Complexity and Disorder at Ultra-Low Temperatures
30th Annual Conference of LANL Center for Nonlinear Studies
SantaFe, 2010 June 25

Quantum metrology: Dynamics vs. entanglement

I. Quantum noise limit and Heisenberg limit
II. Quantum metrology and resources
III. Beyond the Heisenberg limit
IV. Two-component BECs for quantum metrology

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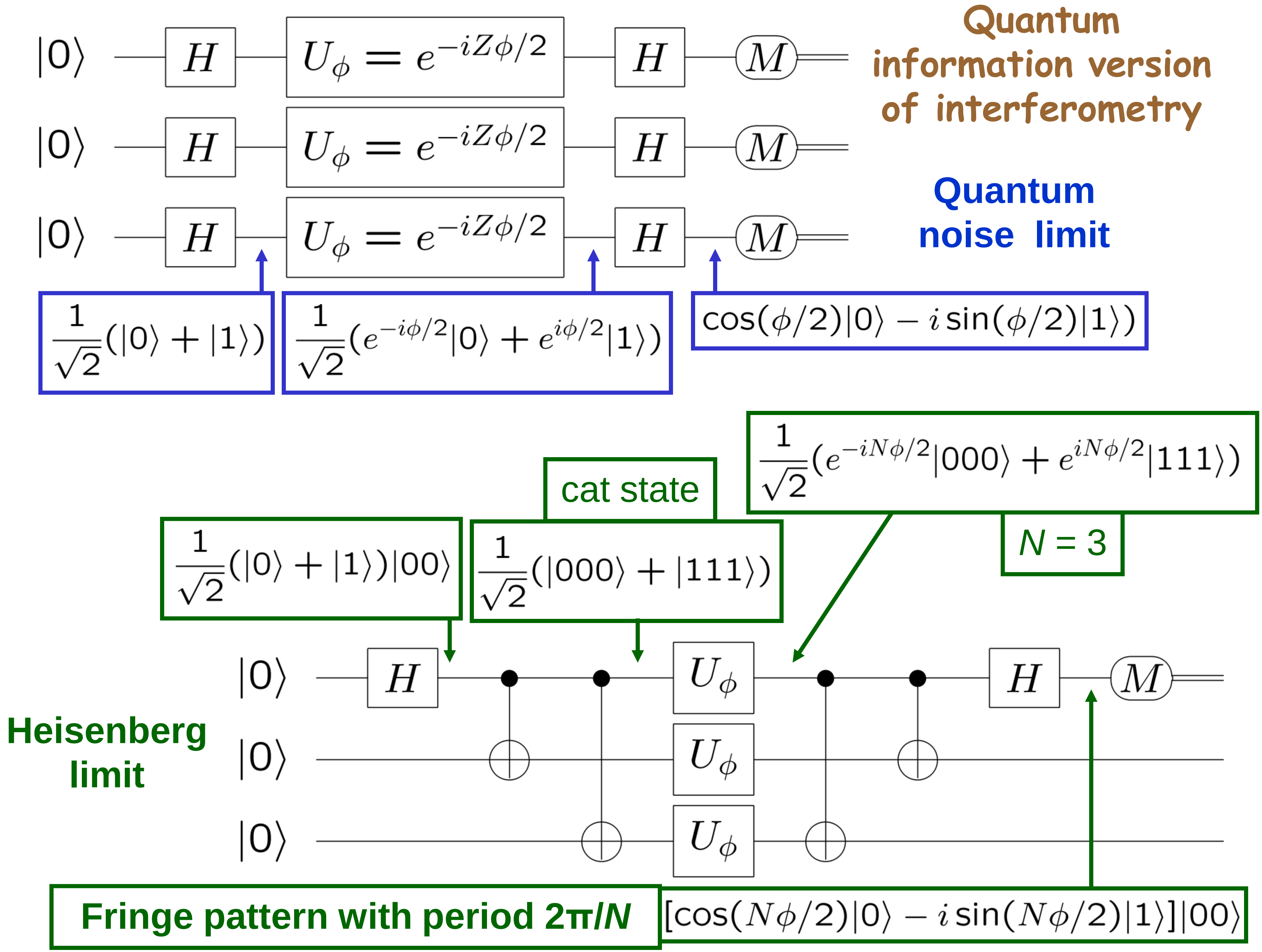
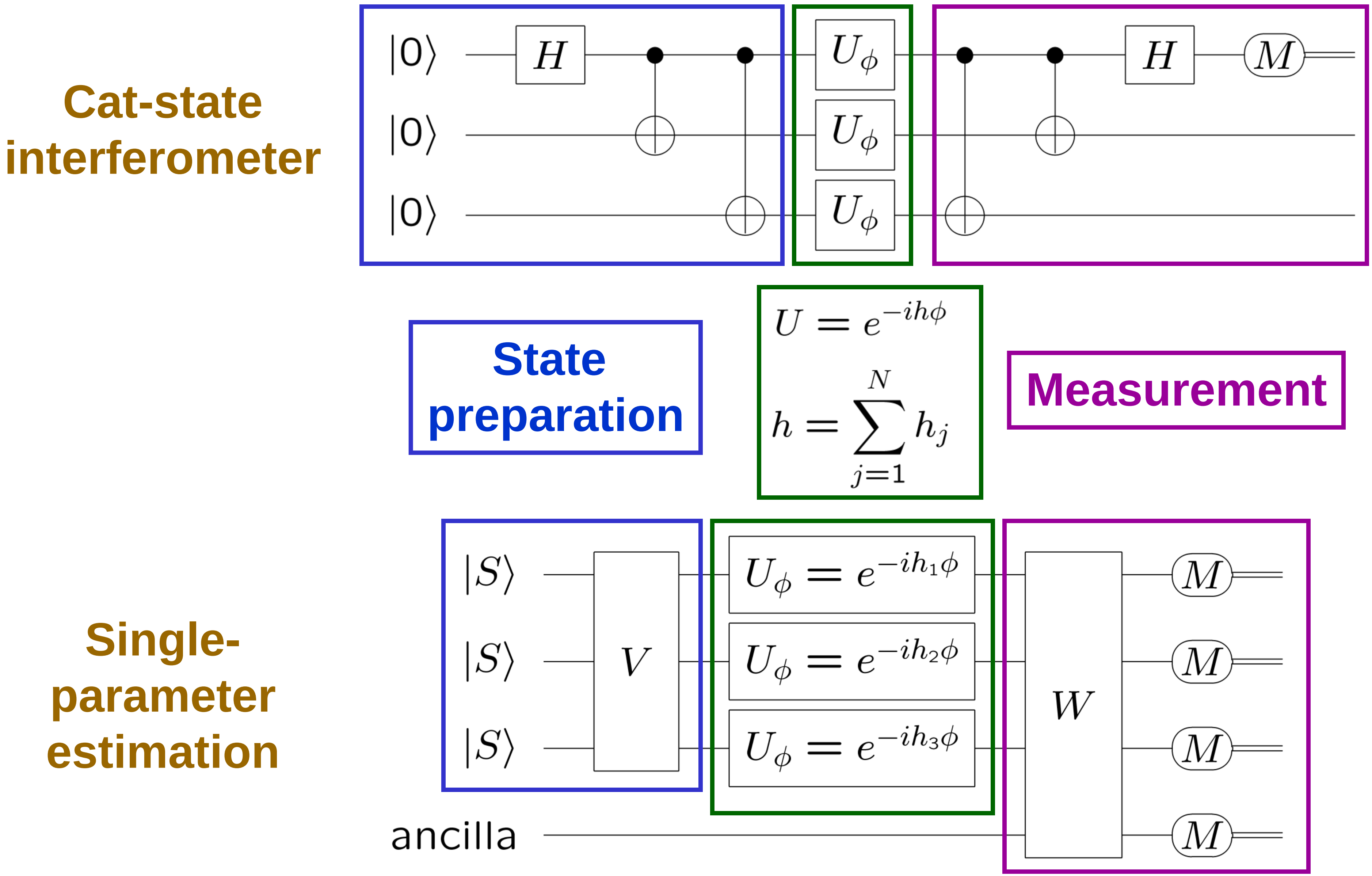
I. Quantum noise limit and Heisenberg limit

