Everything I know I learned by doing physics homework problems

I. Tips for solving physics homework problems
II. How long does something last? Predicting future duration from present age
III. Temporal Copernican principle and Gott's rule
IV. Doom and Bayesian inference
V. Parting shots

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http://info.phys.unm.edu/~caves

C. M. Caves, "Predicting future duration from present age: A critical assessment," Contemporary Physics **41**, 143 (2000).

C. M. Caves, "Predicting future duration from present age: Revisiting a critical assessment of Gott's rule, arXiv:0806.3538 [astro-ph].

I. Tips for solving physics homework problems

Oljeto Wash
Southern Utah

Tips for solving physics homework problems

http://info.phys.unm.edu/~caves/courses/probtech.pdf

Getting started

- **Try to develop a picture—something you can visualize—that captures the essence of the problem.**
- **Use symmetries to simplify your work.**
- **Identify the important scales—i.e., the important quantities with dimensions—before you start.**
- Reason backward from the answer you are asked to supply to the concepts and information you need.
- **Guess the answer, using any technique at your disposal, before you begin.**
- If you don't need to know the answer exactly (you never really do), think about whether an easier approximate technique can be used.
- Before using advanced techniques, think whether the problem—or at least part of the problem—can be solved using elementary methods.
- **Pause before you start work on a problem to try to think of a clever way to do it.**

Tips for solving physics homework problems

http://info.phys.unm.edu/~caves/courses/probtech.pdf

Doing the problem

- Don't put in numerical values till the end.
- Always use vector signs on vectors.

Checking your answer

- **Test your solution in any limiting case where you know the right answer from other considerations.**
- **Do the problem in two or more independent ways.**
- **Check that your answer has the proper units.**
- **Think about your answer critically, to see if it makes sense in light of other things you know.**

Real-life problems are often hard because they are poorly formulated and nobody knows the answer.   It is even more important to know how to formulate the problem and to check your answer critically. Thinking critically about what you do is the foundation of scientific integrity.

II. How long does something last? Predicting future duration from present age

Cape Hauy
Tasman Peninsula

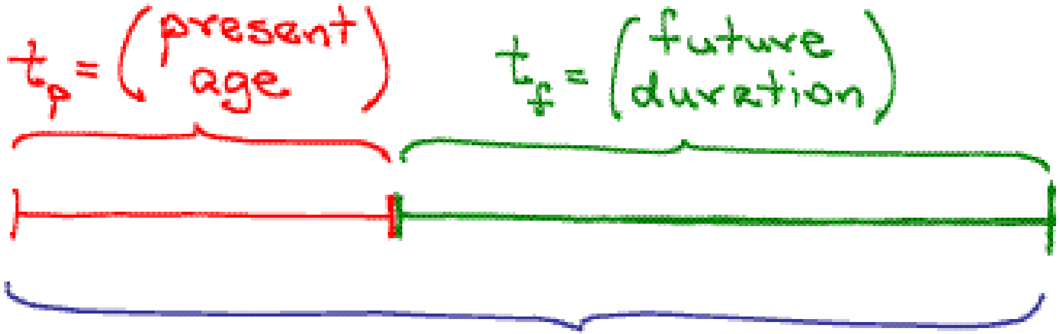
Predicting future duration from present age

You observe a *phenomenon* some time after it begins. How long do you predict it will last?

Phenomenon:

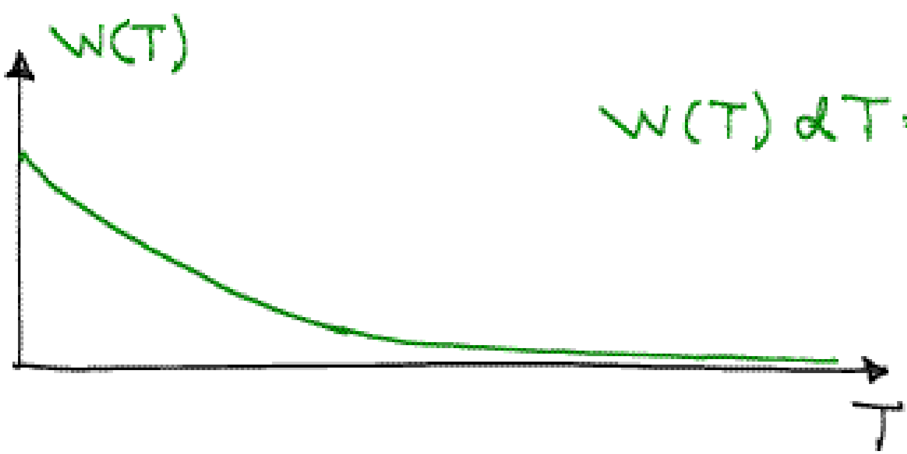
- Decaying atom
- Egg timer set for 10 minutes
- Person
- *Homo sapiens*

Predicting future duration from present age



$$T = (\text{total duration}) = t_p + t_f$$

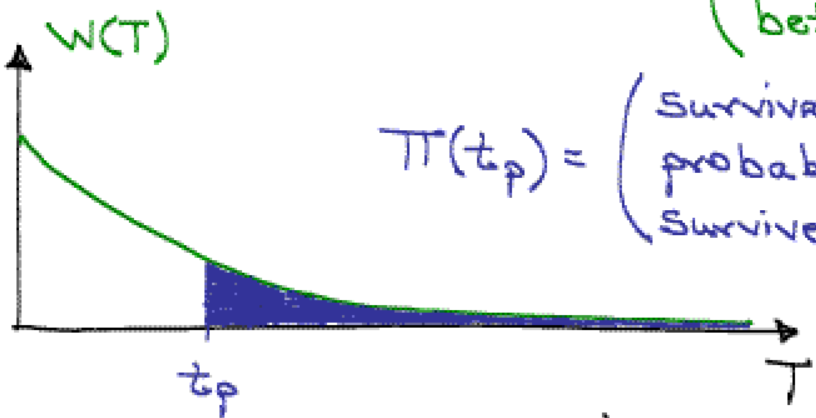
Try to develop a picture—something you can visualize—that captures the essence of the problem.



$$W(T)\,dT = \left(\begin{array}{l}\text{probability that the}\\ \text{phenomenon lasts a time}\\ \text{between } T \text{ and } T + dT\end{array}\right)$$

Predicting future duration from present age

$$w(T)\,dT = \left(\begin{array}{l}\text{probability that the}\\ \text{phenomenon lasts a time}\\ \text{between } T \text{ and } T+dT\end{array}\right)$$



