Quantum information and computation: Why, what, and how

I. Introduction
II. Qubitology and quantum circuits
III. Entanglement and teleportation
IV. Quantum algorithms
V. Quantum error correction
VI. Physical implementations

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Quantum circuits in this presentation were set using the LaTeX package Qcircuit, developed by Bryan Eastin and Steve Flammia. The package is available at http://info.phys.unm.edu/Qcircuit/ .

I. Introduction

In the Sawtooth range
Central New Mexico

Quantum information science

A new way of thinking

Computer science

Computational complexity depends on physical law.

New physics

Quantum mechanics as liberator.

What can be accomplished with quantum systems that can't be done in a classical world?

Explore what can be done with quantum systems, instead of being satisfied with what Nature hands us.

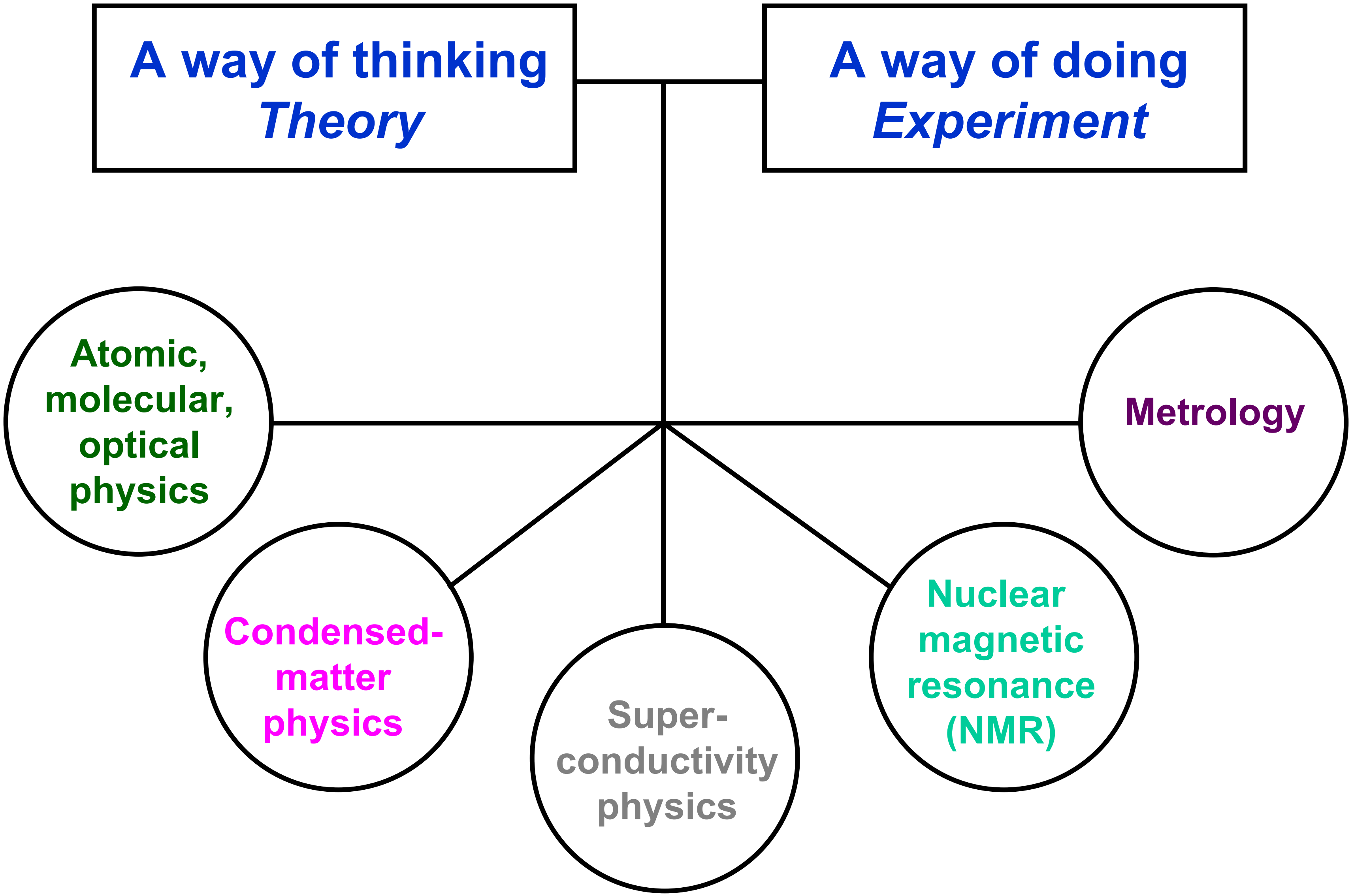
Quantum engineering

~~**Old physics**~~

~~*Quantum mechanics as nag.*~~

~~*The uncertainty principle restricts what can be done.*~~

Quantum information science



Classical information

Stored as string of bits

0100110

Quantum information

Stored as quantum state of string of qubits

$|\Psi\rangle$

whole story

Physical system with two distinguishable states

Few electrons on a capacitor

Pit on a compact disk

0 or 1 on the printed page

Smoke signal on a distant mesa

Spin-1/2 particle $|\uparrow\rangle = |0\rangle \quad |\downarrow\rangle = |1\rangle$

Two-level atom

$|e\rangle = |1\rangle$

$|g\rangle = |0\rangle$

Photon polarization

$$|R\rangle = |0\rangle \quad |L\rangle = |1\rangle$$

$$|V\rangle = \big(|R\rangle + |L\rangle\big)/\sqrt{2}$$

$$|H\rangle = -i\big(|R\rangle - |L\rangle\big)/\sqrt{2}$$

much more

Qubits

$|0\rangle$ and $|1\rangle$ are *orthogonal* and thus *distinguishable*. They make up the *computational basis*.

Pure quantum state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

Classical information | Quantum information

Stored as string of bits 0100110

Stored as quantum state of string of qubits $|\Psi\rangle$

Manipulation of (qu)bits (computation, dynamics)

Bit transformations (function computation)
All functions can be computed reversibly.

Unitary operations *U* (reversible)

Bit states can be copied.

Qubit states cannot be copied, except for *orthogonal* states

$$|\psi\rangle|0\rangle \underset{U}{\rightarrow} |\psi\rangle|\psi\rangle$$

$$\langle\psi|\phi\rangle^2 = \langle\psi|\phi\rangle$$

Transmission of (qu)bits (communication, dynamics)

Readout of (qu)bits (measurement)

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$
$$p_0 = |\alpha|^2 = |\langle 0|\psi\rangle|^2$$
$$p_1 = |\beta|^2 = |\langle 1|\psi\rangle|^2$$

Distinguishability of bit states

Quantum states are not distinguishable, except for *orthogonal* states

Classical information

Stored as string of bits 0100110

Manipulation of (qu)bits (computation, dynamics) Unitary operations U

Transmission of (qu)bits (communication, dynamics)

Readout of (qu)bits (measurement)

Quantum information

Stored as quantum state of string of qubits $|\Psi\rangle$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$
$$p_0 = |\alpha|^2 = |\langle 0|\psi\rangle|^2$$
$$p_1 = |\beta|^2 = |\langle 1|\psi\rangle|^2$$

Quantum mechanics as liberator.

Classical information processing is quantum information processing restricted to distinguishable (orthogonal) states. Superpositions are the additional freedom in quantum information processing.

Classical information | Quantum information

Stored as string of bits 0100110

Stored as quantum state of string of qubits $|\Psi\rangle$

Manipulation of (qu)bits (computation, dynamics) Unitary operations U

Transmission of (qu)bits (communication, dynamics)

Readout of (qu)bits (measurement)

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$
$$p_0 = |\alpha|^2 = |\langle 0|\psi\rangle|^2$$
$$p_1 = |\beta|^2 = |\langle 1|\psi\rangle|^2$$

Correlation of bit states

Quantum correlation of qubit states (entanglement)

Bell inequalities

01 or 10 ← **anticorrelation** → $|\Psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$

Error correction (copying and redundancy OR nonlocal storage of information)

Quantum error correction (entanglement OR nonlocal storage of quantum information)

Analogue vs. digital

II. Qubitology and quantum circuits

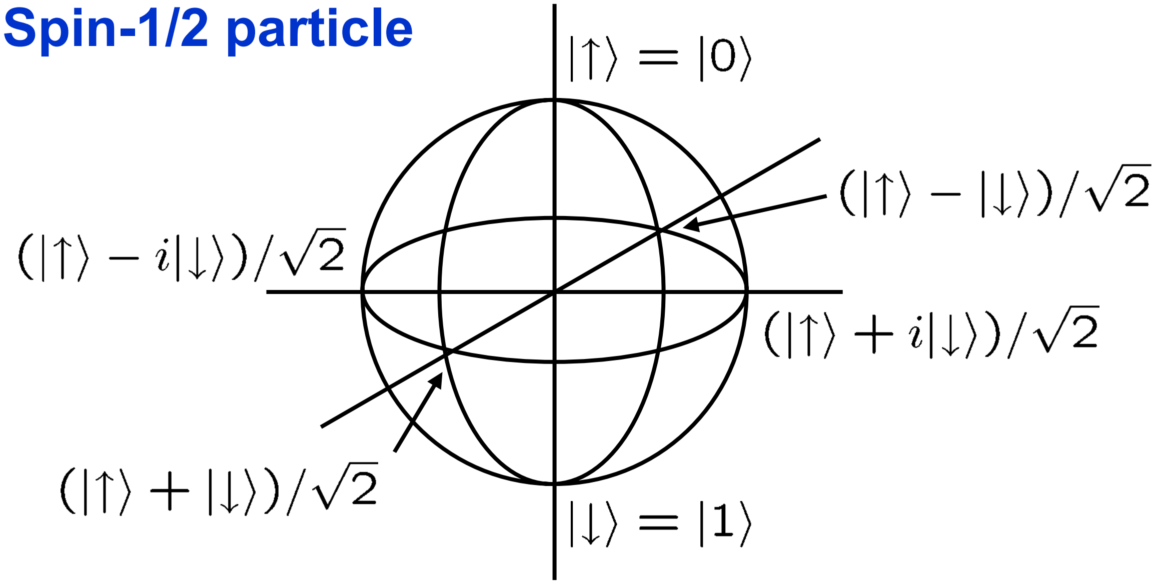
Albuquerque International Balloon Fiesta

Qubitology. States

$$|\psi\rangle = \cos(\theta/2)|0\rangle + e^{i\phi}\sin(\theta/2)|1\rangle = |\mathbf{n}\rangle$$

Direction of spin

Spin-1/2 particle



Bloch sphere

$$\begin{aligned} |\mathbf{n}\rangle\langle\mathbf{n}| &= \frac{1}{2}(I + \sigma_x n_x + \sigma_y n_y + \sigma_z n_z) \\ &= \frac{1}{2}(I + \mathbf{n}\cdot\sigma) \end{aligned}$$

Pauli representation

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = X$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = Y$$

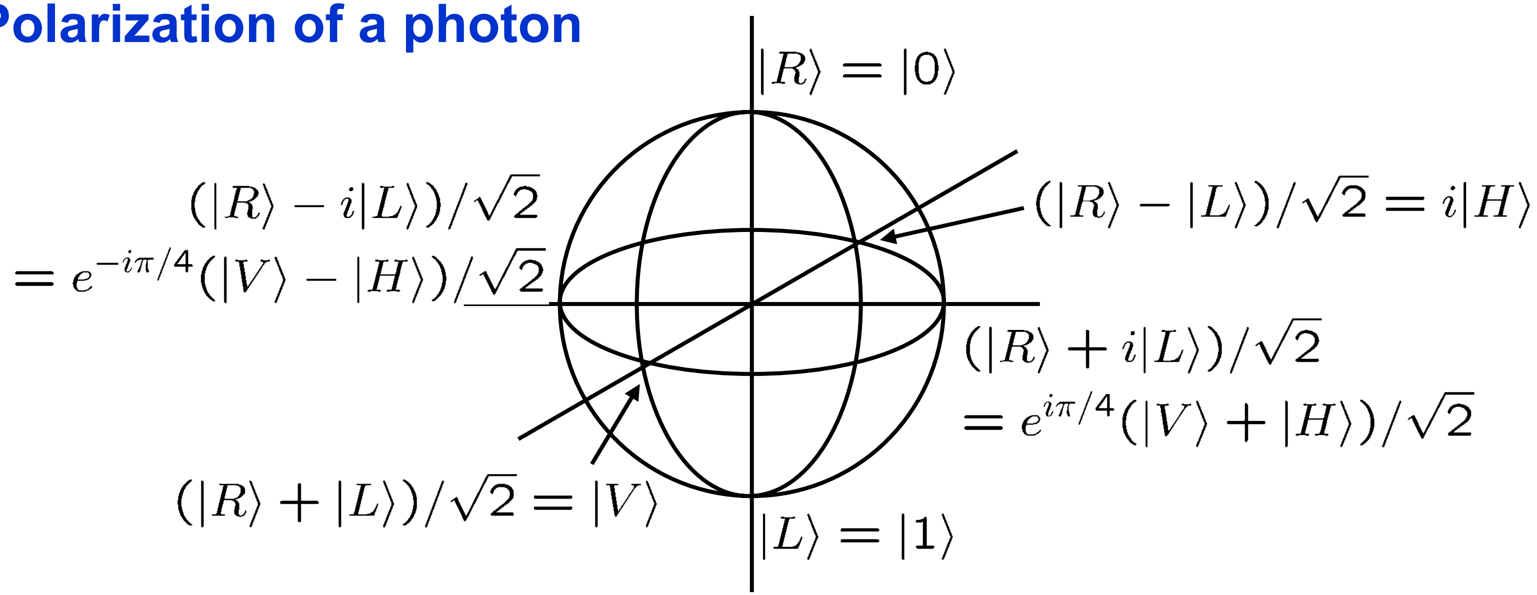
$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = Z$$

Qubitology. States

$$|\psi\rangle = \cos(\theta/2)|0\rangle + e^{i\phi}\sin(\theta/2)|1\rangle = |\mathbf{n}\rangle$$

Abstract "direction"

