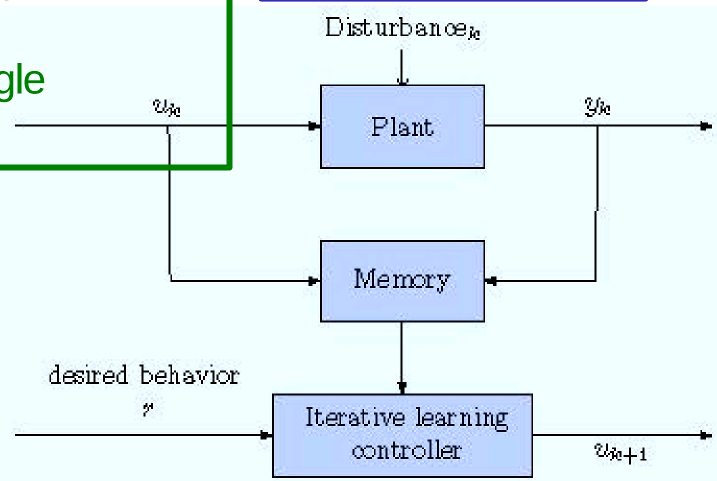
Classical control and feedback

Classical control diagrams *do not* (quantum circuit diagrams *do*)

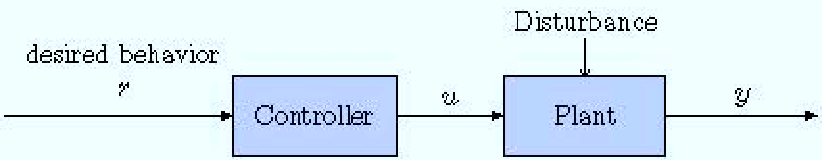
1. Cleanly distinguish systems from their interactions.
2. Display both temporal (causal) and spatial (subsystem) relations.

A quantum circuit displays the interactions between the single degrees of freedom that process quantum information.

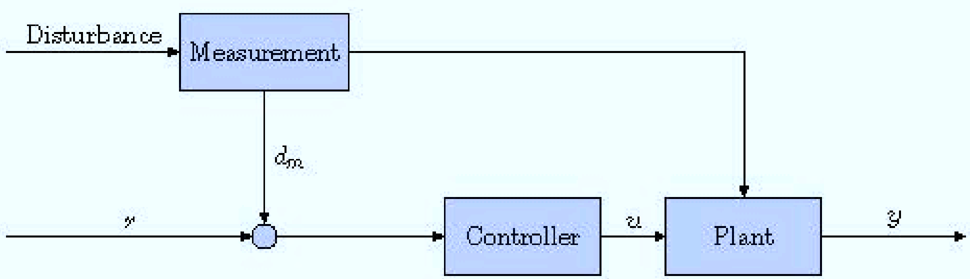
Iterative learning



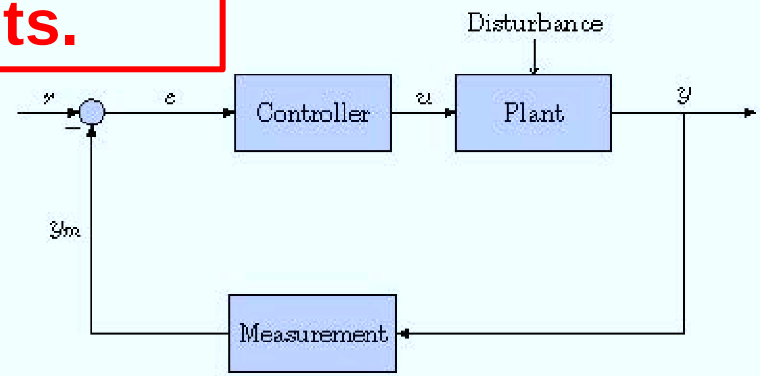
Open-loop control



Michael Nielsen (c. 1998): Carl, you should learn how to use quantum circuits.



Feedforward



Feedback control

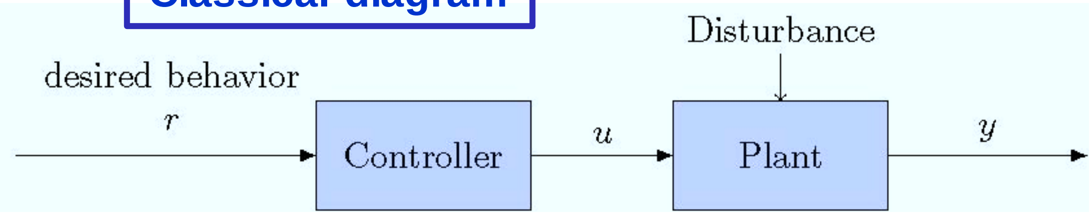
Messages

1. **All measurement-based control and feedback can be converted to coherent quantum control.**
2. **Not all coherent quantum control can be converted to measurement-based control: control on *noncommuting* observables cannot be so converted and is something different.**
3. **Quantum feedback is distinguished from feedforward by the presence in a quantum circuit of interfering quantum paths that begin and end on the plant.**