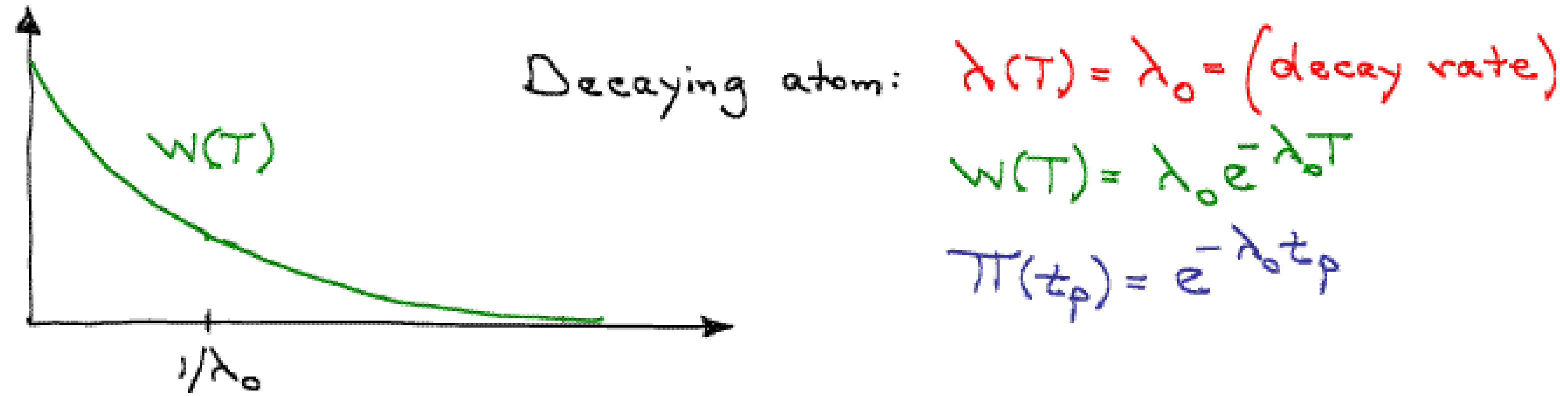
$$\Pi(t_p) = \left(\begin{array}{l}\text{survival probability, i.e.,}\\ \text{probability that phenomenon}\\ \text{survives to age } t_p\end{array}\right) = \int_{t_p}^{\infty} dT\, w(T)$$

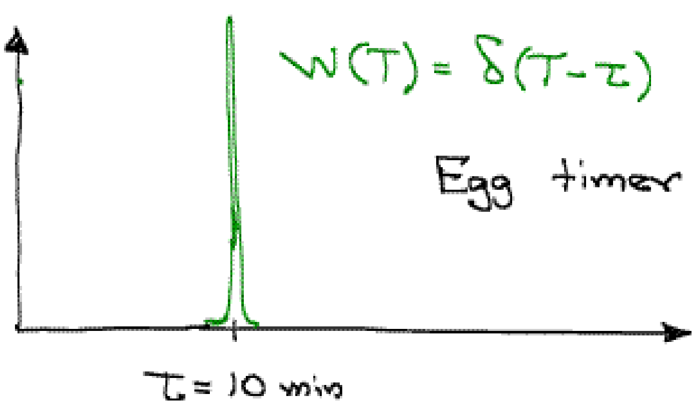
$$\lambda(T) = \left(\begin{array}{l}\text{death rate, i.e., probability}\\ \text{that having survived to } T,\\ \text{phenomenon ends in next } dT\end{array}\right)$$

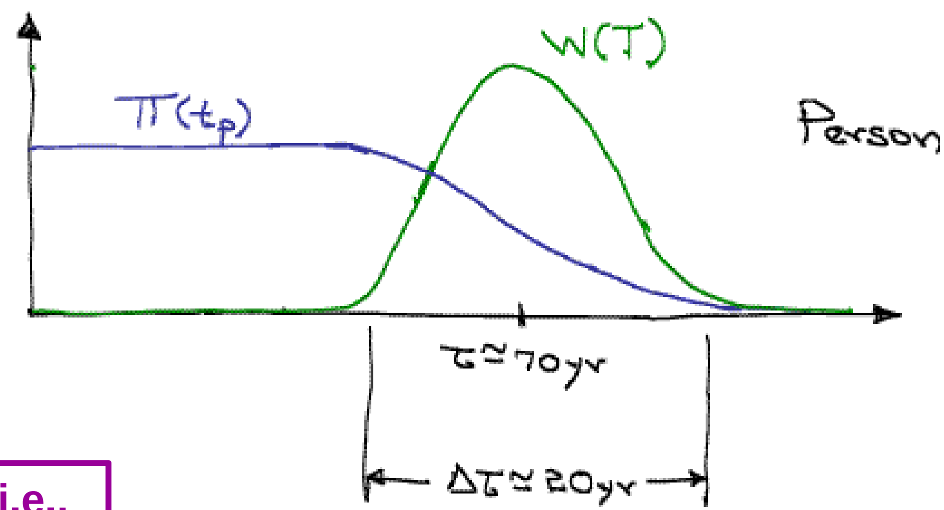
$$\frac{d\Pi}{dT} = -w(T) = -\Pi(T)\,\lambda(T) \Rightarrow \begin{array}{l} w(T) = \lambda(T)\exp\left(-\int_0^T dT'\,\lambda(T')\right) \\ \Pi(t_p) = \exp\left(-\int_0^{t_p} dT\,\lambda(T)\right)\end{array}$$

Check that your answer has the proper units.

Predicting future duration from present age



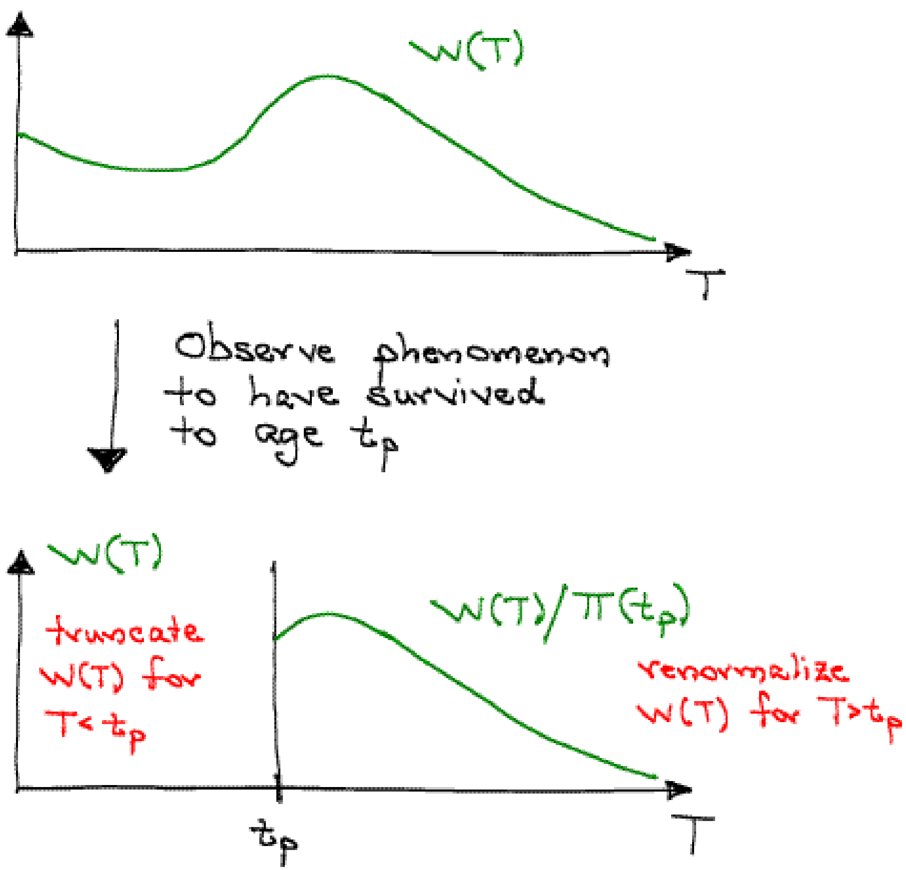




Identify the important scales—i.e., the important quantities with dimensions—before you start.