Polarization of a photon



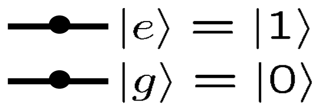
Poincare sphere

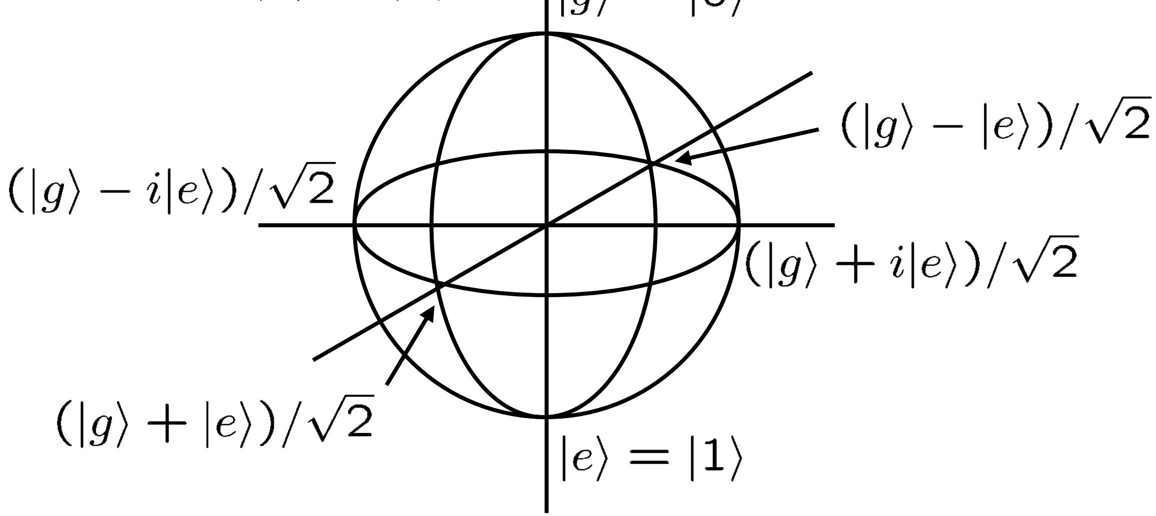
Qubitology. States

$$|\psi\rangle = \cos(\theta/2)|0\rangle + e^{i\phi}\sin(\theta/2)|1\rangle = |\mathbf{n}\rangle$$

Abstract "direction"

Two-level atom





Bloch sphere

Qubitology

Single-qubit **states** are points on the Bloch sphere.

Single-qubit **operations (unitary operators)** are rotations of the Bloch sphere.

Single-qubit **measurements** are rotations followed by a measurement in the computational basis (measurement of *z* spin component).

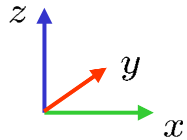
$$p_0 = |\langle 0|\mathbf{n}\rangle|^2 = \frac{1}{2}(1 + n_z)$$

$$p_1 = |\langle 1|\mathbf{n}\rangle|^2 = \frac{1}{2}(1 - n_z)$$

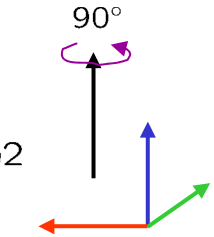
Platform-independent description:
Hallmark of an information theory

Qubitology. Gates and quantum circuits

Single-qubit gates

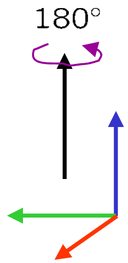


$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} = T^2$$



$$|a\rangle \text{ — } \boxed{S} \text{ — } i^a|a\rangle$$

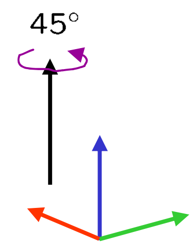
$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = S^2$$



Phase flip

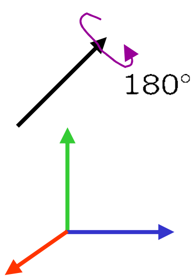
$$|a\rangle \text{ — } \boxed{Z} \text{ — } (-1)^a|a\rangle$$

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$



$$|a\rangle \text{ — } \boxed{T} \text{ — } e^{ia\pi/4}|a\rangle$$

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$



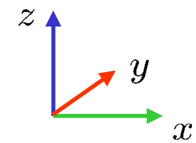
Hadamard

$$|a\rangle \text{ — } \boxed{H} \text{ — } (|0\rangle + (-1)^a|1\rangle)/\sqrt{2}$$

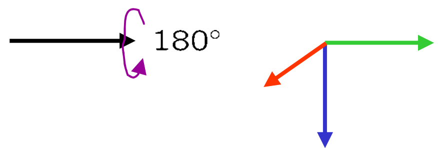
$$Z^2 = H^2 = I$$

Qubitology. Gates and quantum circuits

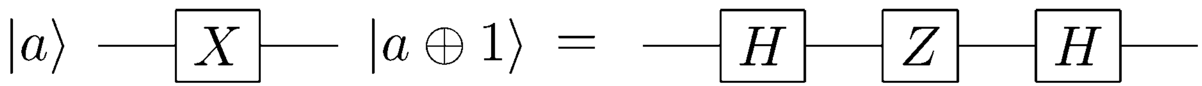
More single-qubit gates



$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = HZH$$

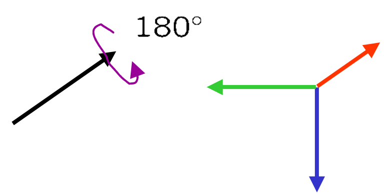


Bit flip

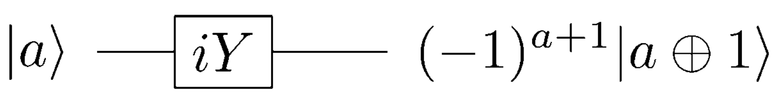


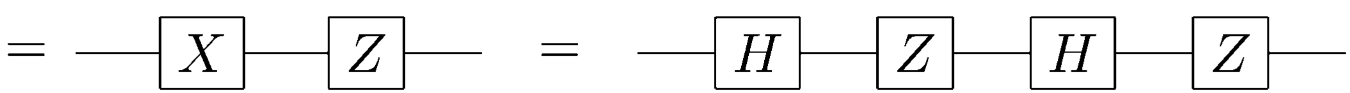
$$X^2 = Y^2 = I$$

$$iY = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = ZX$$



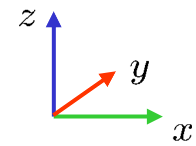
Phase-bit flip

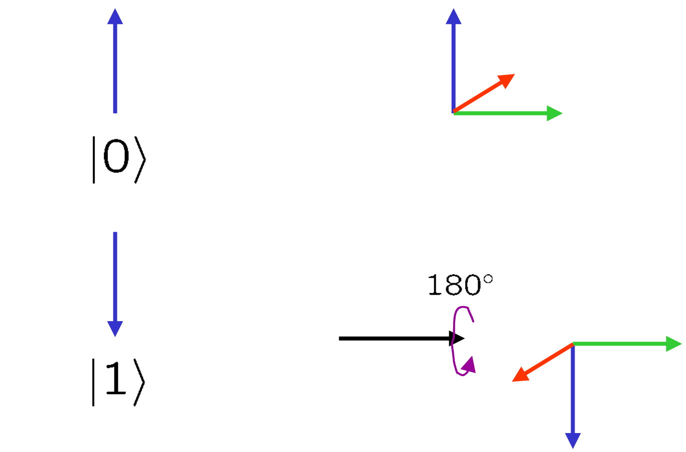




Qubitology. Gates and quantum circuits

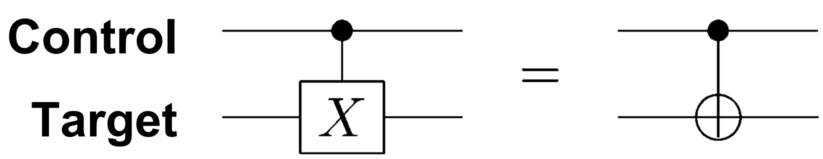
Control-target two-qubit gate

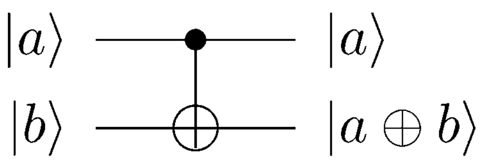




$$\text{C-NOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$= |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X$$





$$(\text{C-NOT})^2 = I$$

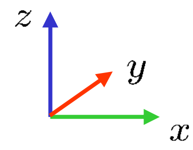
Qubitology. Gates and quantum circuits

Universal set of quantum gates

- *T* (45-degree rotation about *z*)
- *H* (Hadamard)
- C-NOT

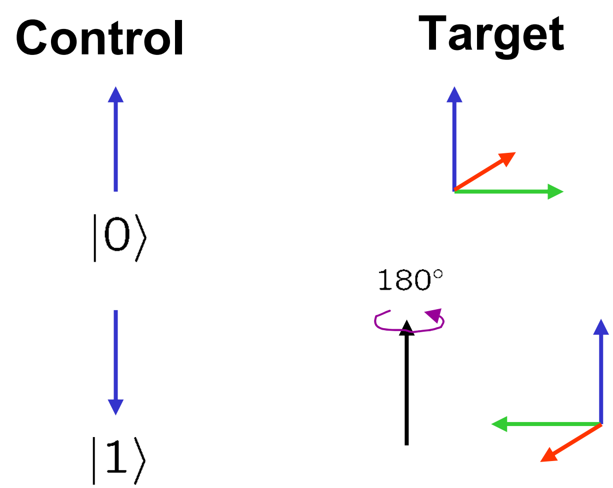
Qubitology. Gates and quantum circuits

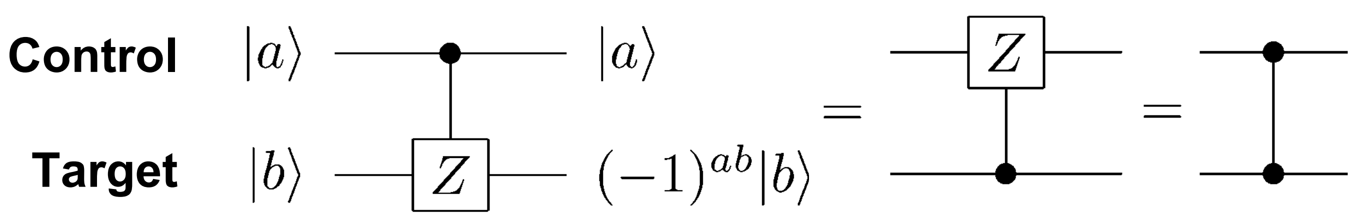
Another two-qubit gate

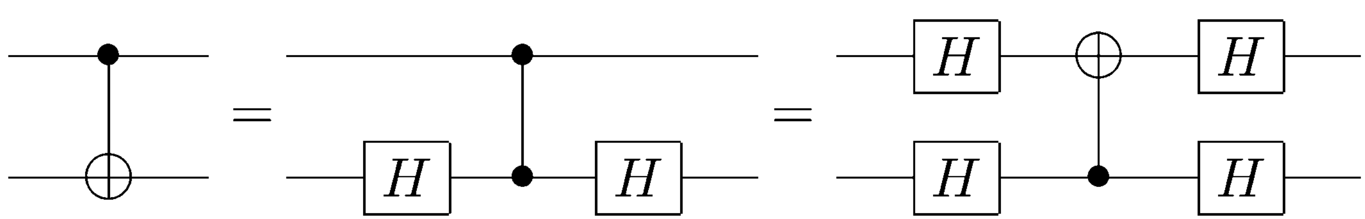


$$\text{C-PHASE} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$= |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes Z$$