II. Measurement-based and coherent control

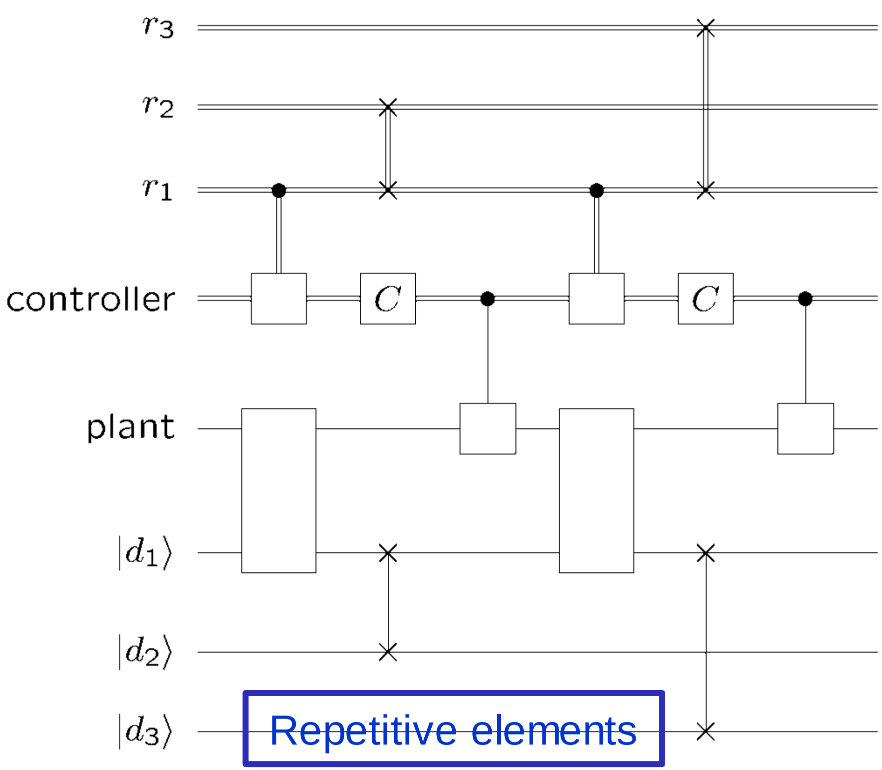
Open-loop control

Classical diagram



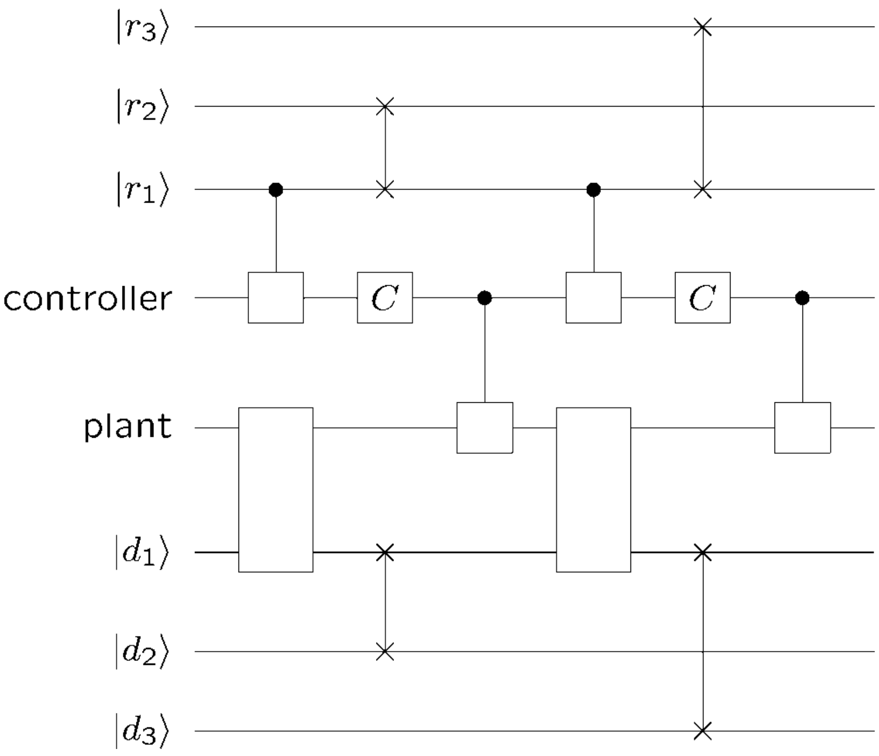
Plant: persisting quantum system
Controller: classical information processor
Desired behavior: successive inputs
Disturbances: successive quantum systems

Quantum circuit



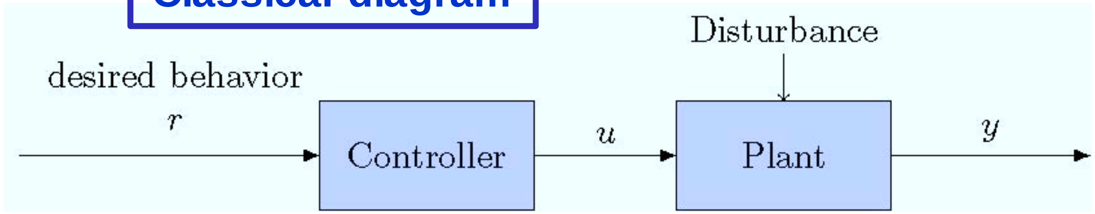
Repetitive elements

Coherent version

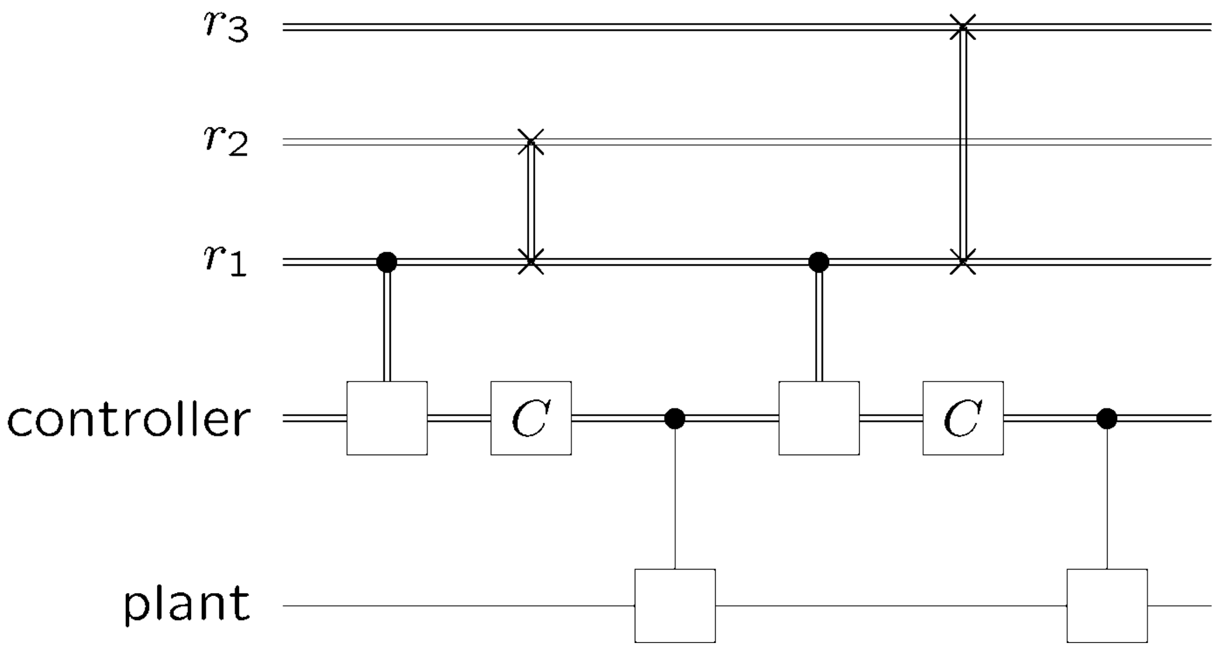


Open-loop control

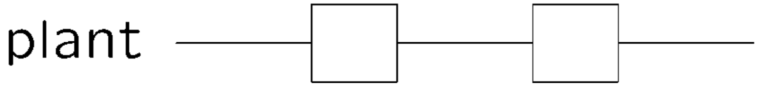
Classical diagram



Quantum circuit (disturbances omitted)