Predicting future duration from present age

Guess the answer, using any technique at your disposal, before you begin.



Bayes's theorem

$$p(A|B)p(B) = p(A,B) = p(B|A)p(A)$$

Two ways of writing a joint probability

- Let O denote that you observe the phenomenon to have survived to age t_p. The *a priori* probability for O to occur is

$$P(O) = \Pi(t_p)\ ,$$

and the probability for O to occur given total duration T is

$$P(O|T) = \Theta(T - t_p) = \begin{cases} 1\ , & \text{if } t_p < T, \\ 0\ , & \text{if } t_p > T. \end{cases}$$

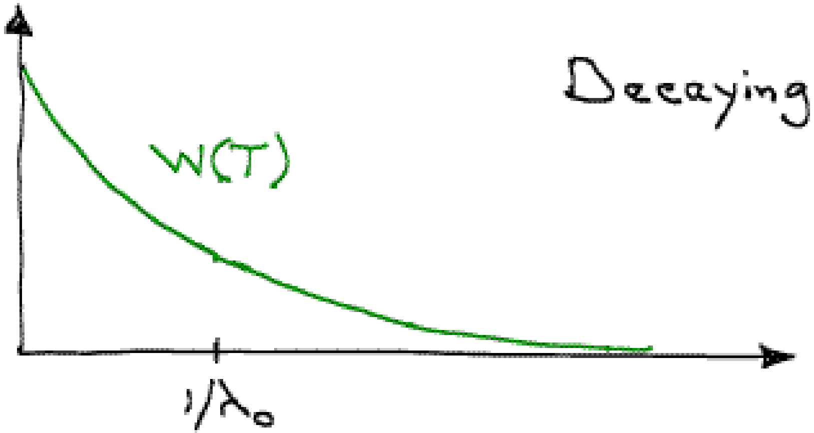
- *Bayes's theorem* says that

$$w(T|O)P(O) = P(O|T)w(T)\ ,$$

which gives

$$w(T|O) = \begin{cases} 0\ , & \text{if } T < t_p, \\ w(T)/\Pi(t_p)\ , & \text{if } T > t_p. \end{cases}$$

Predicting future duration from present age



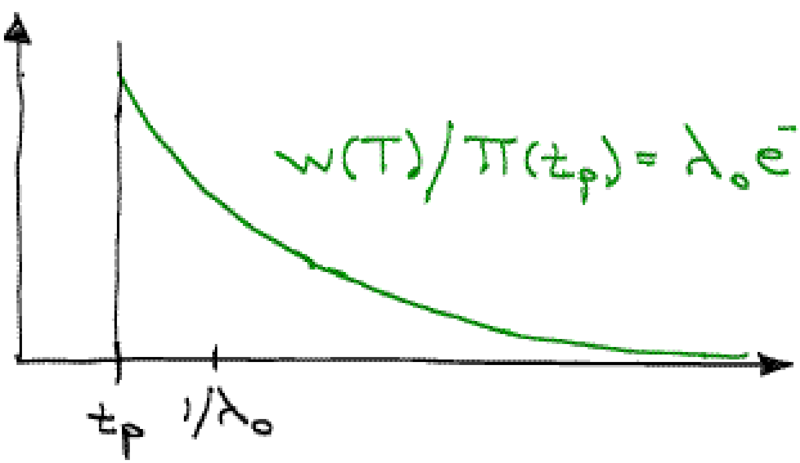
Decaying atom: $\lambda(T) = \lambda_0 = \text{(decay rate)}$

$$w(T) = \lambda_0 e^{-\lambda_0 T}$$

$$\pi(t_p) = e^{-\lambda_0 t_p}$$

Observe atom not to have decayed at age t_p

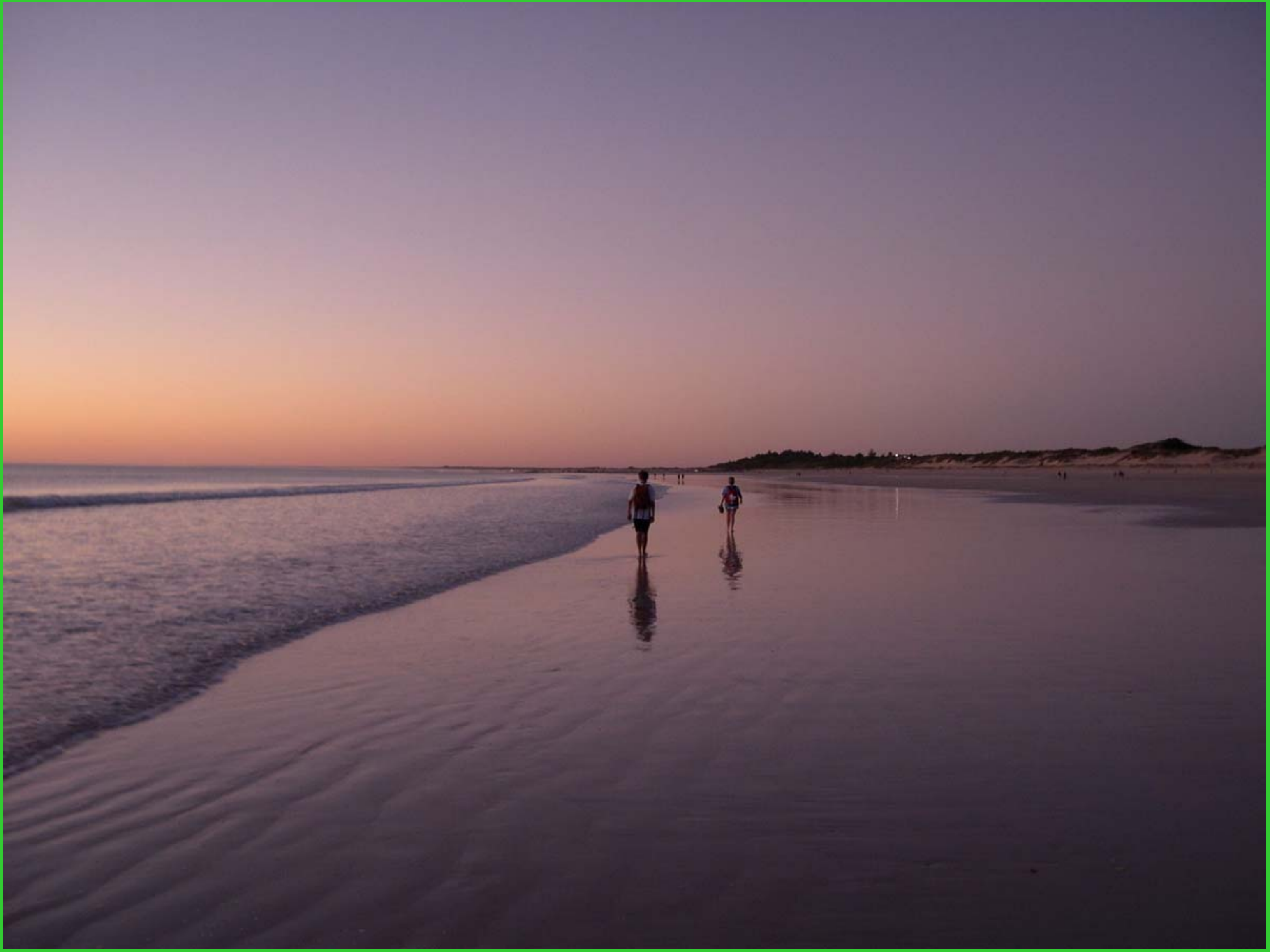
$$w(T)/\pi(t_p) = \lambda_0 e^{-\lambda_0 (T - t_p)}$$



Test your solution in any limiting case where you know the right answer from other considerations.