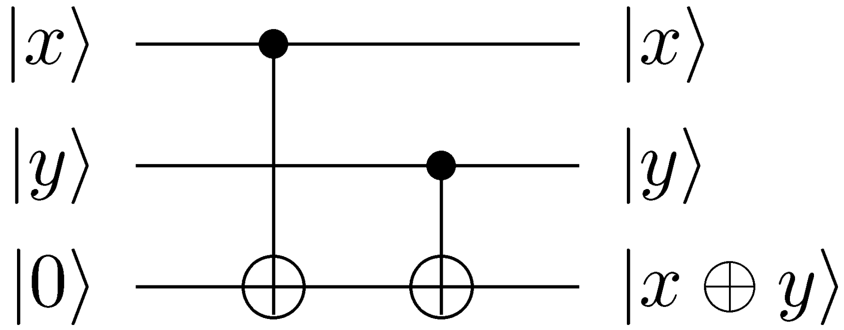




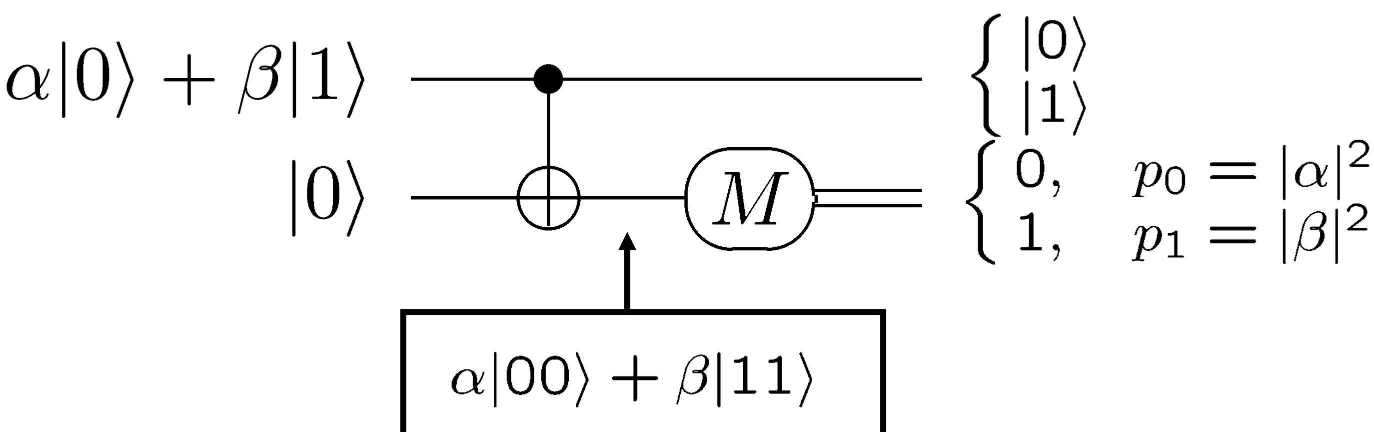


Qubitology. Gates and quantum circuits

C-NOT as parity check



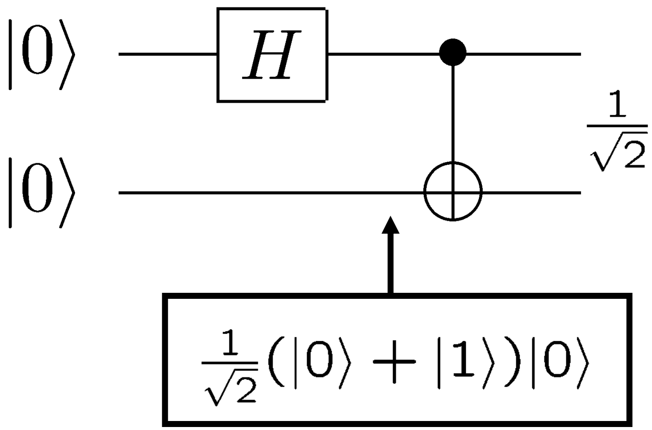
C-NOT as measurement gate



Circuit identity

Qubitology. Gates and quantum circuits

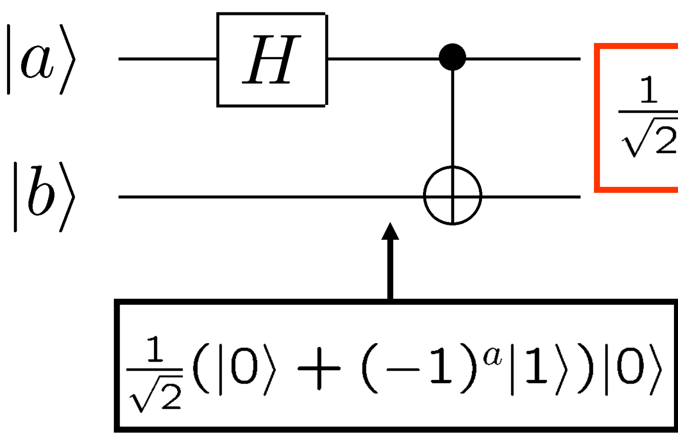
Making Bell states using C-NOT



$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

Bell states

$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$
$$|\beta_{10}\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$
$$|\beta_{01}\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$
$$|\beta_{11}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$



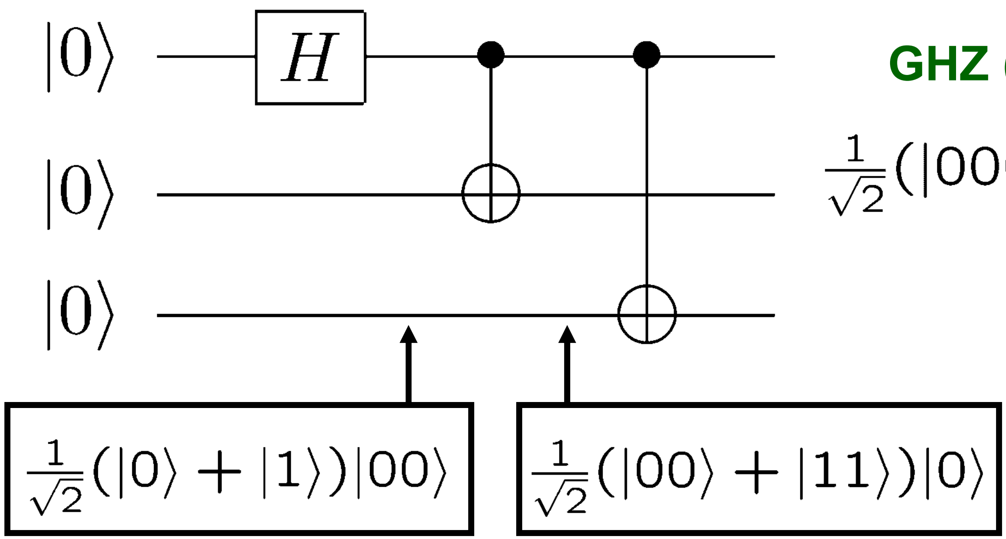
$$\frac{1}{\sqrt{2}}(|0b\rangle + (-1)^a|1, b \oplus 1\rangle) \equiv |\beta_{ab}\rangle$$

phase bit

parity bit

Qubitology. Gates and quantum circuits

Making cat states using C-NOT



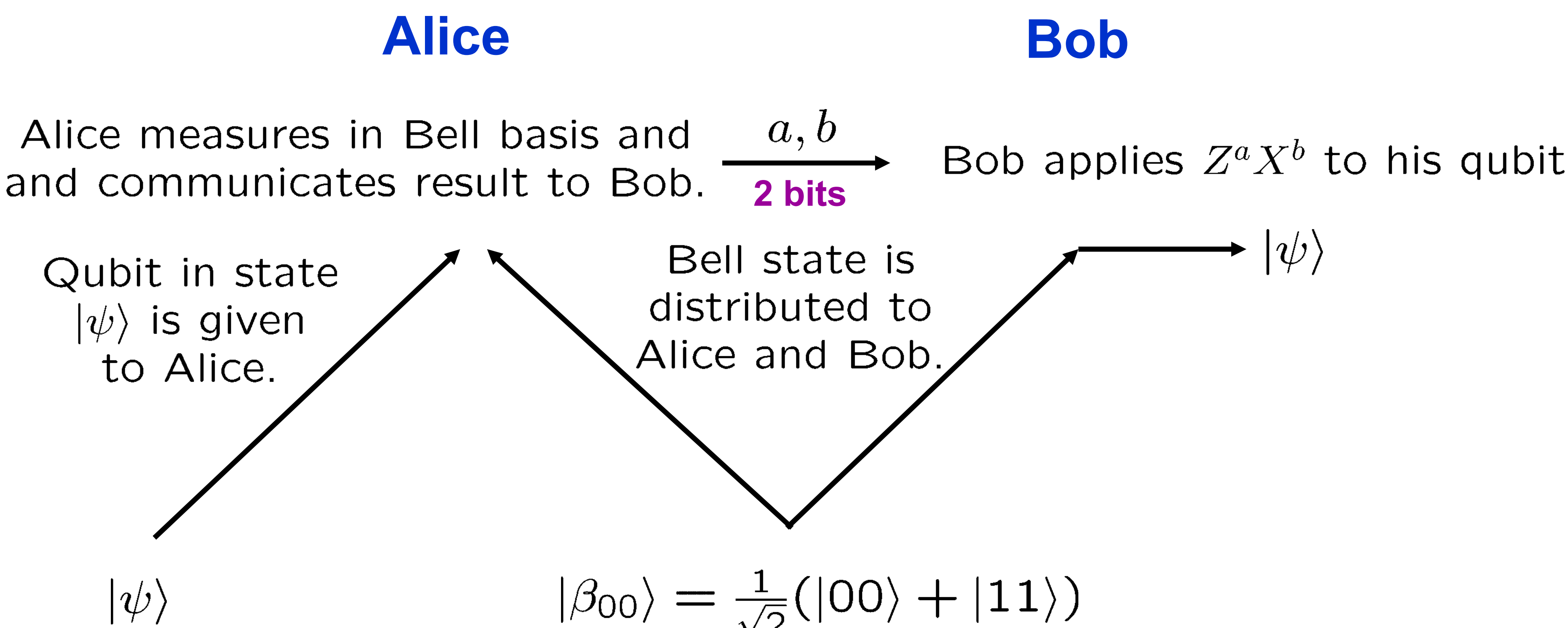
GHZ (cat) state

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

III. Entanglement and teleportation

Oljeto Wash
Southern Utah

Entanglement and teleportation



$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|\Psi\rangle = |\psi\rangle \otimes |\beta_{00}\rangle = \frac{1}{2}\sum_{a,b} |\beta_{ab}\rangle \otimes X^b Z^a |\psi\rangle$$

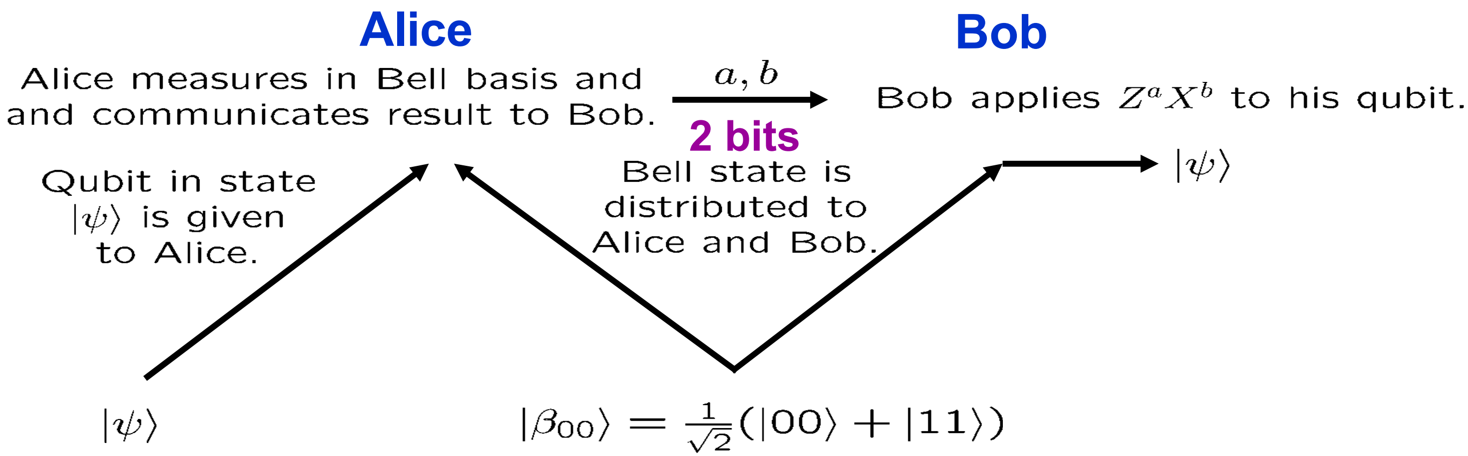
$$\begin{aligned} |\beta_{00}\rangle &= (|00\rangle + |11\rangle)/\sqrt{2} \\ |\beta_{10}\rangle &= (|00\rangle - |11\rangle)/\sqrt{2} \\ |\beta_{01}\rangle &= (|01\rangle + |10\rangle)/\sqrt{2} \\ |\beta_{11}\rangle &= (|01\rangle - |10\rangle)/\sqrt{2} \end{aligned}$$