Quantum circuits generally omit classical open-loop controls, so this circuit reduces to

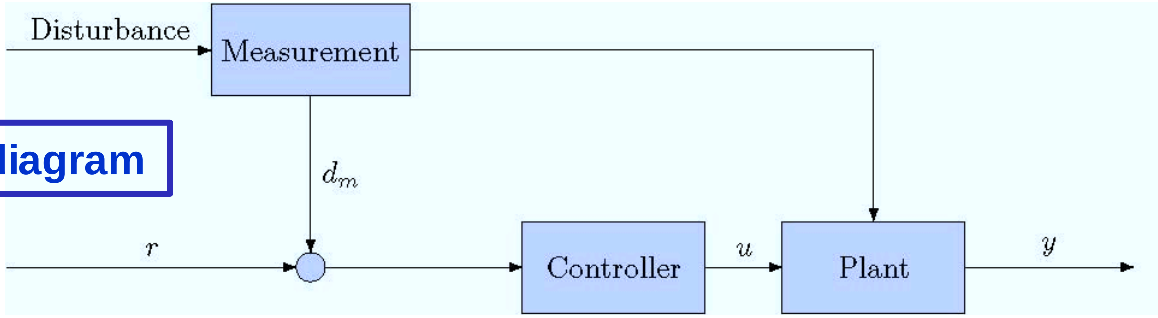


Objective: synthesize plant state or unitary or quantum operation

Measures of efficacy: efficiency, robustness

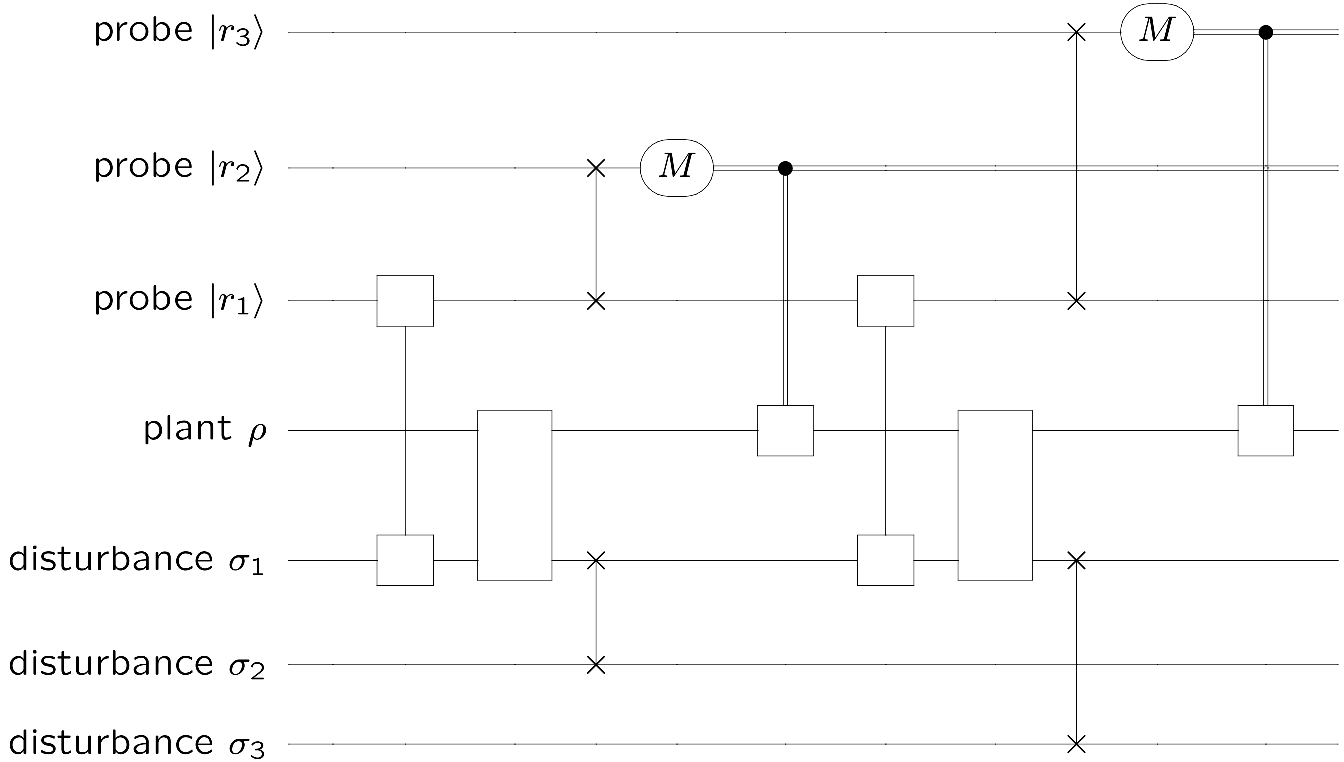
Feedforward

Classical diagram



Plant: atoms, optical or microwave cavity, mechanical oscillator
Probes and disturbances: successive quantum systems that interact with one another and with the plant, e.g., field modes, atoms, qubits, qudits (Markovicity?)
Controller: omitted