Think about your answer critically, to see if it makes sense in light of other things you know.

III. Temporal Copernican principle and Gott's rule



Cable Beach
Western Australia

Gott's temporal Copernican principle

Copernican principle:
We are not at a special place.

Temporal Copernican principle:
We are not at a special time.

J. R. Gott III, "Implications of the Copernican principle for our future prospects," Nature **363**, 315 (1993).

Temporal Copernican principle and Gott's rule

Standing at the (Berlin) Wall in 1969, I made the following argument, using the Copernican principle. I said, Well, there's nothing special about the timing of my visit. I'm just travelling—you know, Europe on five dollars a day—and I'm observing the Wall because it happens to be here. My visit is random in time. So if I divide the Wall's total history, from the beginning to the end, into four quarters, and I'm located randomly somewhere in there, there's a fifty-per-cent chance that I'm in the middle two quarters—that means, not in the first quarter and not in the fourth quarter.

Let's suppose that I'm at the beginning of that middle fifty per cent. In that case, one quarter of the Wall's ultimate history has passed and there are three quarters left in the future. In that case, the future's three times as long as the past. On the other hand, if I'm at the other end, then three quarters have happened already, and there's one quarter left in the future. In that case, the future is one-third as long as the past. …

(The Wall was) eight years (old in 1969). So I said to a friend, "There's a fifty-per-cent chance that the Wall's future duration will be between (two and) two-thirds of a year and twenty-four years." Twenty years later, in 1989, the Wall came down, within those two limits that I had predicted. I thought, Well, you know, maybe I should write this up.

T. Ferris, "How to predict everything: Has the physicist J. Richard Gott found a way?" The New Yorker **75**(18) 35 (1999 July 12).

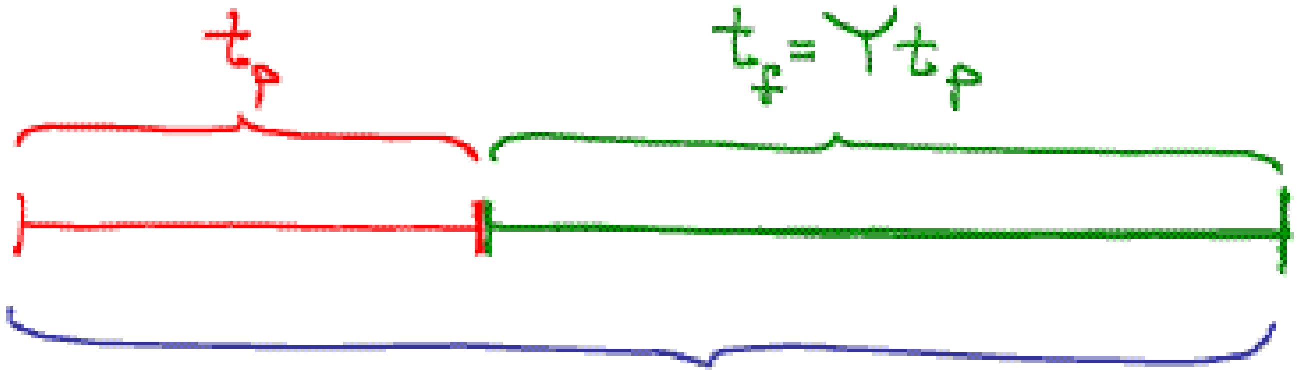
Temporal Copernican principle and Gott's rule

"*Homo sapiens* has been around for two hundred thousand years," Gott said. … "That's how long our past is. Two and half per cent is equal to one-fortieth, so the future is probably at least one-thirty-ninth as long as the past but not more than thirty-nine times the past. If we divide two hundred thousand years by thirty-nine, we get about fifty-one hundred years. If we multiply it by thirty-nine, we get 7.8 million years. So if our location in human history is not special, there's a ninety-five-per-cent chance we're in the middle ninety-five per cent of it. Therefore the human future is probably going to last longer than fifty-one hundred years but less than 7.8 million years.

"Now, those numbers are interesting, because they give us a total longevity that's comparable to that of other species."

T. Ferris, "How to predict everything: Has the physicist J. Richard Gott found a way?" The New Yorker **75**(18) 35 (1999 July 12).

Gott's rule



$$T = t_p + t_f = (1+Y)\,t_p$$

Gott's rule

$$\underbrace{G\big(T \geq (1+Y)\,t_p\big)}_{\text{Gott's probability that } T \geq (1+Y)t_p} = \underbrace{G\big(t_f \geq Y t_p\big)}_{\text{Gott's probability that } t_f \geq Y t_p} = \frac{1}{1+Y}$$

Gott's rule

Gott's rule: $G(T \geq (1+Y)t_p) = G(t_f \geq Y t_p) = \dfrac{1}{1+Y} = \dfrac{t_p}{T}$

$$g(T) = \begin{pmatrix} \text{probability density} \\ \text{for } T \end{pmatrix} = -\frac{dG(T)}{dT} = \begin{cases} 0\,, & T < t_p \\ t_p/T^2\,, & T > t_p \end{cases}$$

Picture? Clever approach? Units?

Cases: Decaying atom? Egg timer? Person?
Scales? Two ways? Think critically?

This is a professor who didn't do his homework. Should we try to figure out where he went wrong?