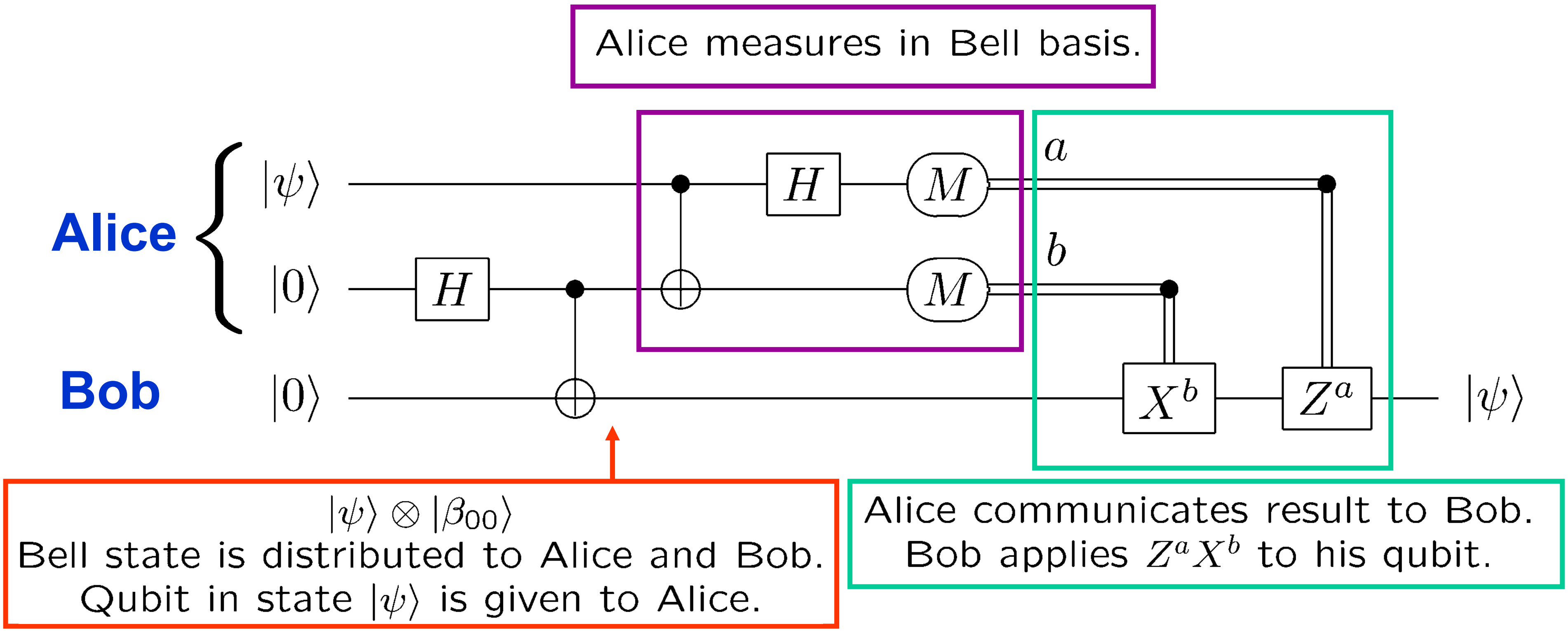
Classical teleportation

Teleportation of probabilities

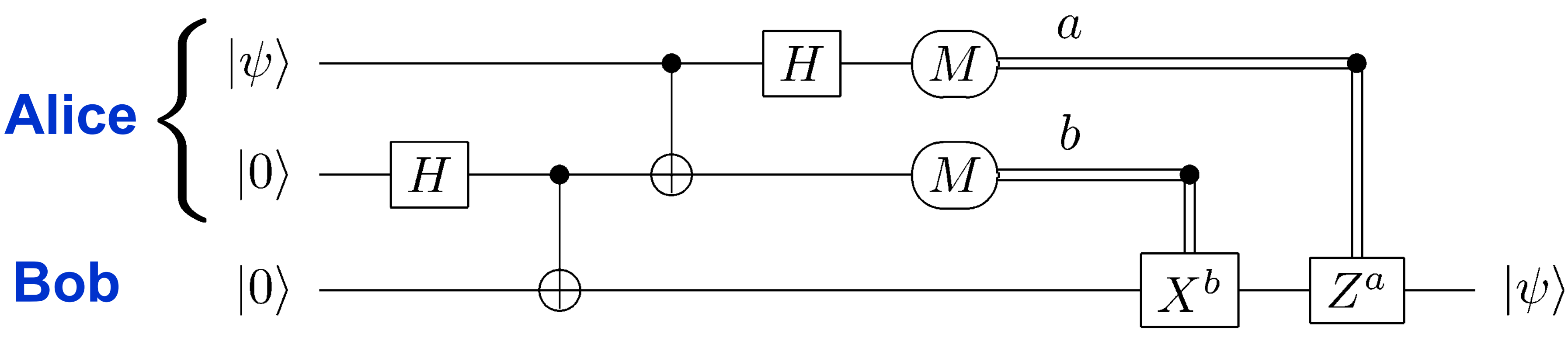
Demonstration

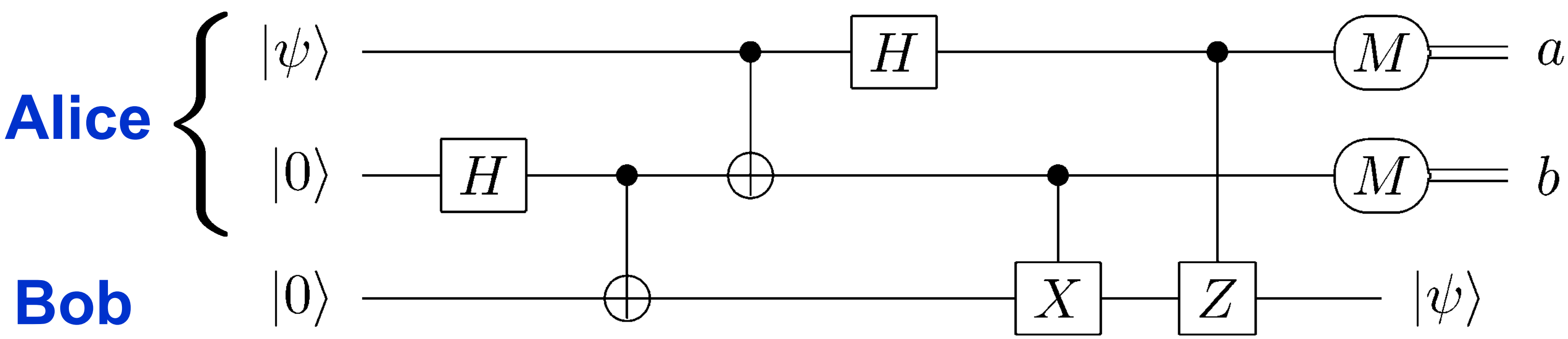
Entanglement and teleportation





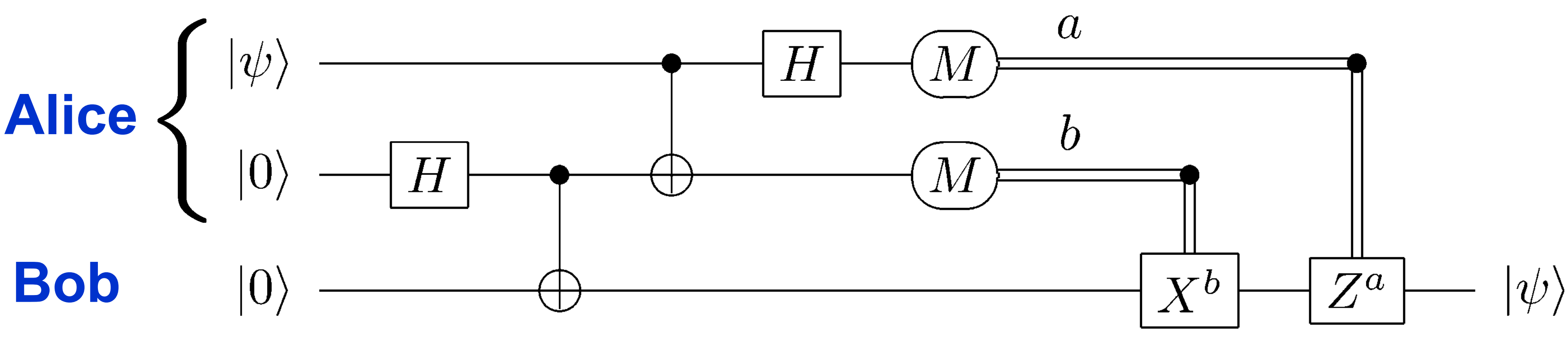
Entanglement and teleportation

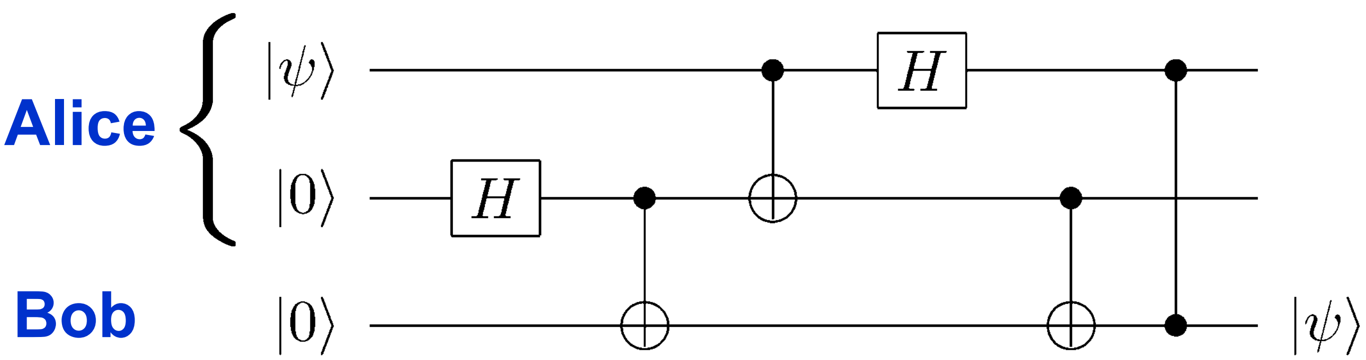




Entanglement and teleportation

Standard teleportation circuit





Coherent teleportation circuit

Error correction

IV. Quantum algorithms

Truchas from East Pecos Baldy
Sangre de Cristo Range
Northern New Mexico

Quantum algorithms. Deutsch-Jozsa algorithm

Boolean function $f : \{0,1\}^N \to \{0,1\}$

Promise: ***f*** **is constant or balanced.**

Problem: Determine which.

Classical: Roughly 2^{N-1} function calls are required to be certain.

Quantum: Only 1 function call is needed.

$$N = 3: \; U_f|x,y,z\rangle|w\rangle = |x,y,z\rangle\Big|w \oplus f(x,y,z)\Big\rangle$$

work qubit

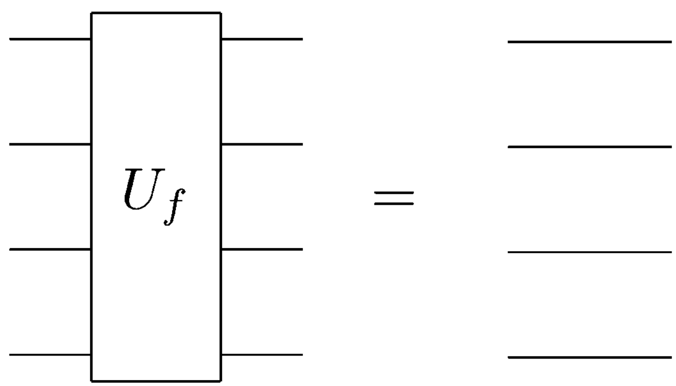
Quantum algorithms. Deutsch-Jozsa algorithm

$$N = 3: \; U_f|x,y,z\rangle|w\rangle = |x,y,z\rangle\Big|w \oplus f(x,y,z)\Big\rangle$$

work qubit

Example: Constant function

$$\begin{aligned}
f(0,0,0) &= 0 \\
f(0,0,1) &= 0 \\
f(0,1,0) &= 0 \\
f(0,1,1) &= 0 \\
f(1,0,0) &= 0 \\
f(1,0,1) &= 0 \\
f(1,1,0) &= 0 \\
f(1,1,1) &= 0
\end{aligned}$$



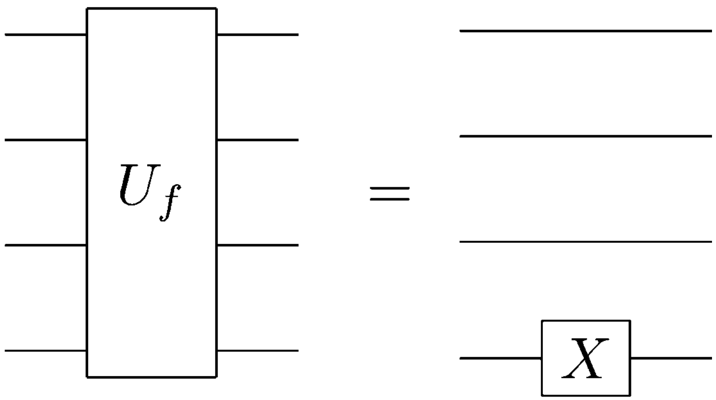
Quantum algorithms. Deutsch-Jozsa algorithm

$$N = 3: \; U_f|x,y,z\rangle|w\rangle = |x,y,z\rangle\Big|w \oplus f(x,y,z)\Big\rangle$$

work qubit

Example: Constant function

$$\begin{aligned} f(0,0,0) &= 1 \\ f(0,0,1) &= 1 \\ f(0,1,0) &= 1 \\ f(0,1,1) &= 1 \\ f(1,0,0) &= 1 \\ f(1,0,1) &= 1 \\ f(1,1,0) &= 1 \\ f(1,1,1) &= 1 \end{aligned}$$



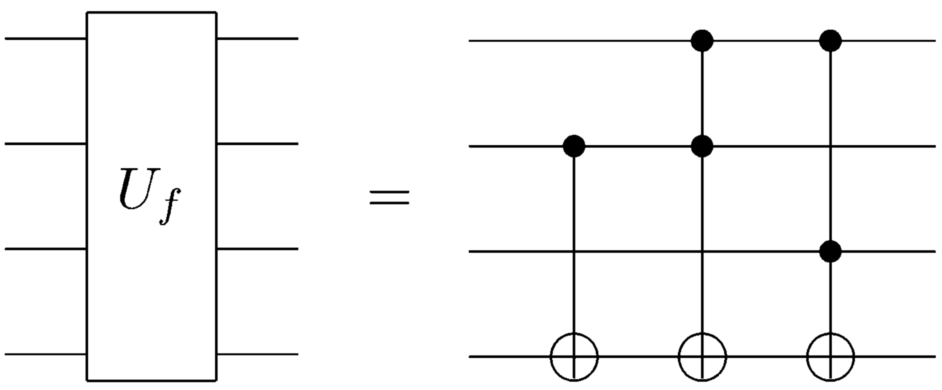
Quantum algorithms. Deutsch-Jozsa algorithm

$$N = 3: \; U_f|x,y,z\rangle|w\rangle = |x,y,z\rangle\Big|w \oplus f(x,y,z)\Big\rangle$$

work qubit

Example: Balanced function

$$\begin{aligned} f(0,0,0) &= 0 \\ f(0,0,1) &= 0 \\ f(0,1,0) &= 1 \\ f(0,1,1) &= 1 \\ f(1,0,0) &= 0 \\ f(1,0,1) &= 1 \\ f(1,1,0) &= 0 \\ f(1,1,1) &= 1 \end{aligned}$$



$$f(x,y,z) = y \oplus xy \oplus xz$$

Quantum algorithms. Deutsch-Jozsa algorithm

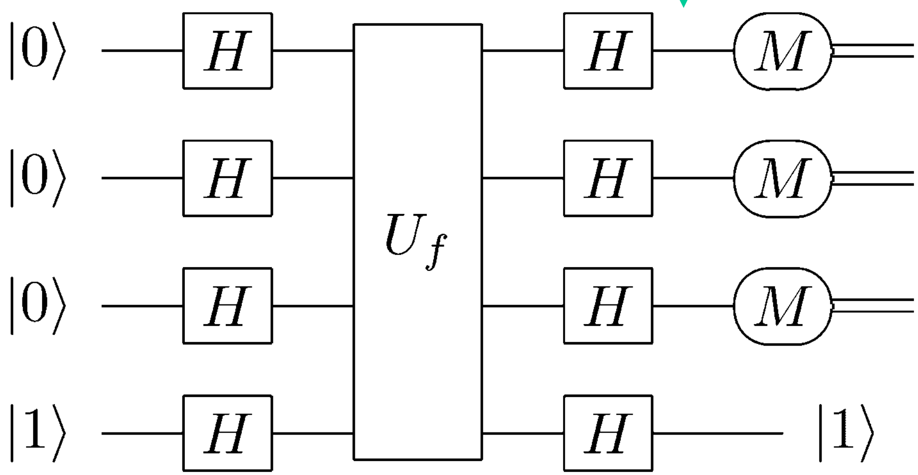
$f : \{0,1\}^N \to \{0,1\}$ **Problem: Determine whether f is constant or balanced.**

$N = 3$

$$\begin{cases} \pm|000\rangle \,, & f \text{ constant} \\ \text{state orthogonal to } |000\rangle, & f \text{ balanced} \end{cases}$$

quantum interference

work qubit



all zeroes, f constant
not all zeroes, f balanced

$$\frac{1}{2\sqrt{2}}\sum_{x,y,z}|x,y,z\rangle\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)$$

$$\langle\Phi_{\text{balanced}}|\Phi_{\text{constant}}\rangle = \pm\frac{1}{8}\sum_{x,y,z}(-1)^{f_b(x,y,z)} = 0$$