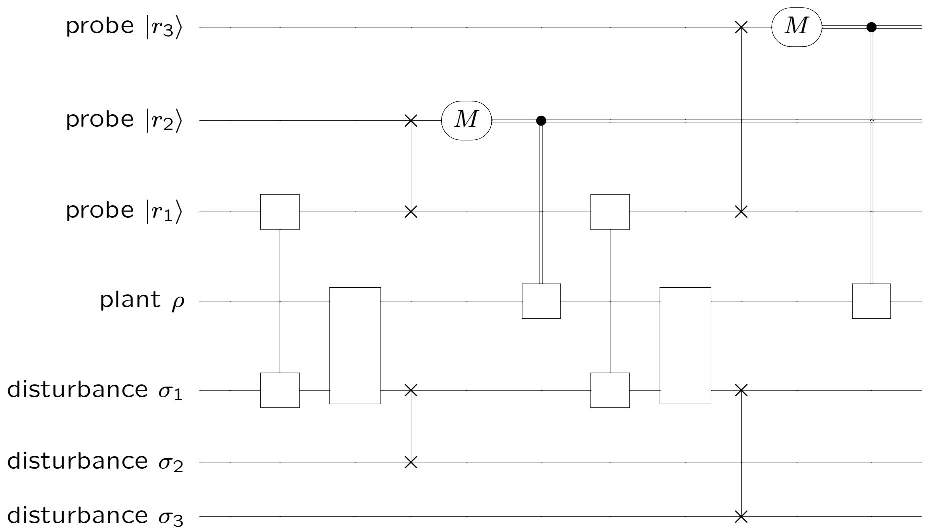
Measurement-based quantum circuit



Feedforward

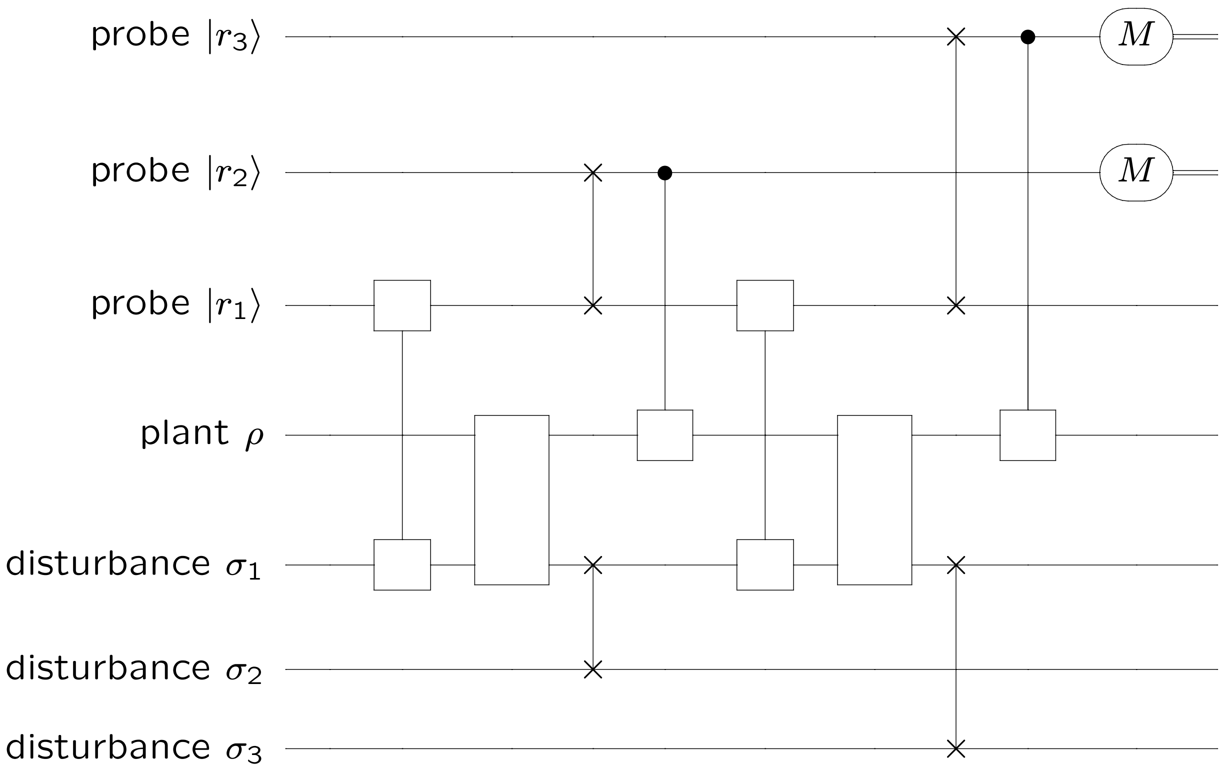
Use the ***Principle of Deferred Measurement*** to push the measurements through the associated controls to the end of the circuit, where they can be omitted.