Himself

Christianity

The former Soviet Union

The Third Reich

The United States

Canada

World leaders

Nature

Wall Street Journal

The New York Times

Stonehenge

The Seven Wonders of the World

The Pantheon

The Great Wall of China

The Berlin Wall

The Astronomical Society of the Pacific

The 44 Broadway and off-Broadway plays open and running on 1993 May 27

Thatcher-Major government in the UK

The New York Stock Exchange

Oxford University

The Internet

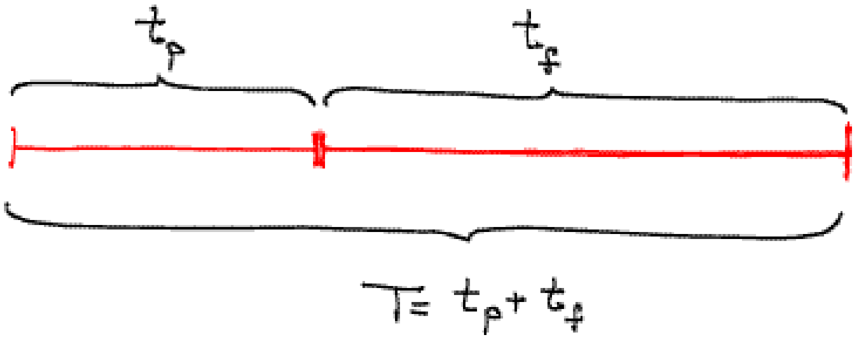
Microsoft

General Motors

The human spaceflight program

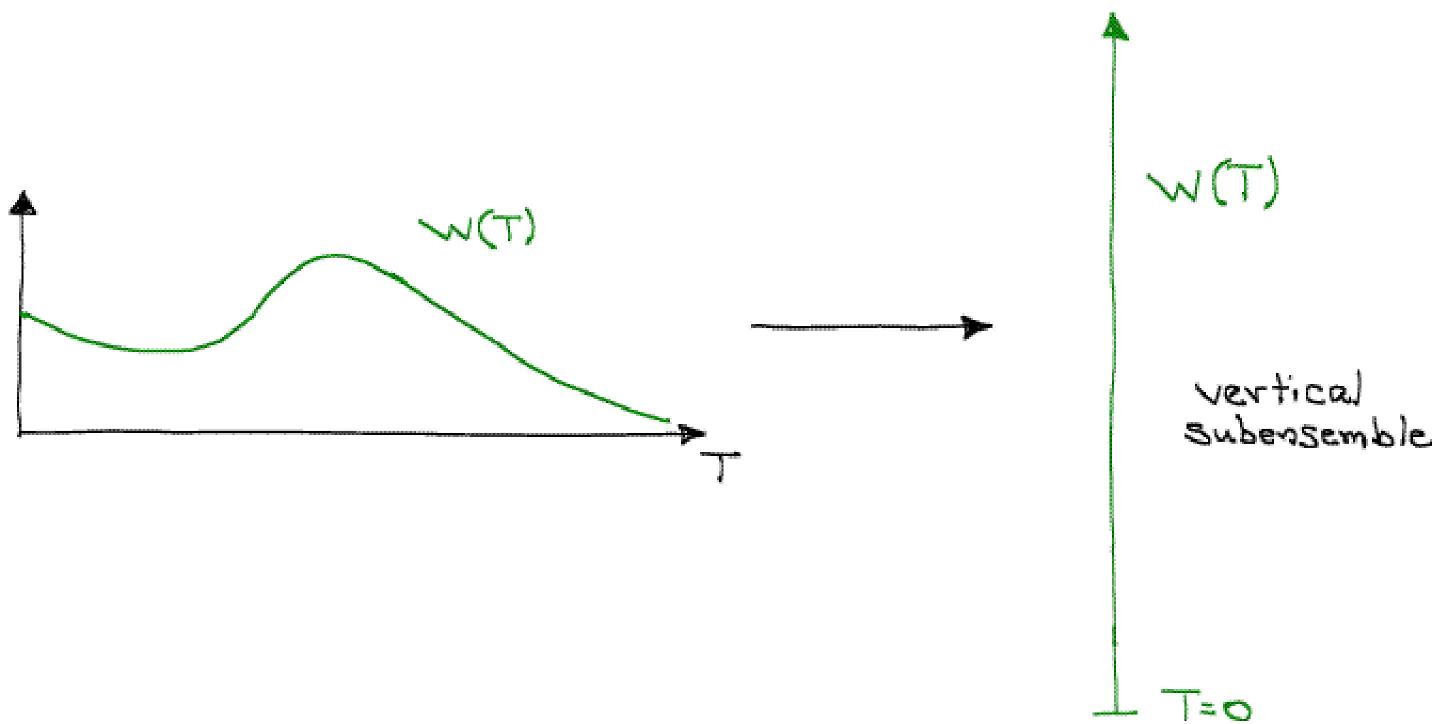
Homo sapiens

Explicating Gott's rule

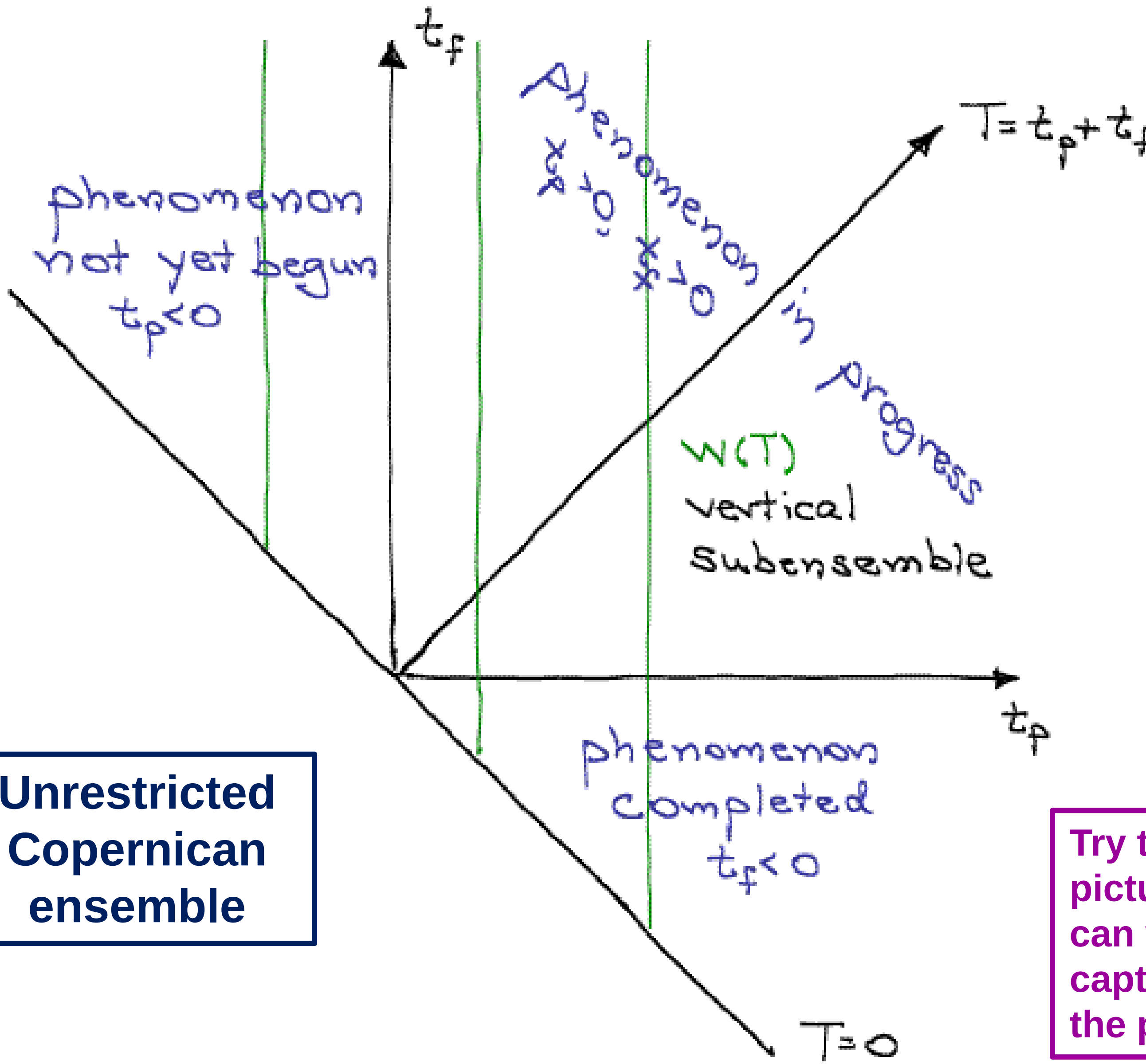


The temporal Copernican principle is equivalent to saying that all start times are equally likely; i.e., t_p is a uniformally distributed random variable.