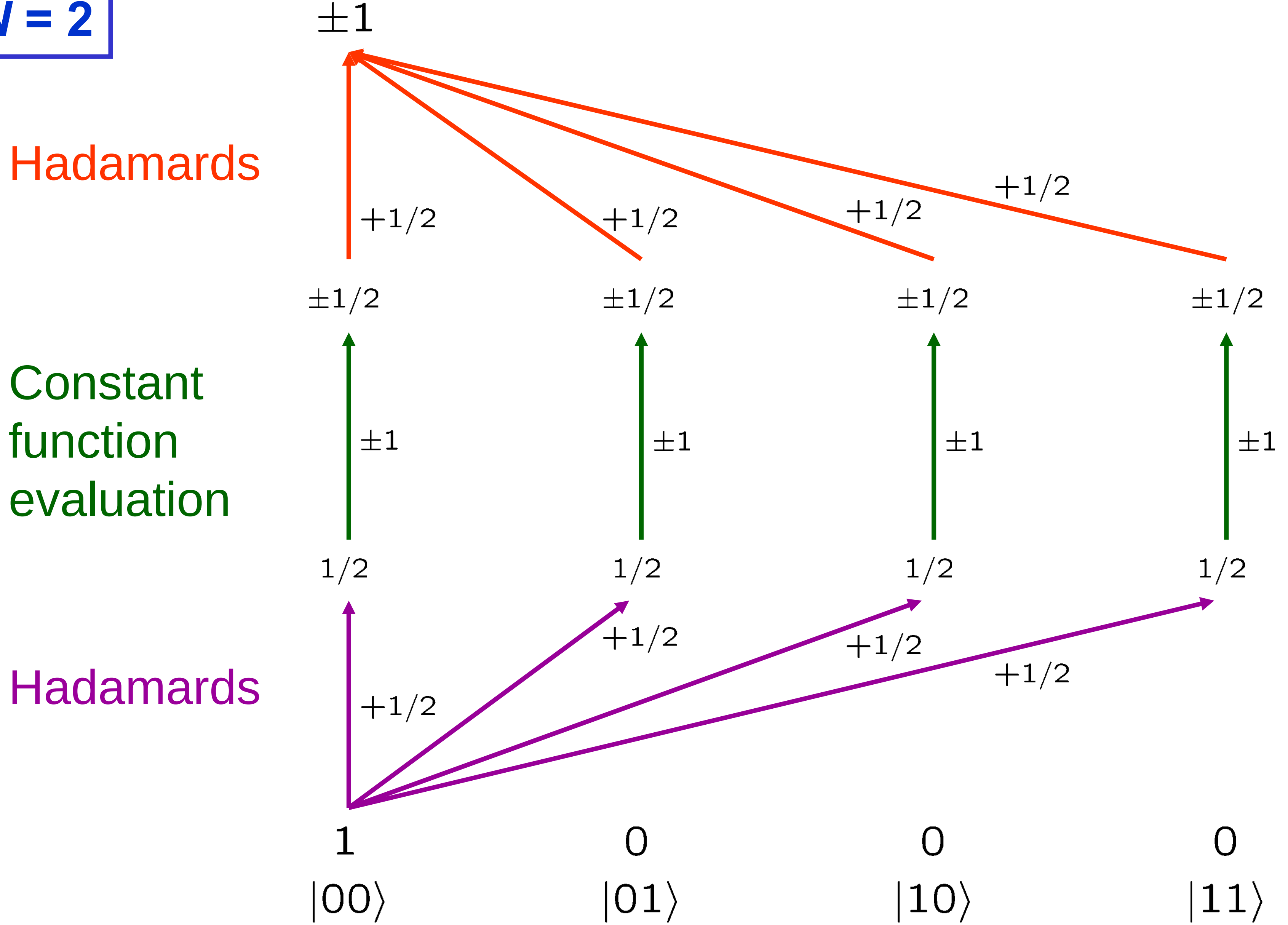
$$\frac{1}{2\sqrt{2}}\sum_{x,y,z}|x,y,z\rangle\frac{1}{\sqrt{2}}(|f(x,y,z)\rangle-|1\oplus f(x,y,z)\rangle) = \underbrace{\frac{1}{2\sqrt{2}}\sum_{x,y,z}(-1)^{f(x,y,z)}|x,y,z\rangle}_{\equiv|\Phi\rangle}\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)$$

quantum parallelism

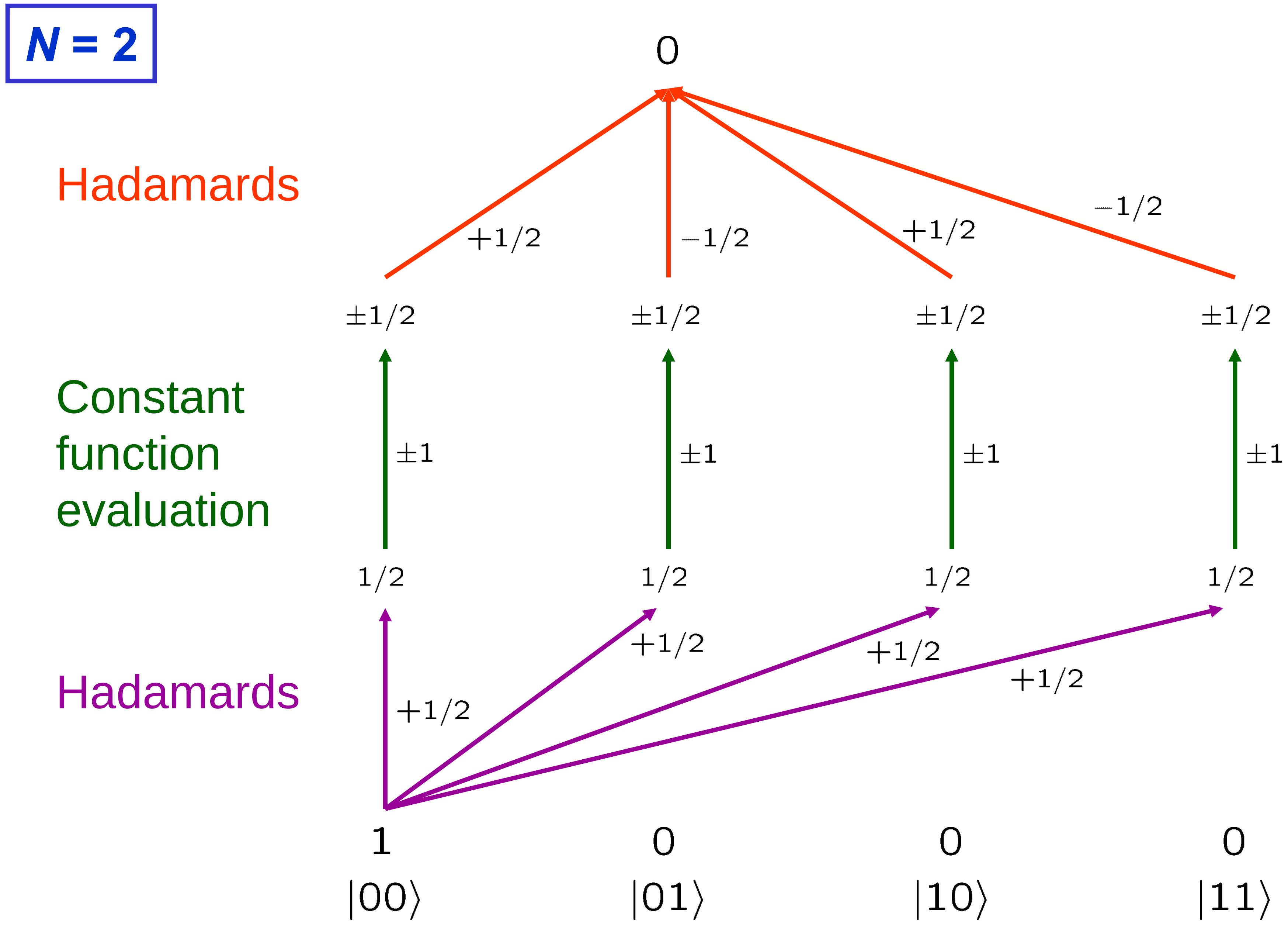
phase "kickback"

Quantum interference in the Deutsch-Jozsa algorithm

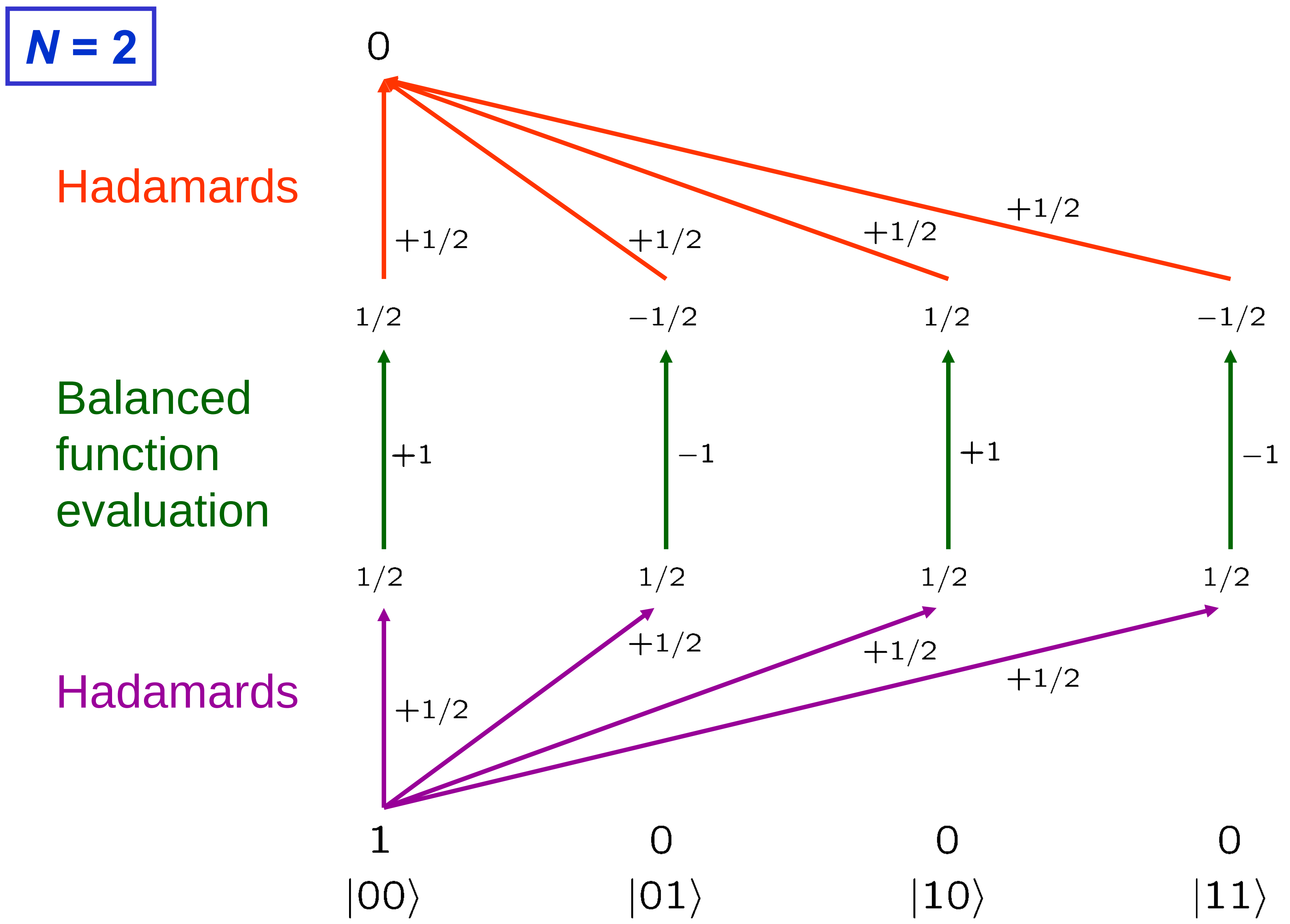
***N* = 2**



Quantum interference in the Deutsch-Jozsa algorithm



Quantum interference in the Deutsch-Jozsa algorithm



Quantum interference in the Deutsch-Jozsa algorithm

Quantum interference allows one to distinguish the situation where half the amplitudes are +1 and half -1 from the situation where all the amplitudes are +1 or -1 (this is the information one wants) without having to determine all amplitudes (this information remains inaccessible).

Entanglement in the Deutsch-Jozsa algorithm

N = 3

$$|\Phi\rangle = \frac{1}{2\sqrt{2}} \sum_{x,y,z} (-1)^{f(x,y,z)} |x, y, z\rangle$$

This state is globally entangled for some balanced functions.

Example

$$\begin{aligned} f(0,0,0) &= 0 \\ f(0,0,1) &= 0 \\ f(0,1,0) &= 1 \\ f(0,1,1) &= 1 \\ f(1,0,0) &= 0 \\ f(1,0,1) &= 1 \\ f(1,1,0) &= 0 \\ f(1,1,1) &= 1 \end{aligned}$$

$$\begin{aligned} |\Phi\rangle &= \frac{1}{2\sqrt{2}} \big(|000\rangle + |001\rangle - |010\rangle - |011\rangle \\ &\quad + |100\rangle - |101\rangle + |110\rangle - |111\rangle \big) \end{aligned}$$

Implementations

V. Quantum error correction

Aspens
Sangre de Cristo Range
Northern New Mexico

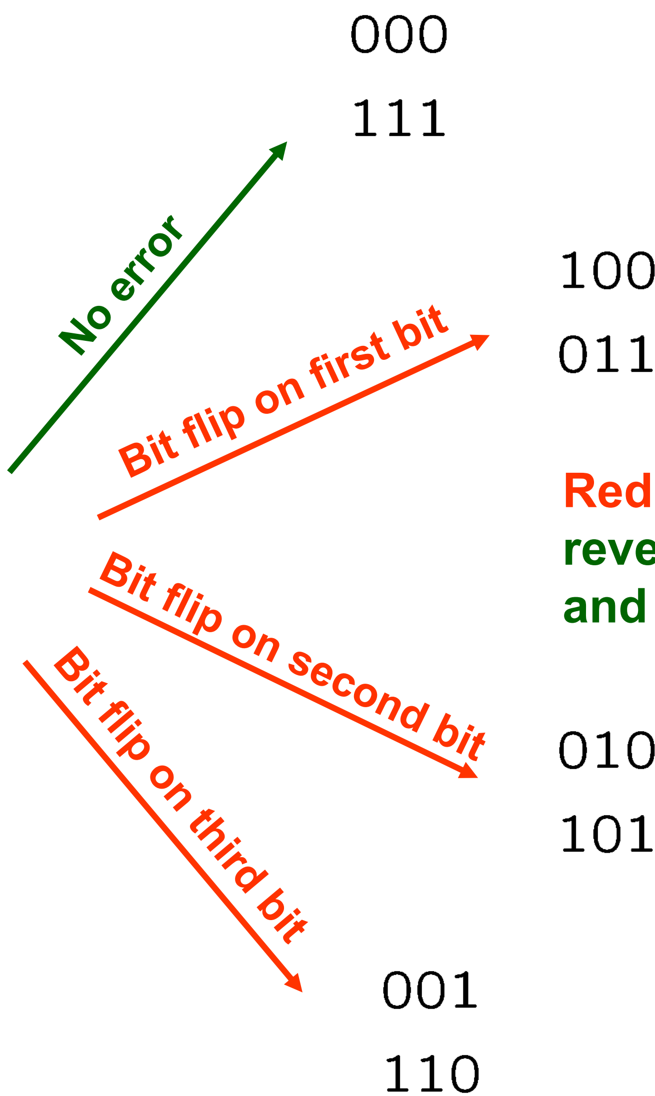
Classical error correction

Correcting single bit flips

$$0_L = 000$$
$$1_L = 111$$

code words

Copying



Redundancy: majority voting reveals which bit has flipped, and it can be flipped back.