R. B. Griffiths and C.-S. Niu, PRL 76, 3228 (1996).



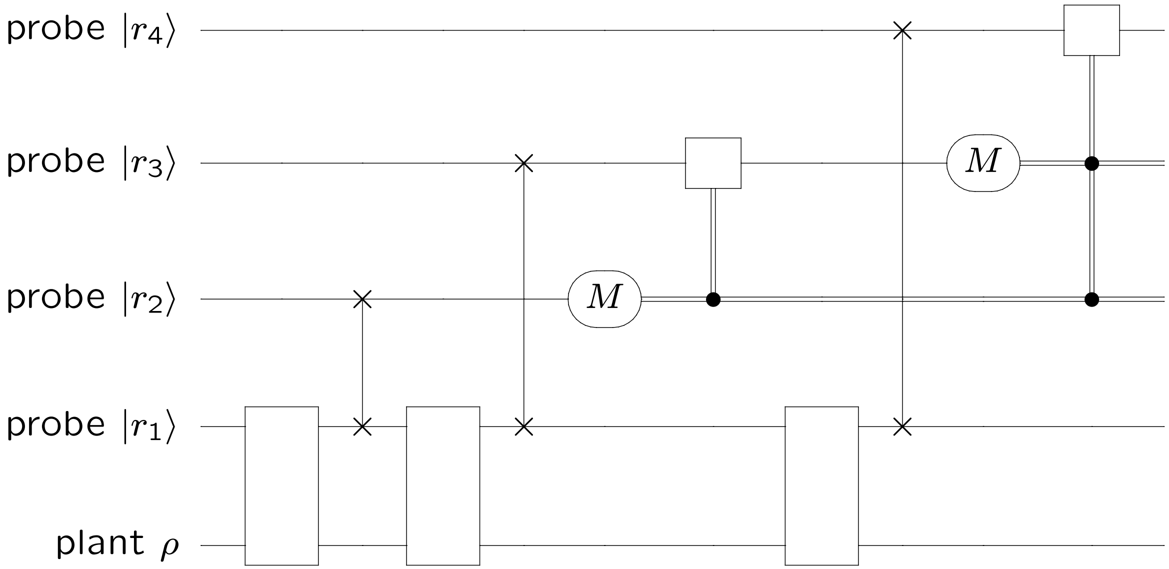
Measurement-based quantum circuit

Coherent version



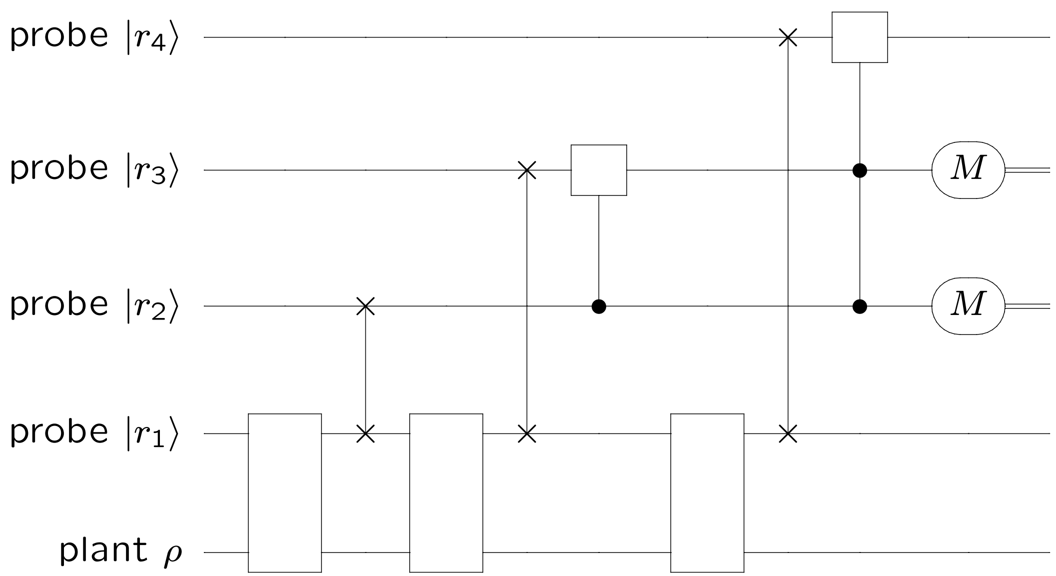
No feedback onto plant: no multiple (interfering quantum) paths that begin and end on the plant.