Gott's line



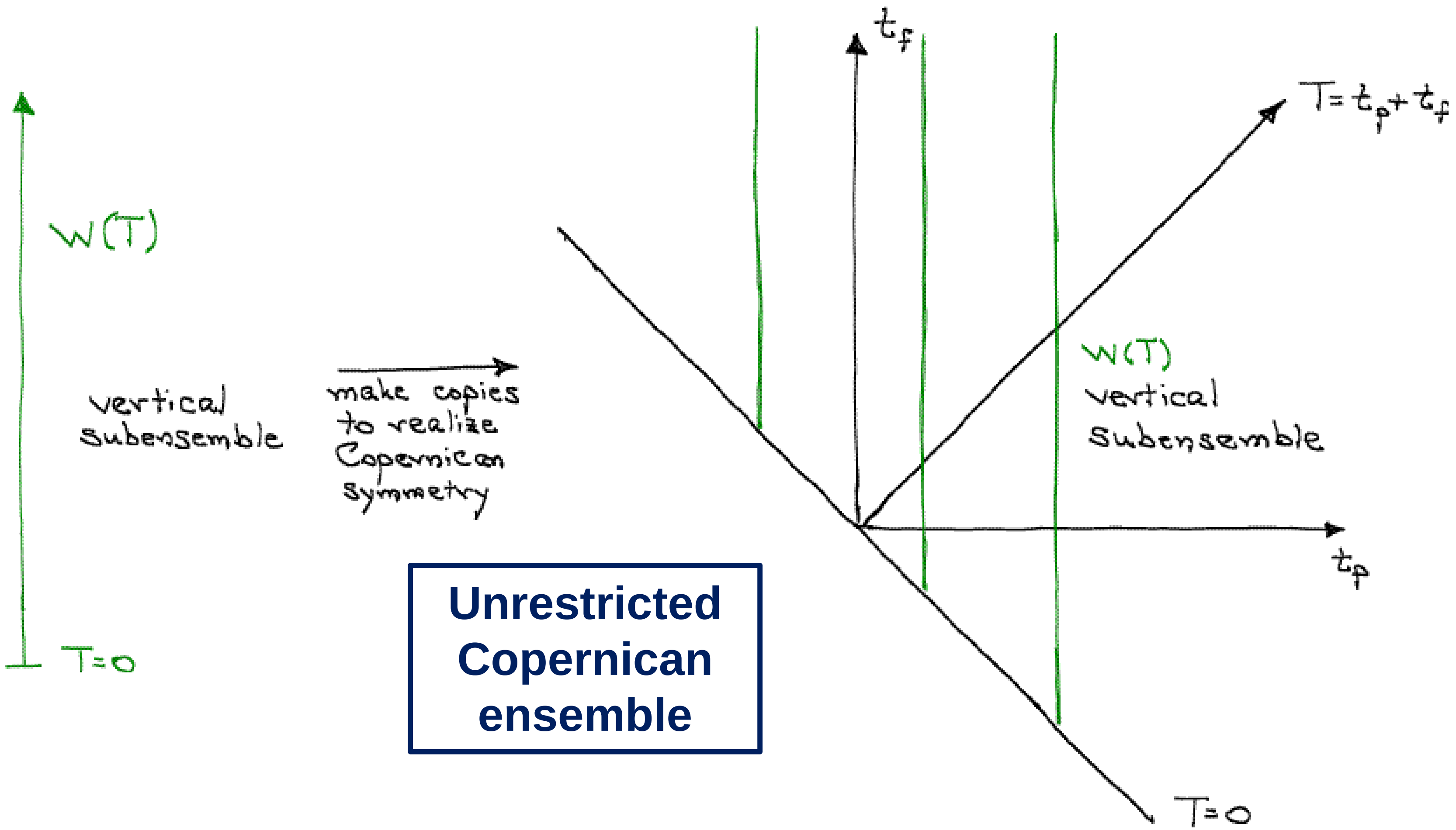
Explicating Gott's rule



Unrestricted Copernican ensemble

Try to develop a picture—something you can visualize—that captures the essence of the problem.