Quantum error correction

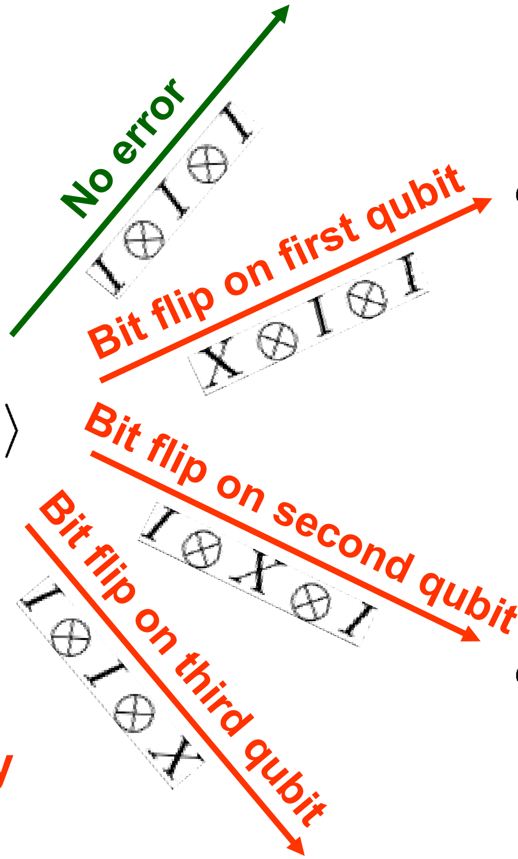
Correcting single bit flips

Four errors map the code subspace unitarily to four orthogonal subspaces.

$$|0\rangle_L = |000\rangle$$
$$|1\rangle_L = |111\rangle$$

code states

$$|\psi\rangle = \alpha|000\rangle + \beta|111\rangle$$



$\alpha|000\rangle + \beta|111\rangle$ **Even Even**

$\alpha|100\rangle + \beta|011\rangle$ **Odd Even**

Parity of pairs 12 and 23

$Z \otimes Z \otimes I$ and $I \otimes Z \otimes Z$

Error syndrome

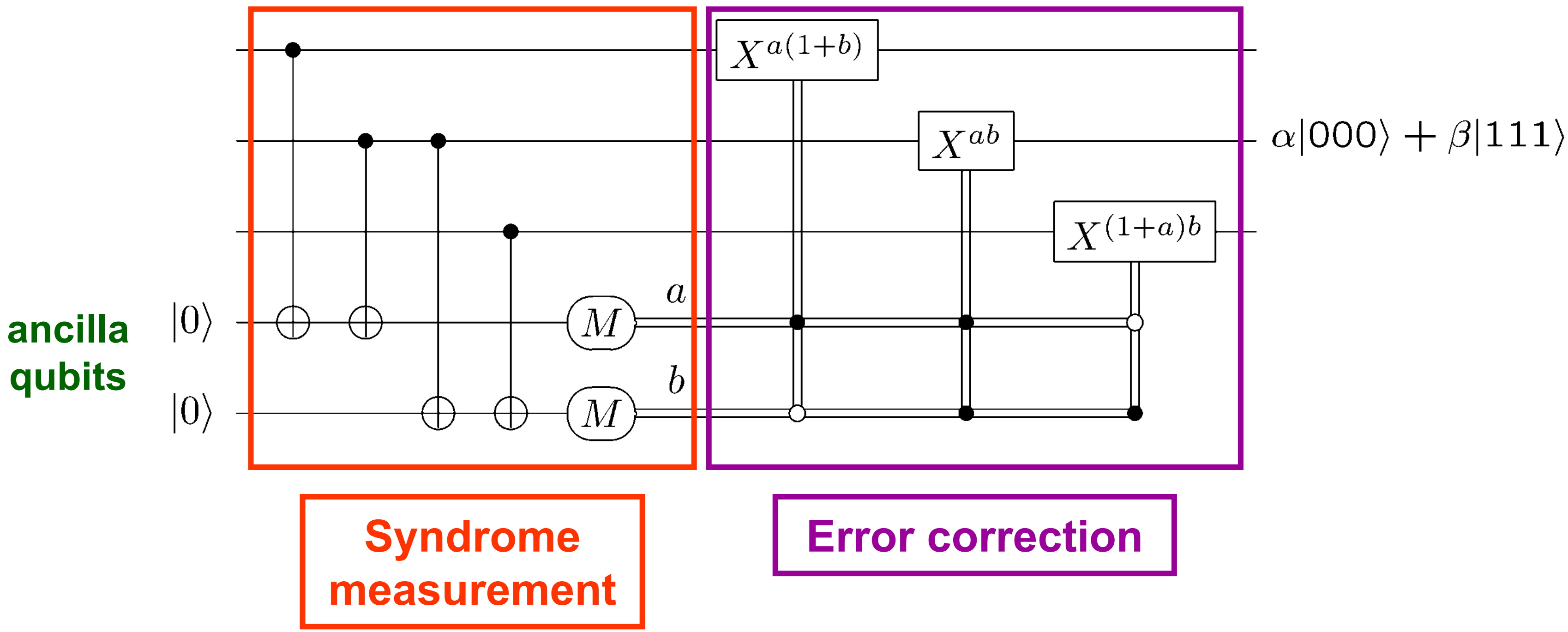
$\alpha|010\rangle + \beta|101\rangle$ **Odd Odd**

$\alpha|001\rangle + \beta|110\rangle$ **Even Odd**

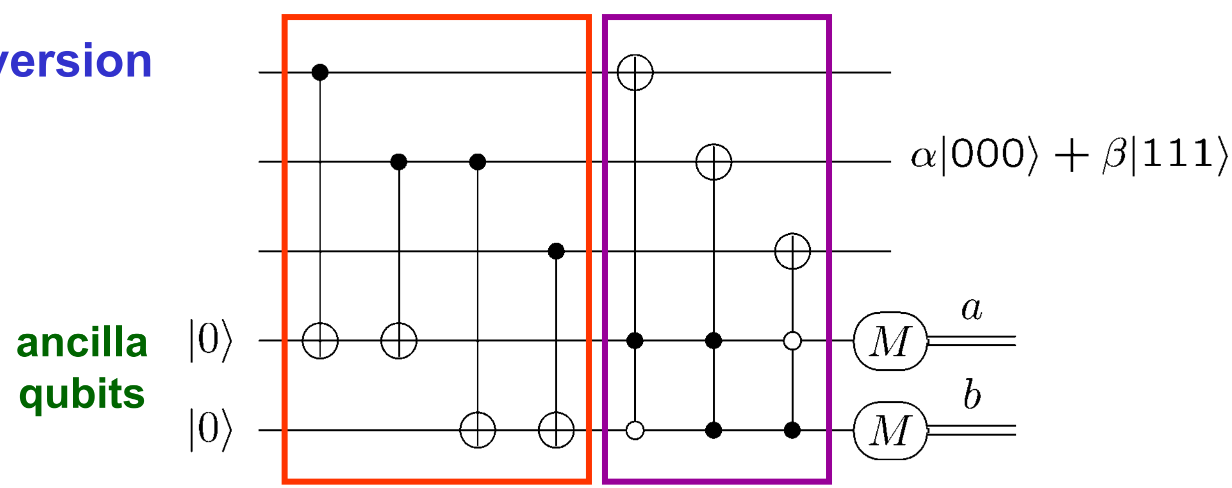
No need for copying. Redundancy replaced by nonlocal storage of information.

Quantum error correction

Single bit flip correction circuit



Coherent version



Quantum error correction

Other quantum errors?

Entanglement

phase error *Z*

code states

$$|+\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$
$$|-\rangle = \frac{1}{\sqrt{2}}(|000\rangle - |111\rangle)$$

$$\begin{pmatrix} Z\otimes I\otimes I \\ I\otimes Z\otimes I \\ I\otimes I\otimes Z \end{pmatrix} \begin{pmatrix} |+\rangle \\ |-\rangle \end{pmatrix} = \begin{pmatrix} |+\rangle \\ |-\rangle \end{pmatrix}$$

A phase error on any of the three qubits flips $|+\rangle$ and $|-\rangle$.

Shor's 9-qubit code

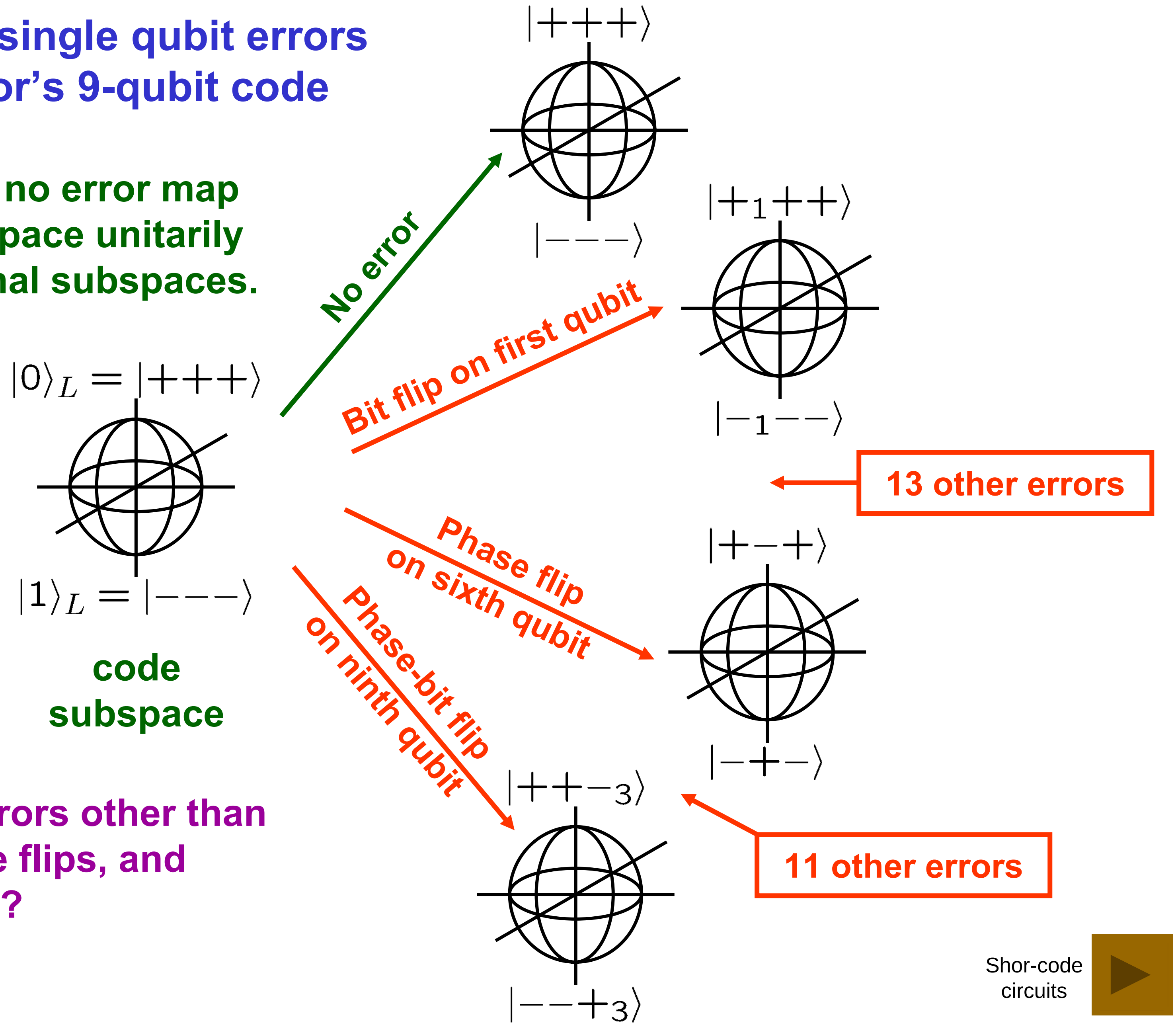
$$|0\rangle_L = |{+}{+}{+}\rangle = \frac{1}{2\sqrt{2}}(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)$$
$$|1\rangle_L = |{-}{-}{-}\rangle = \frac{1}{2\sqrt{2}}(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)$$