Feedforward onto probes



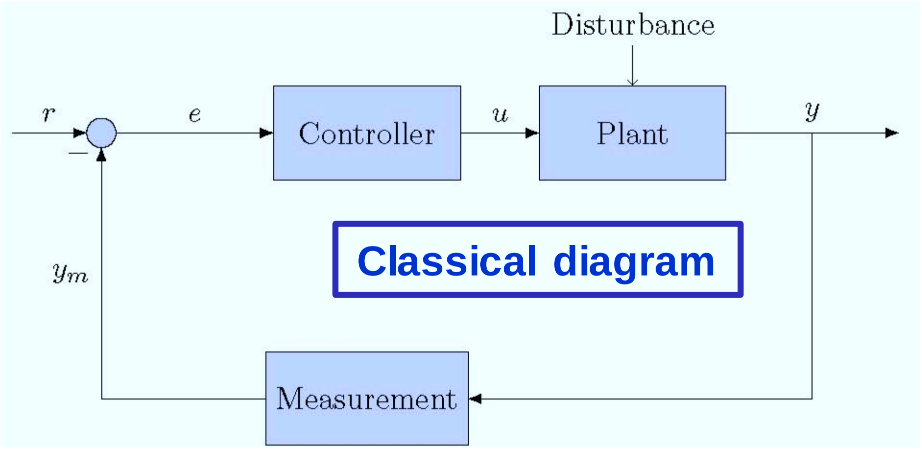
Measurement-based quantum circuit

Coherent version

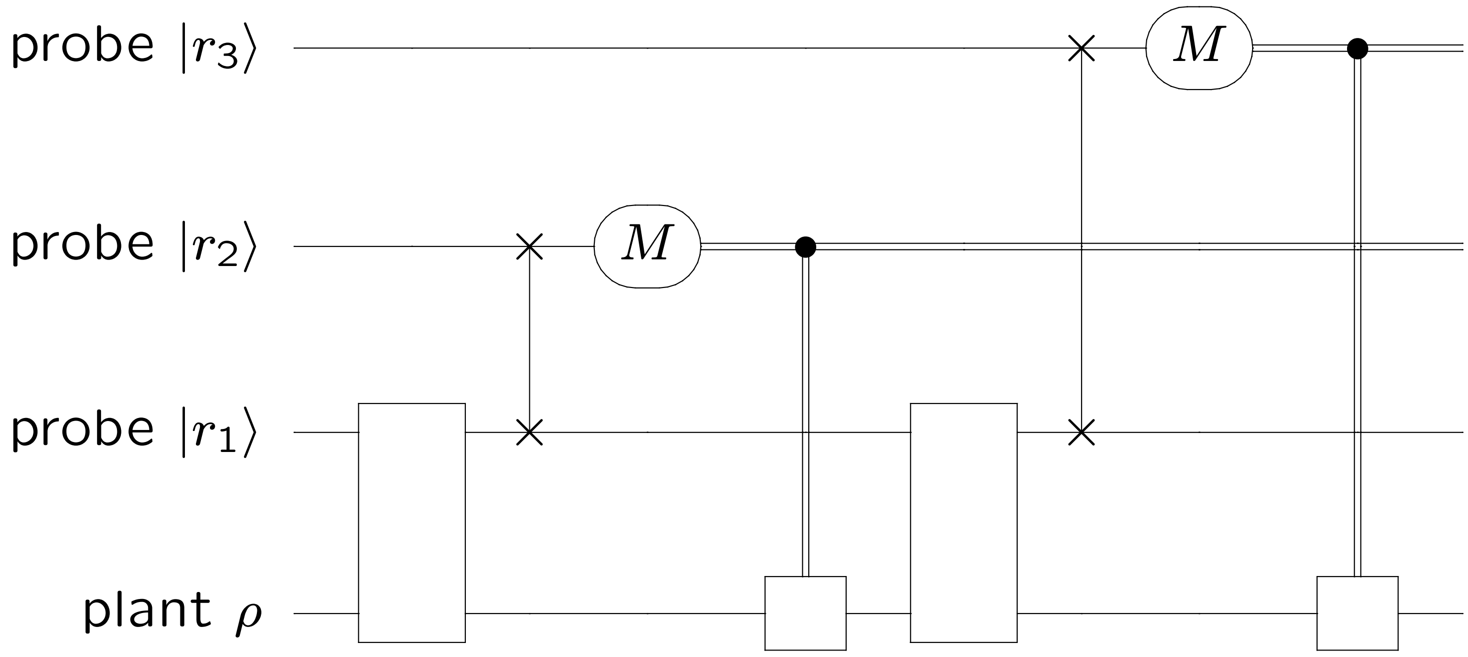


Direct feedback

Plant: atoms, optical or microwave cavity, mechanical oscillator
Probes: successive quantum systems that interact with one another and the plant, e.g., field modes, atoms, qubits, qudits
Disturbances and controller: omitted



Measurement-based quantum circuit



Direct feedback

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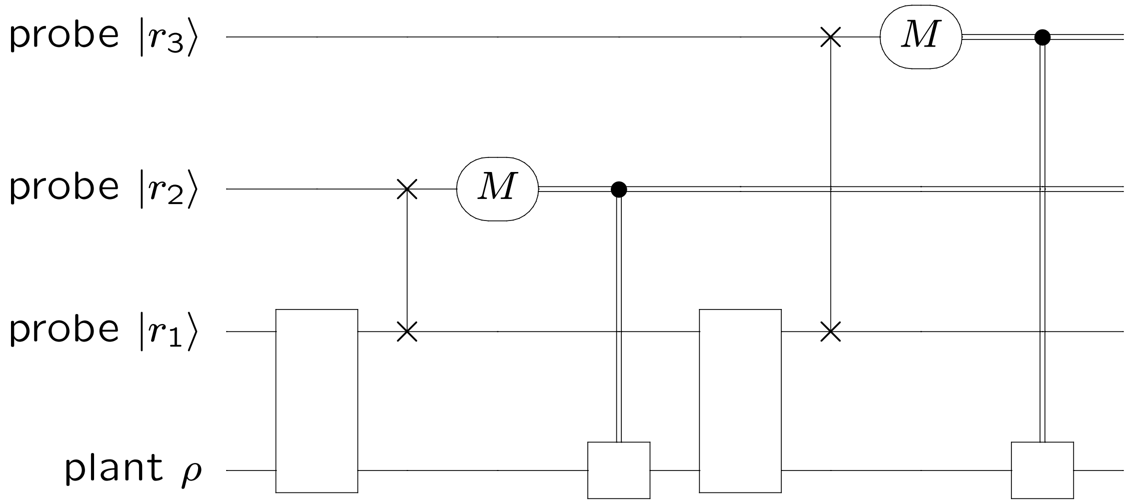
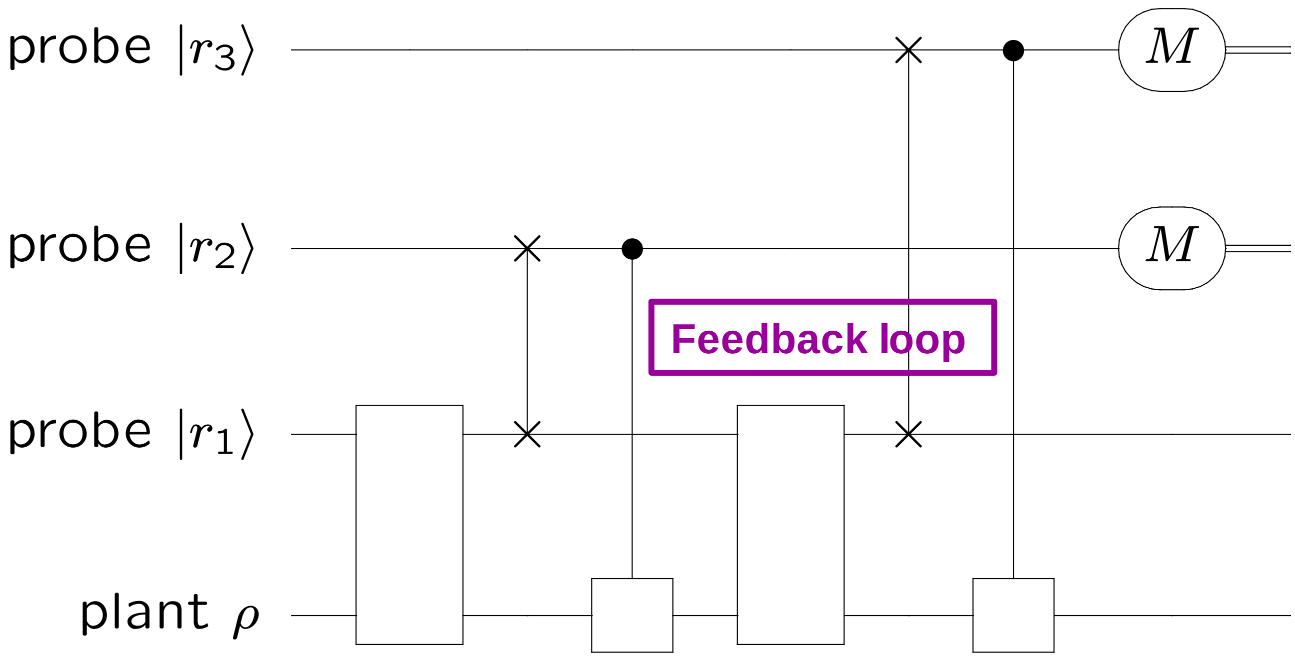