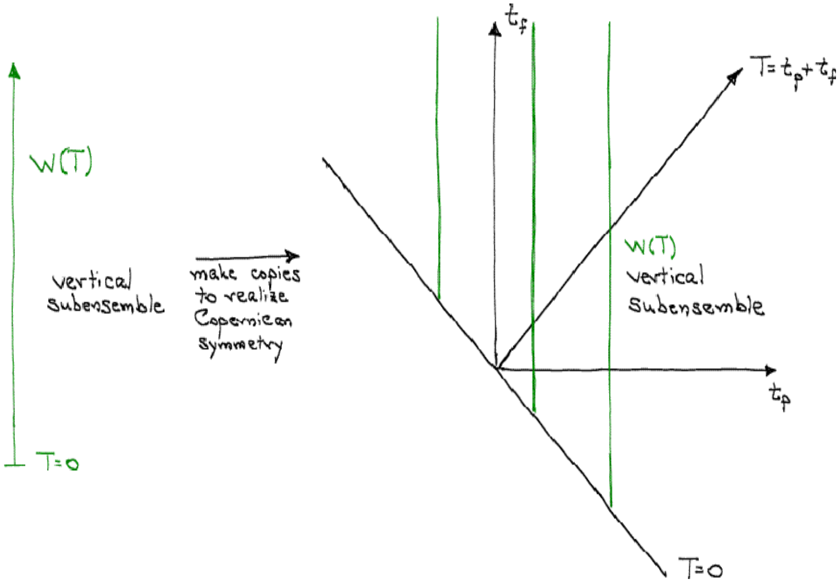
Explicating Gott's rule

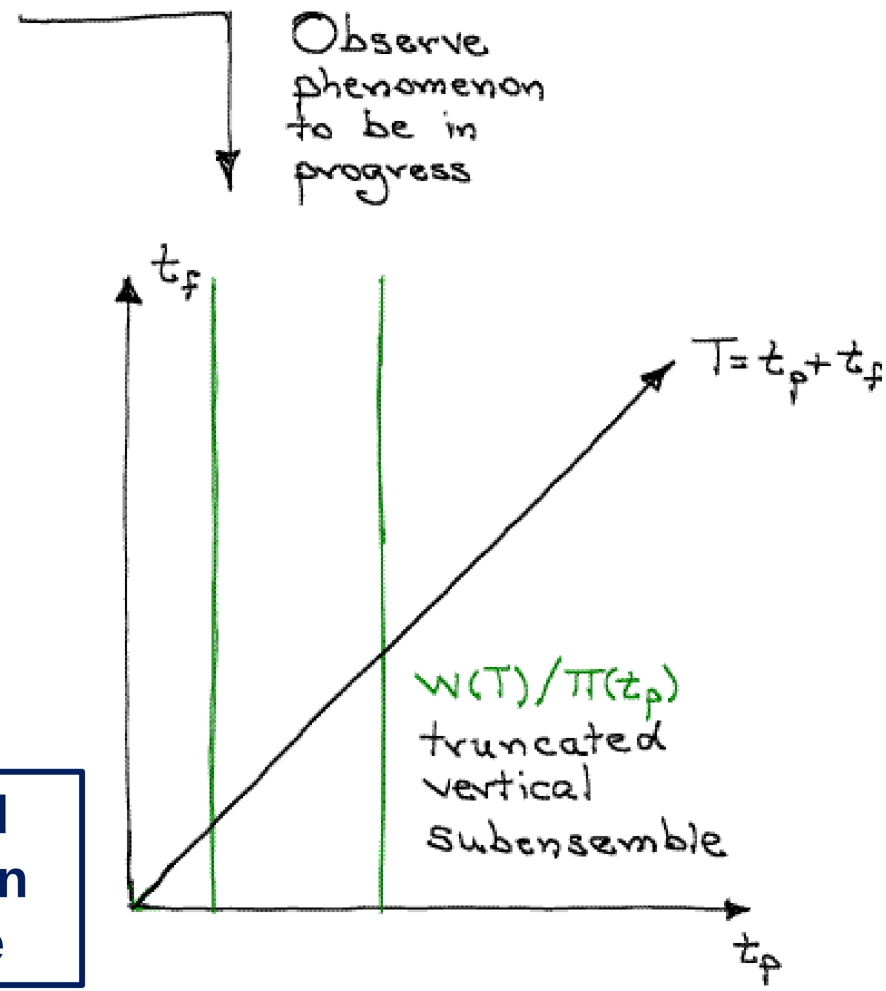


Unrestricted Copernican ensemble

Use symmetries to simplify your work.