Corrects all single-qubit errors

Quantum error correction

Correcting single qubit errors using Shor's 9-qubit code

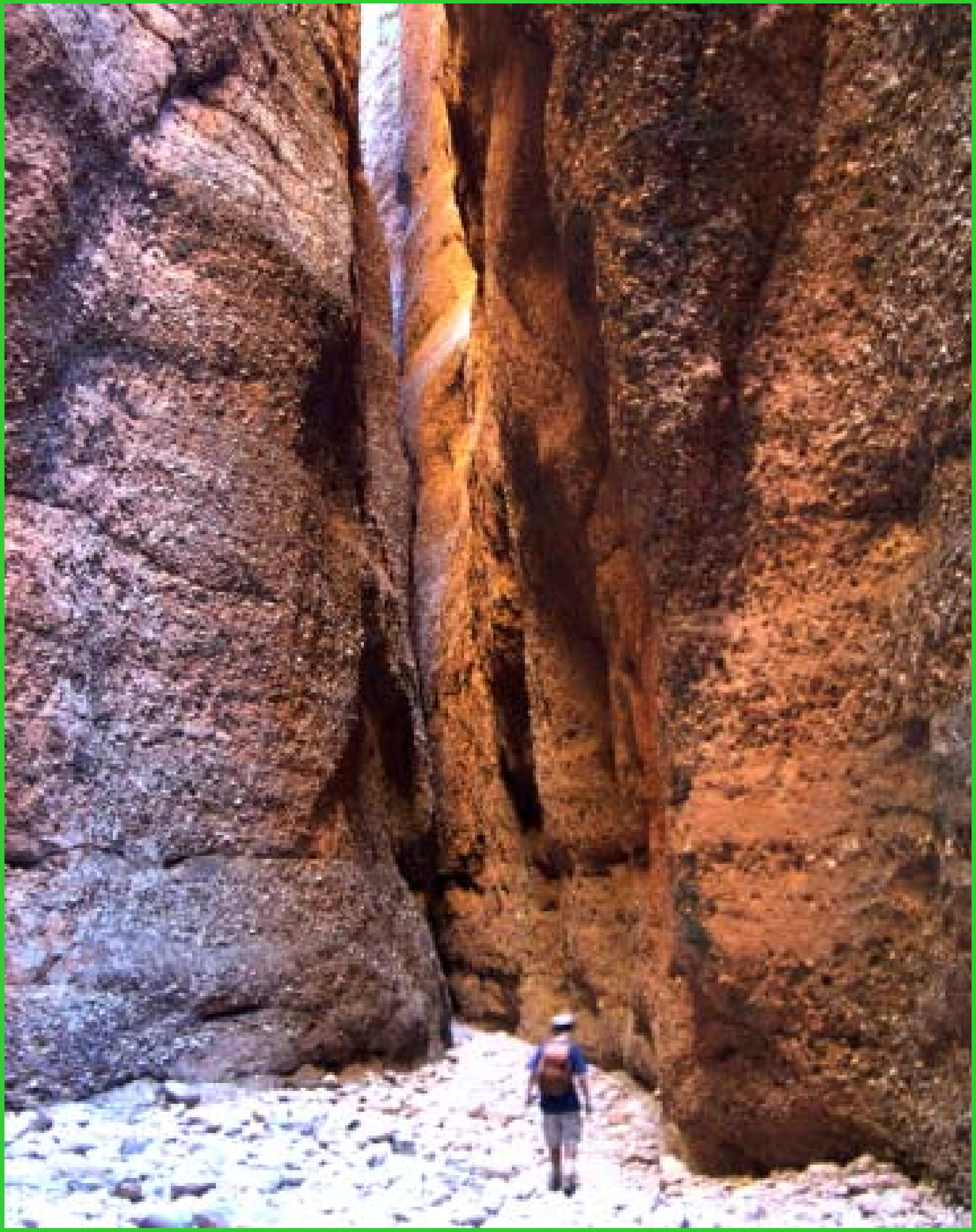
27 errors plus no error map the code subspace unitarily to 22 orthogonal subspaces.



What about errors other than bit flips, phase flips, and phase-bit flips?

Shor-code circuits

VI. Physical implementations



Echidna Gorge
Bungle Bungle Range
Western Australia

Implementations: DiVincenzo criteria

1. ***Scalability:*** A scalable physical system made up of well characterized parts, usually qubits.

2. ***Initialization:*** The ability to initialize the system in a simple fiducial state.

3. ***Control:*** The ability to control the state of the computer using sequences of elementary universal gates.

4. ***Stability:*** Decoherence times much longer than gate times, together with the ability to suppress decoherence through error correction and fault-tolerant computation.

5. ***Measurement:*** The ability to read out the state of the computer in a convenient product basis.

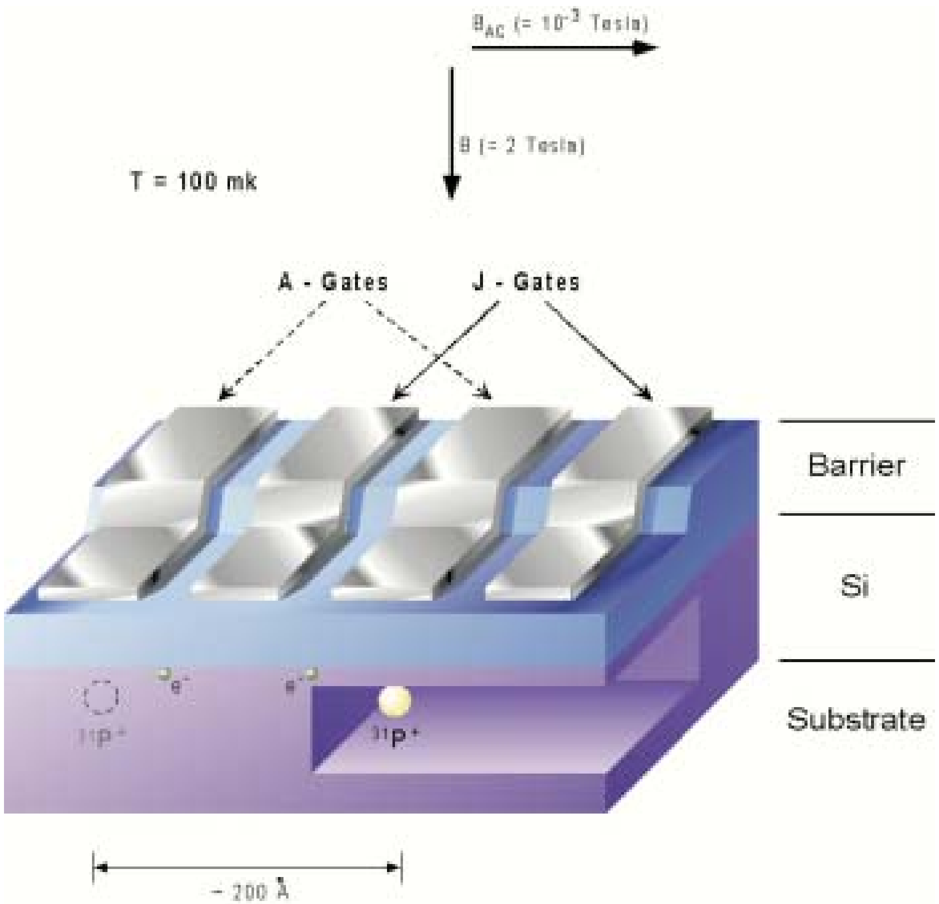
Strong coupling between qubits and of qubits to external controls and measuring devices (→ 2, 3, 5)

Weak coupling to everything else (→ 4)

Many qubits, entangled, protected from error, with initialization and readout for all.

Implementations

Original Kane proposal



Qubits: nuclear spins of P ions in Si; fundamental fabrication problem.

Single-qubit gates: NMR with addressable hyperfine splitting.

Two-qubit gates: electron-mediated nuclear exchange interaction.

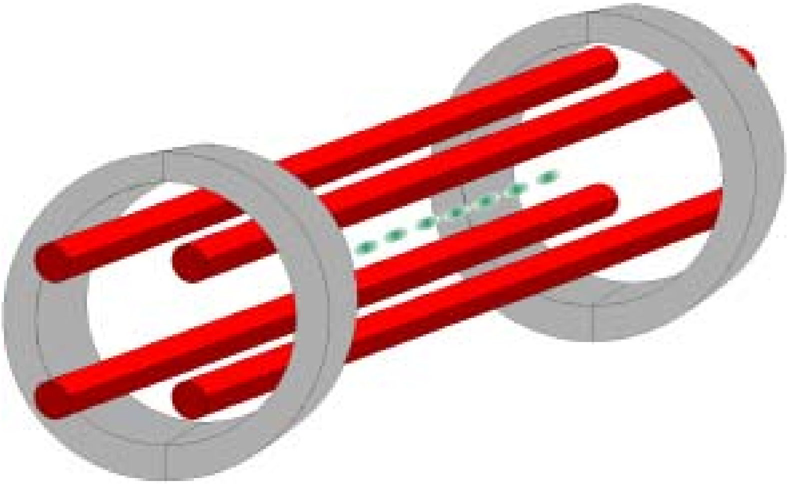
Decoherence: nuclear spins highly coherent, but decoherence during interactions unknown.

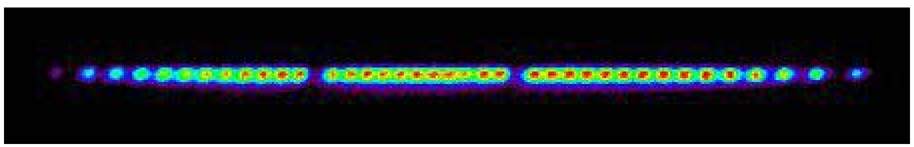
Readout: spin-dependent charge transfer plus single-electron detection.

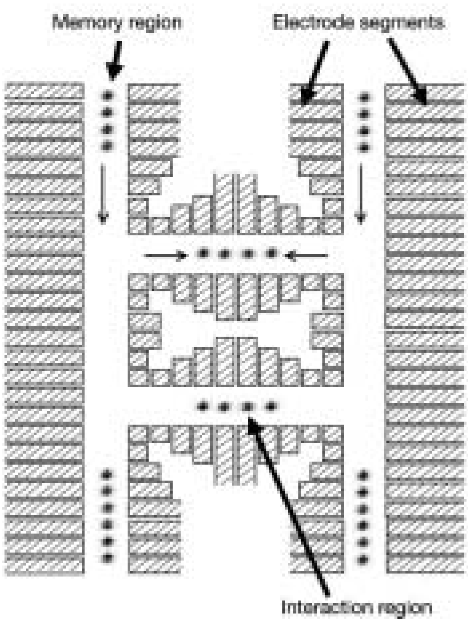
Scalability: if a few qubits can be made to work, scaling to many qubits might be easy.

Implementations

Ion traps







Qubits: electronic states of trapped ions (ground-state hyperfine levels or ground and excited states).

State preparation: laser cooling and optical pumping.

Single-qubit gates: laser-driven coherent transitions.

Two-qubit gates: phonon-mediated conditional transitions.

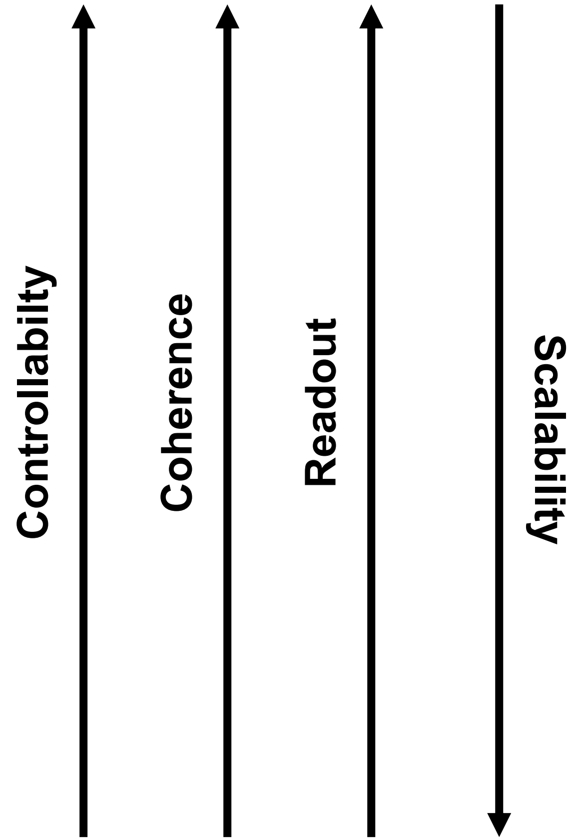
Decoherence: ions well isolated from environment.

Readout: fluorescent shelving.

Scalability: possibly scalable architectures, involving many traps and shuttling of ions between traps, are being explored.

Implementations

		Qubits
	Trapped ions	Electronic states
AMO systems	Trapped neutral atoms	Electronic states
	Linear optics	Photon polarization or spatial mode
	Superconducting circuits	Cooper pairs or quantized flux
Condensed systems	Doped semiconductors	Nuclear spins
	Semiconductor heterostructures	Quantum dots
	NMR	Nuclear spins (not scalable; high temperature prohibits preparation of initial pure state)



Implementations

ARDA Quantum Computing Roadmap, v. 2 (spring 2004)

By the year 2007, to

● encode a single qubit into the state of a logical qubit formed from several physical qubits,

● perform repetitive error correction of the logical qubit,

● transfer the state of the logical qubit into the state of another set of physical qubits with high fidelity, and

by the year 2012, to

● implement a concatenated quantum error correcting code.

It was the unanimous opinion of the Technical Experts Panel that it is too soon to attempt to identify a smaller number of potential “winners;” the ultimate technology may not have even been invented yet.

That's all, folks.