Measurement-based quantum circuit

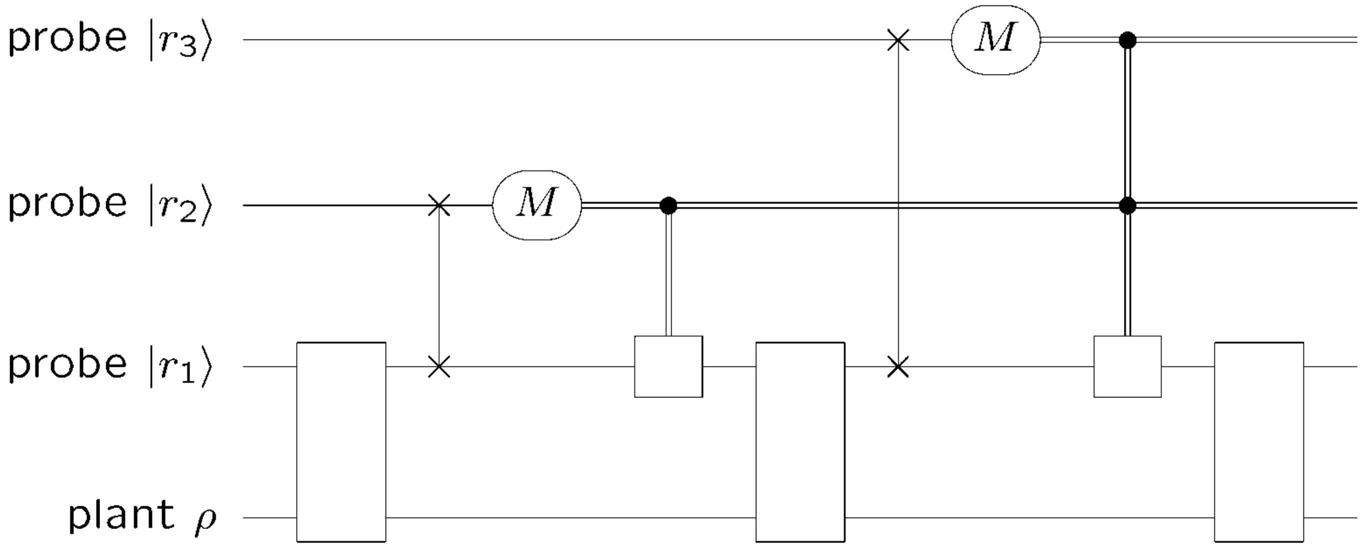
Coherent version



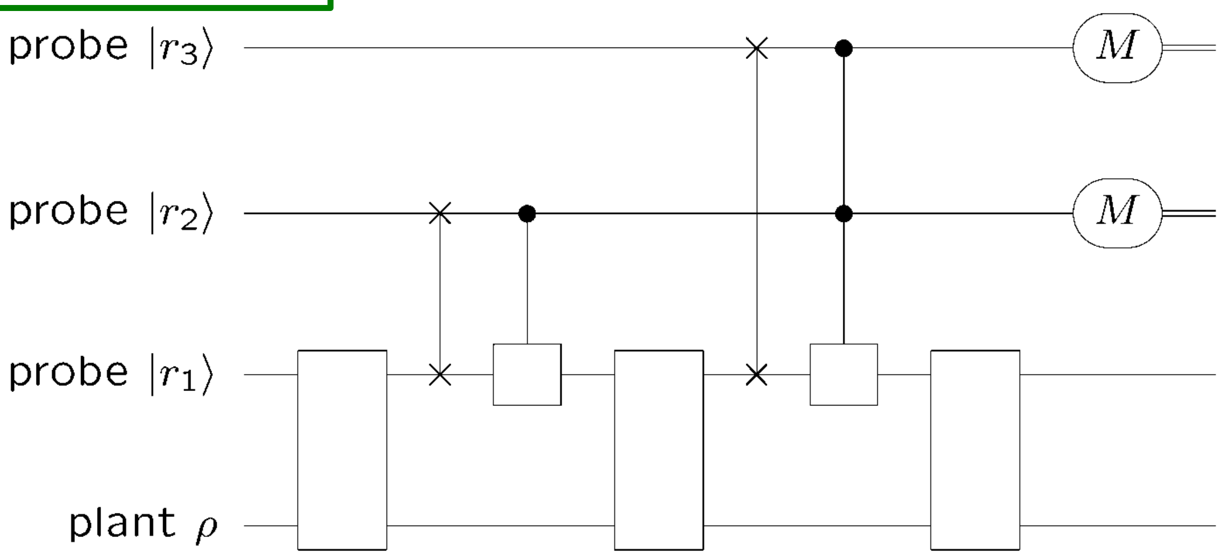
Feedback: multiple (interfering quantum) paths that begin and end on the plant.

Indirect feedback

Measurement-based quantum circuit



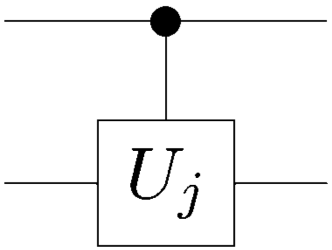
Coherent version



Feedback: multiple (interfering quantum) paths that begin and end on the plant.

III. ~~True quantum~~ Noncommutative control and feedback

Controlled unitaries

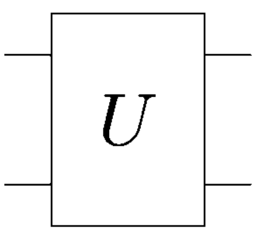


$$\begin{aligned} H &= \sum_j |e_j\rangle\langle e_j| \otimes B_j = \sum_j P_j \otimes B_j \\ U &= e^{-iH} = \sum_j P_j \otimes e^{-iB_j} = \sum_j P_j \otimes U_j \end{aligned}$$

Basis $|e_j\rangle$ controls target unitaries U_j. This sort of unitary can be moved through a measurement in the control basis $|e_j\rangle$.