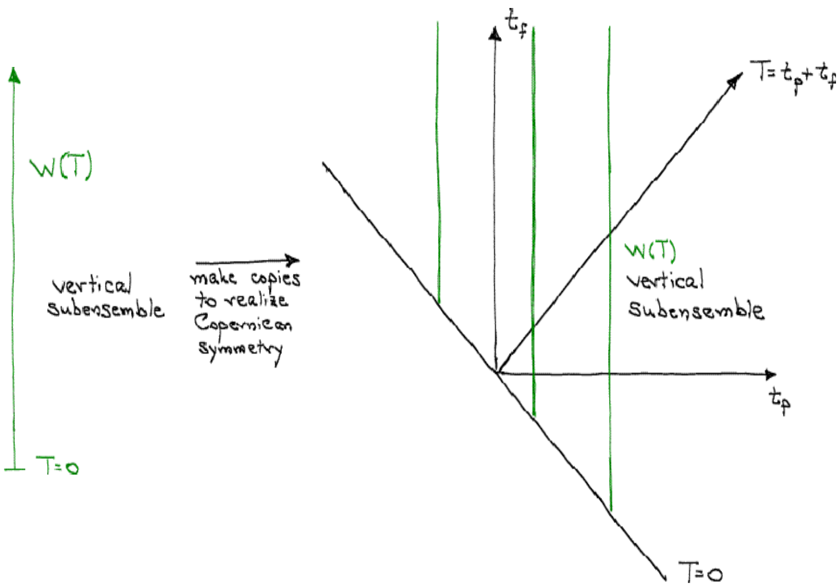
Explicating Gott's rule

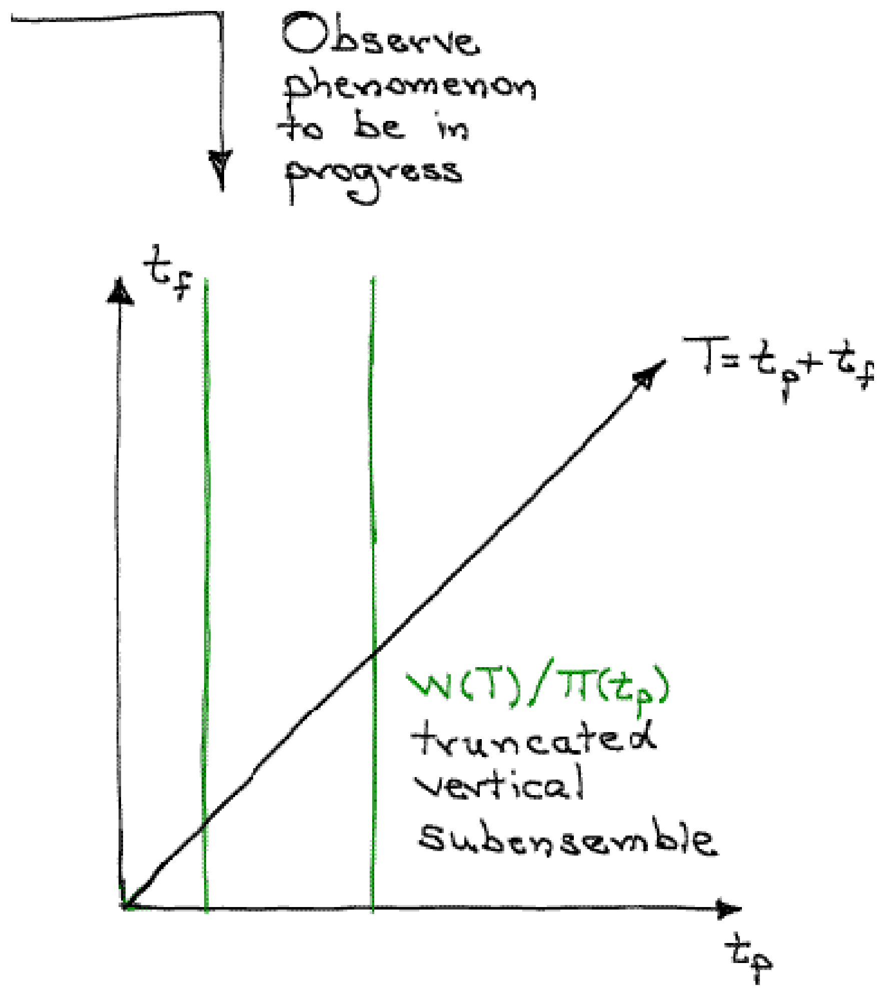


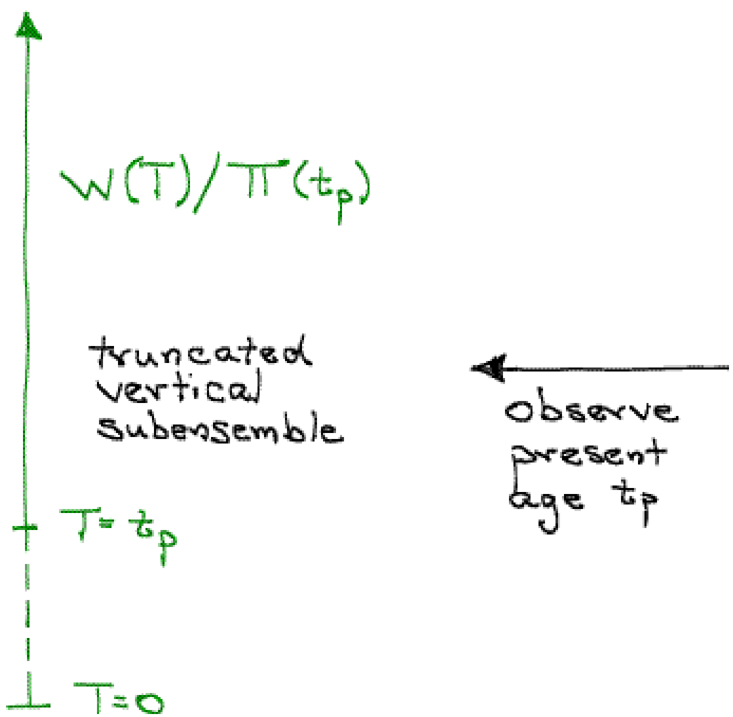


Truncated Copernican ensemble

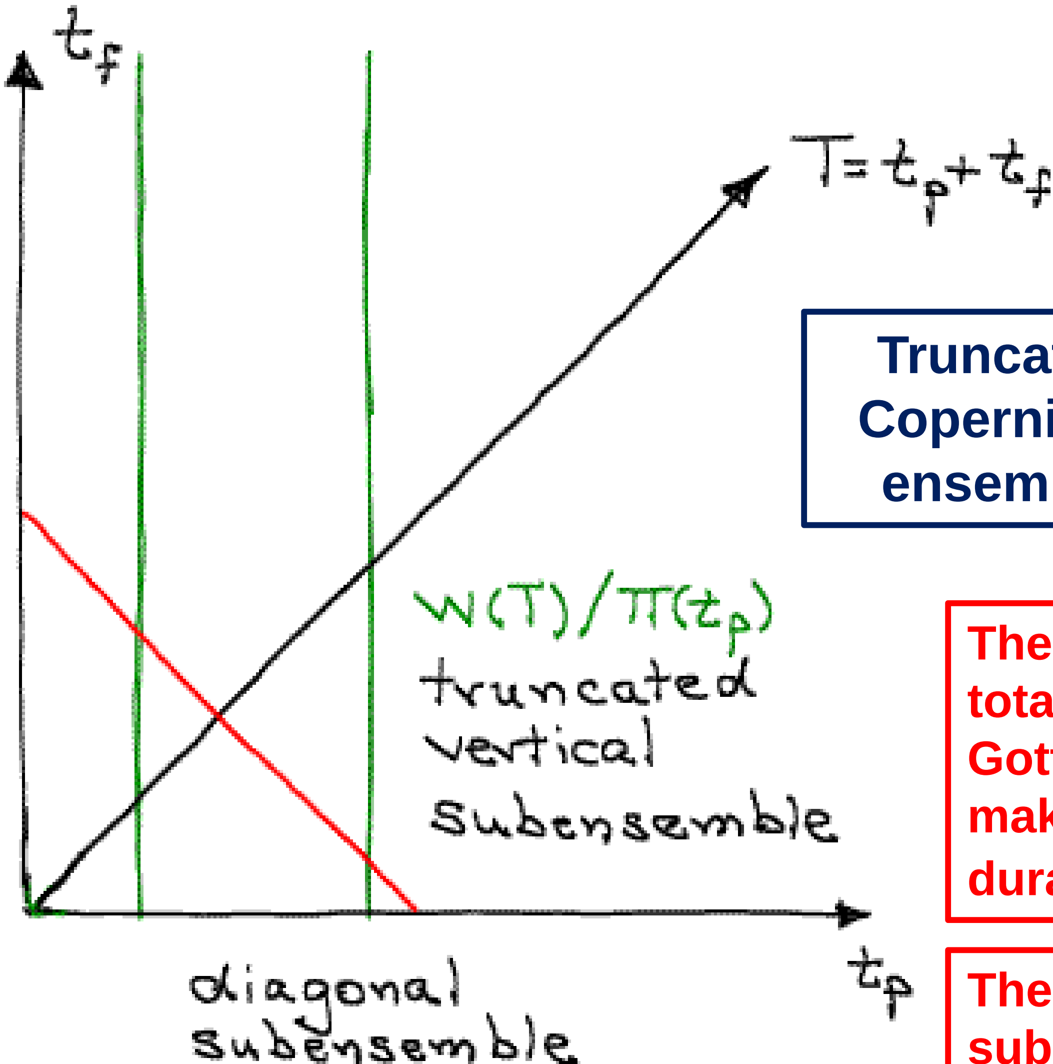
Explicating Gott's rule







Explicating Gott's rule: A baseball game



Truncated Copernican ensemble

The diagonal subensemble for total duration *T* is a realization of Gott's line, but it does not warrant making predictions of future duration based on present age.

The total population of a diagonal subensemble is proportional to *Tw(T)*, not *w(T)*.

IV. Doom and Bayesian inference

Echidna Gorge
Bungle Bungle Range
Western Australia

The doomsday argument

J. Leslie, *The End of the World: The Science and Ethics of Human Extinction* (Routledge, London, 1996)..

- Let $w(T)dT$ be the *a priori* probability that humanity lasts a time between T and $T + dT$.

- Before one knows the present age of humanity, the present age t_p is distributed uniformly between 0 and T; i.e., the probability that the present age lies between t_p and $t_p + dt_p$, given T, is

$$q(t_p|T)dt_p = \begin{cases} dt_p/T\ , & \text{if } t_p < T, \\ 0\ , & \text{if } t_p > T. \end{cases}$$

- *Bayes's theorem* says that

$$w(T|t_p)q(t_p) = q(t_p|T)w(T)\ ,$$

which gives

$$w(T|t_p) = \begin{cases} 0\ , & \text{if } T < t_p, \\ w(T)/\boxed{T}\,q(t_p)\ , & \text{if } T > t_p. \end{cases}$$

Clever approach? Units?

Cases: Decaying atom? Egg timer? Person? Scales? Two ways? Think critically? Updates?

DOOM

The doomsday argument

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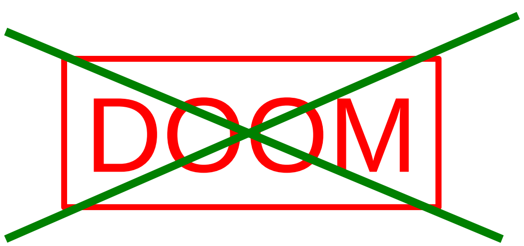
which gives

$$w(T|t_p) = \begin{cases} 0\ , & \text{if } T < t_p, \\ w(T)/\boxed{T}\,q(t_p)\ , & \text{if } T > t_p. \end{cases}$$

The present age is distributed uniformly within the diagonal subensemble of the truncated Copernican ensemble.

But the probability for T is renormalized to be $Tw(T)/\overline{T}$. The additional factor of T compensates for the doom factor and gets rid of the idea that you can enhance doom simply by looking at your watch.



V. Parting shots

Pecos Wilderness
Sangre de Cristo Range
Northern New Mexico

More baseball

White Sox

Gott predicted in 1996 that the Sox, having not won a World Series title since 1917, would, with 95% confidence, win a Series sometime between 1999 and 5077.

Gott would have predicted a World Series title in 2005 or before with probability 0.10, considerably less than the probability, $1 - (29/30)^9 = 0.26$, that comes from assuming that the Sox had the same chance each year as the 30 other major-league ball clubs.

The Sox won the 2005 World Series title.

More baseball

Cubs

The Cubs have now had a World Series drought of a century. Gott would predict that with probability ½ they will not a win a Series for the next century.

Giving the Cubs the same chance each year as all the other clubs gives a probability $(29/30)^{100} = 0.034$ of having a further 100-year drought. Gott's prediction corresponds to giving the Cubs a probability $1 - 1/2^{0.01} = 0.007$ of winning each year, about a factor of 5 less than 1/30.