Bungle Bungle Range
Western Australia

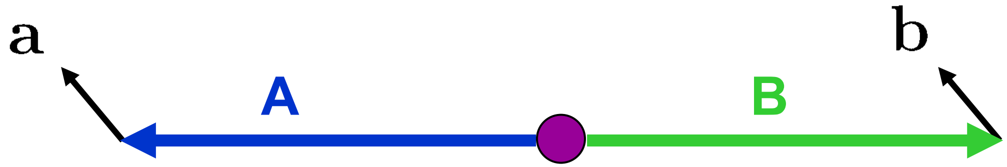
Entanglement, local realism, and Bell inequalities

Entangled state (quantum correlations)

$$|\Psi\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle$$

Bell entangled state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

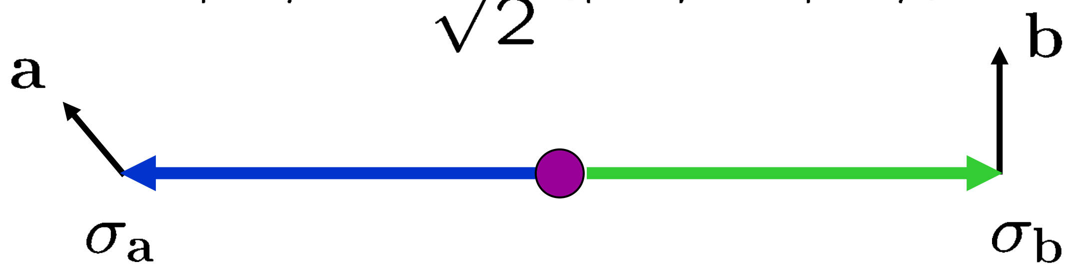


$\sigma_{\mathbf{a}} = \pm 1$ $\sigma_{\mathbf{b}} = \mp 1$

Entanglement, local realism, and Bell inequalities

Bell entangled state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



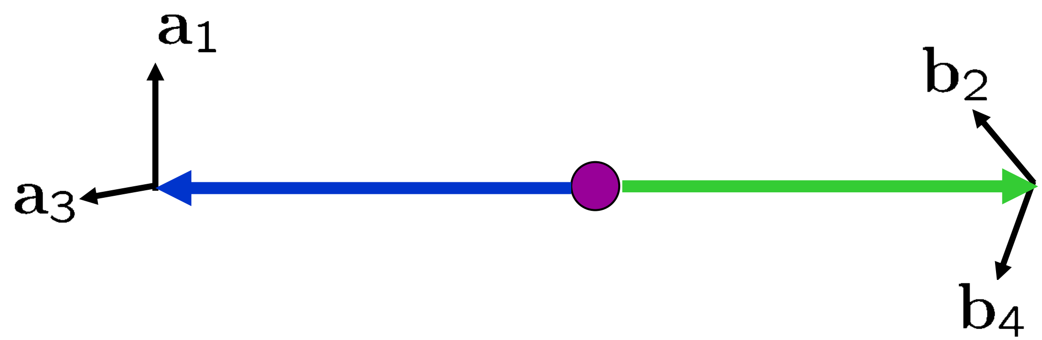
$$C(\mathbf{a},\mathbf{b}) \equiv \langle \sigma_{\mathbf{a}}\sigma_{\mathbf{b}}\rangle = -\mathbf{a}\cdot\mathbf{b} = -\cos\theta_{\mathbf{ab}}$$

Entanglement, local realism, and Bell inequalities

Local hidden variables (LHV) and Bell inequalities

Bell entangled state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



$$S = C(\mathbf{a}_1, \mathbf{b}_2) + C(\mathbf{a}_3, \mathbf{b}_2) + C(\mathbf{a}_3, \mathbf{b}_4) - C(\mathbf{a}_1, \mathbf{b}_4)$$

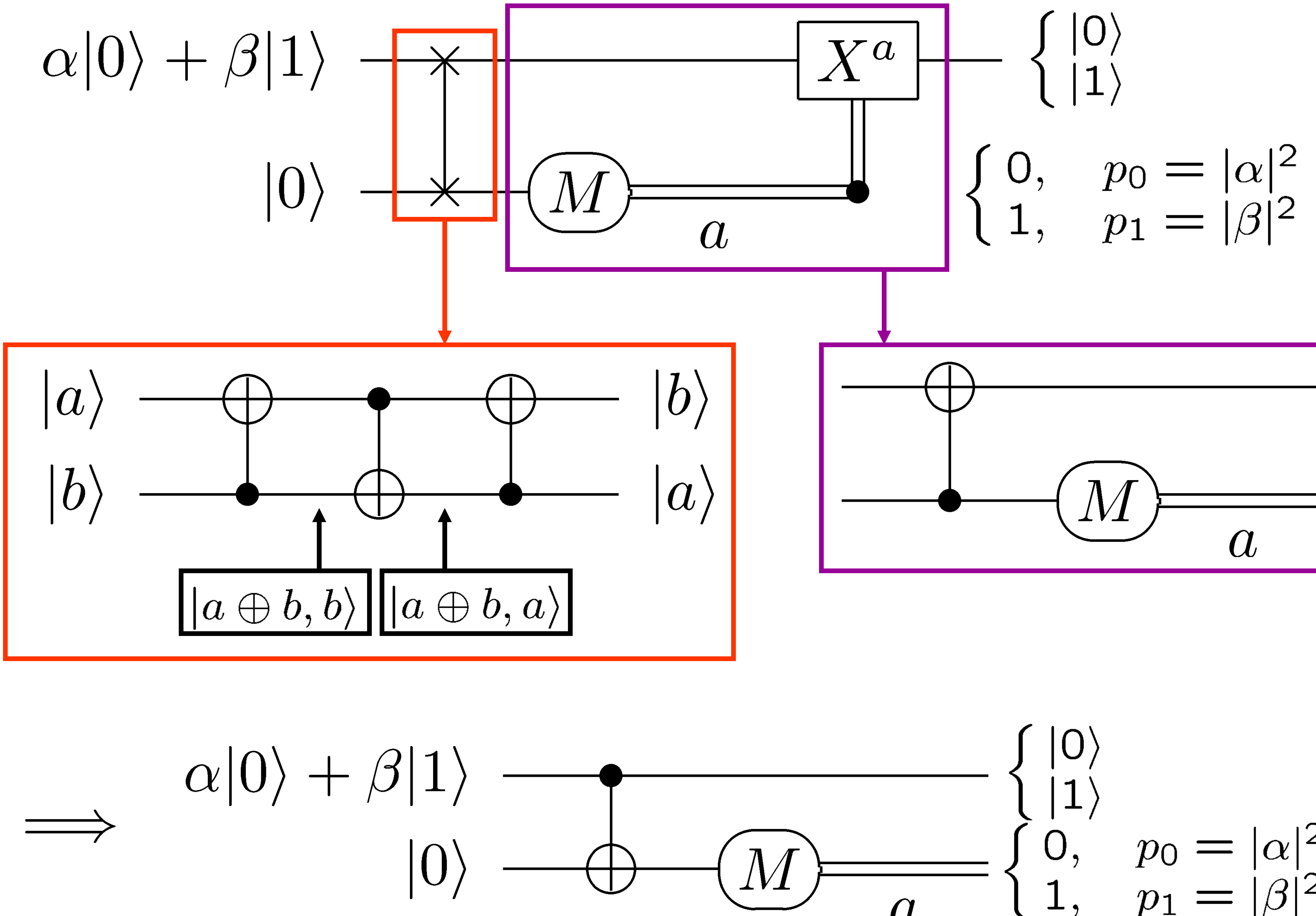
LHV: $$|S| = \left|\left\langle \underbrace{\sigma_{\mathbf{b}_2}(\sigma_{\mathbf{a}_3} + \sigma_{\mathbf{a}_1}) + \sigma_{\mathbf{b}_4}(\sigma_{\mathbf{a}_3} - \sigma_{\mathbf{a}_1})}_{=\pm 2} \right\rangle\right| \leq 2$$

QM: $$S = 2\sqrt{2}$$

The quantum correlations cannot be explained in terms of local, realistic properties.

Back

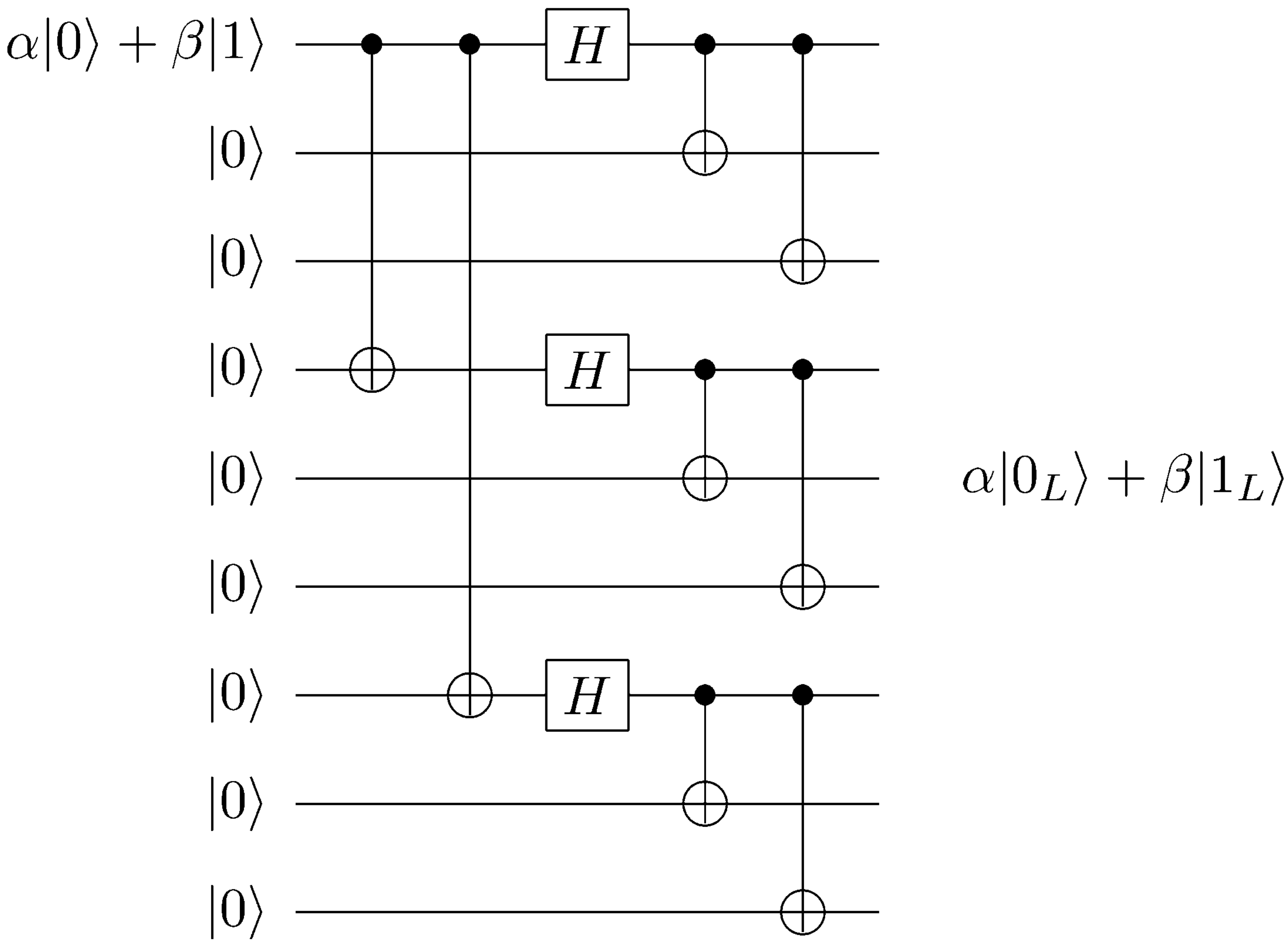
C-NOT as measurement gate: circuit identity



Back

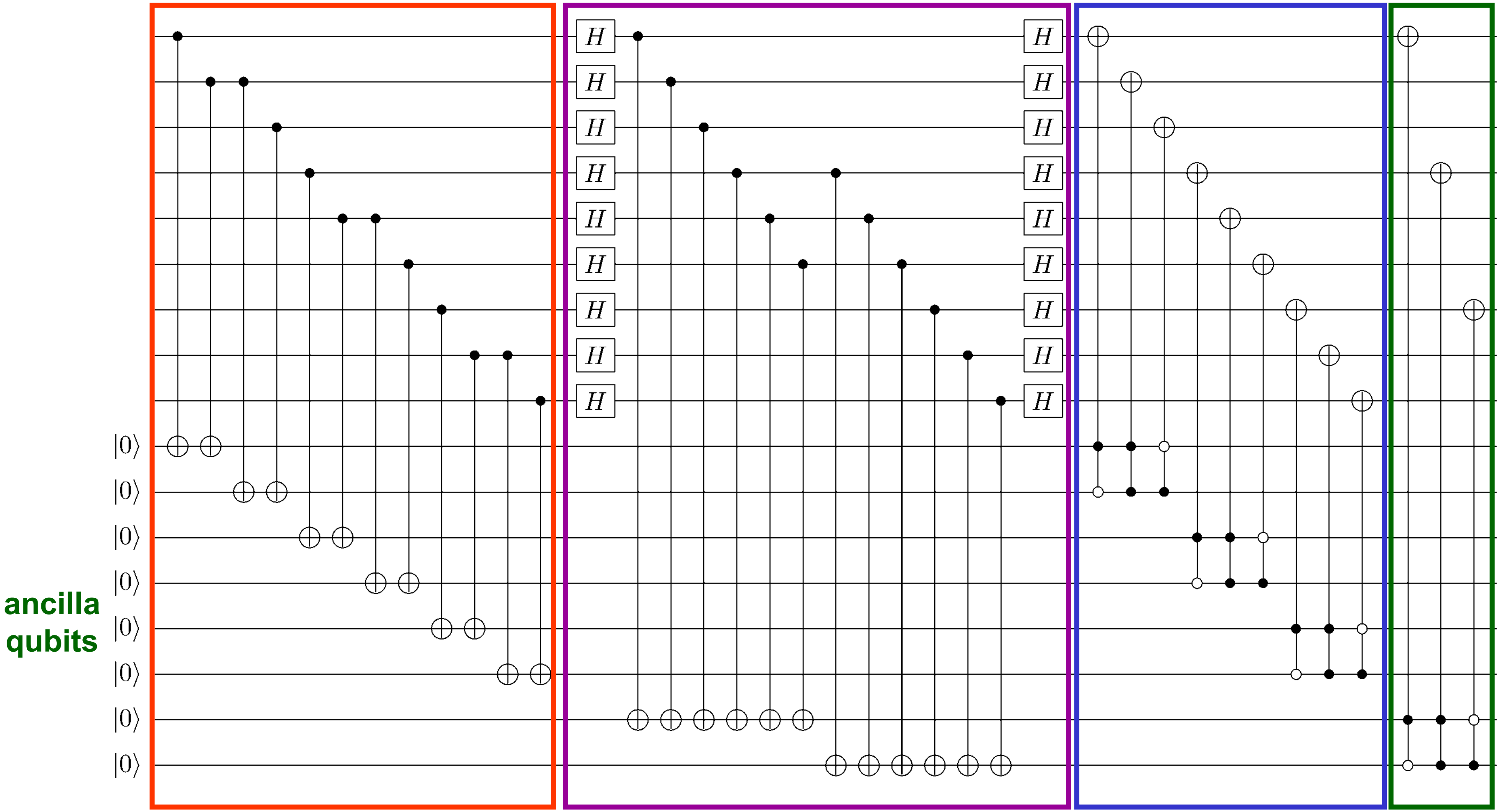
Quantum error correction

Shor code encoding circuit



Quantum error correction

Shor code correction circuit (coherent version)



Bit-flip syndrome detection

Phase-flip syndrome detection

Bit-flip correction

Phase-flip correction

Back