But what about

Noncommuting (say, mutually unbiased, or conjugate) bases

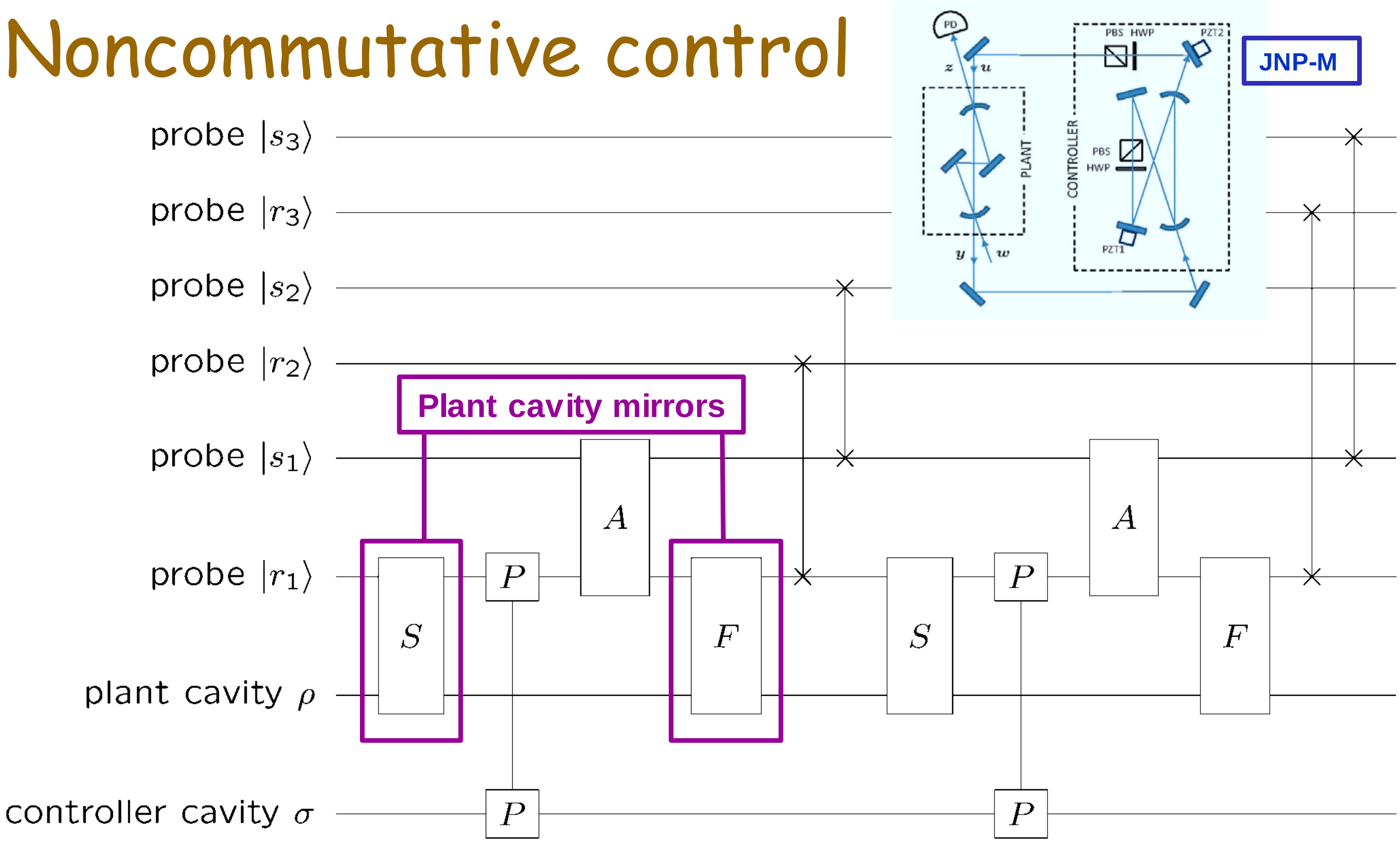


$$\begin{aligned} H &= \sum_j P_j \otimes B_j + Q_j \otimes C_j \\ U &= e^{-iH} \end{aligned}$$

This unitary might be thought of as simultaneous control in two incompatible bases. It cannot be moved through a subsequent projective measurement.

H. M. Wiseman and G. J. Milburn, PRA 49, 4110 (1994).

Noncommutative control



Mirror interaction

$$
\begin{aligned}
U &= e^{\mu[a^\dagger(C+iB)-a(C-iB)]/\sqrt{2}} \\
&= e^{i\mu(xB-pC)}
\end{aligned}
$$

Noncommutative control on x and p

Noncommutative control

Mirror interaction

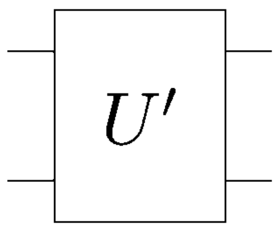
$$U = e^{\mu[a^\dagger(C+iB)-a(C-iB)]/\sqrt{2}}$$
$$= e^{i\mu(xB-pC)}$$

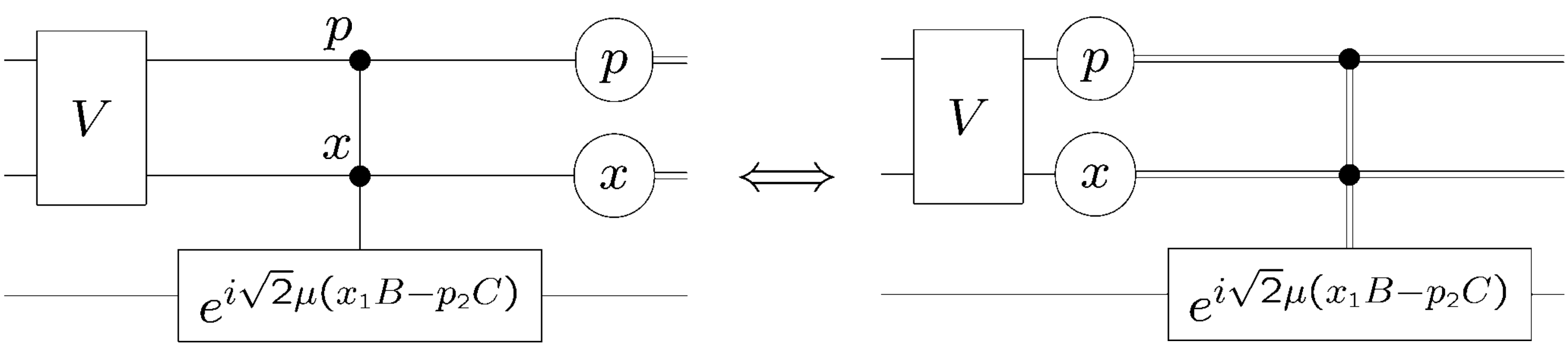
Noncommutative control on x and p

Inequivalent commuting control

$$U' = e^{i\mu[(x_1+x_2)B-(p_1-p_2)C]}$$
$$= V^\dagger e^{i\sqrt{2}\mu(x_1B-p_2C)}V$$

Commutative control on x_1 and p_2





That’s all, folks!
Thanks for your attention.

Western diamondback rattlesnake
Sandia Heights, New Mexico

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