View from Cape Hauy
Tasman Peninsula
Tasmania

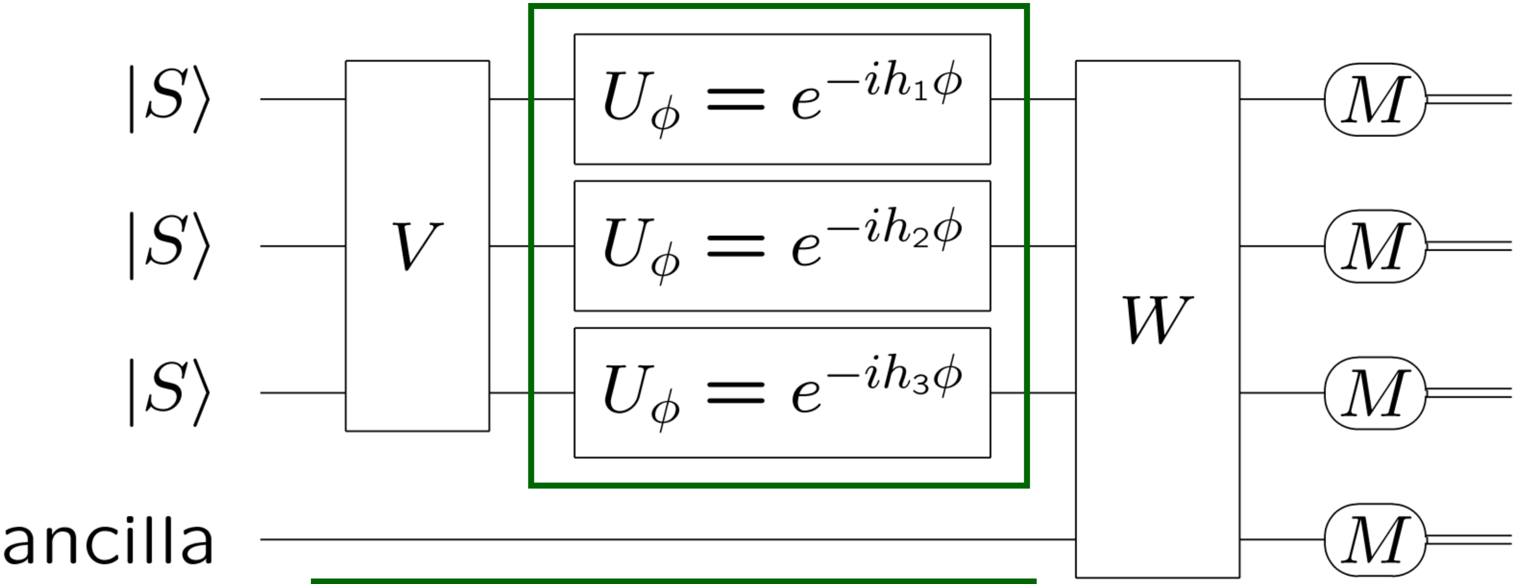

Quantum information version of interferometry
Quantum noise limit
$|0\rangle$
H
$U_\phi = e^{-iZ\phi/2}$
M
$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
$\frac{1}{\sqrt{2}}(e^{-i\phi/2}|0\rangle + e^{i\phi/2}|1\rangle)$
$\cos(\phi/2)|0\rangle - i\sin(\phi/2)|1\rangle)$
Heisenberg limit
$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|00\rangle$
cat state
$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$
$\frac{1}{\sqrt{2}}(e^{-iN\phi/2}|000\rangle + e^{iN\phi/2}|111\rangle)$
$N = 3$
U_ϕ
Fringe pattern with period $2\pi/N$
$[\cos(N\phi/2)|0\rangle - i\sin(N\phi/2)|1\rangle]|00\rangle$



Cat-state interferometer
$|0\rangle$
$|0\rangle$
$|0\rangle$
H
U_ϕ
U_ϕ
U_ϕ
H
M
State preparation
$U = e^{-ih\phi}$
$h = \sum_{j=1}^{N} h_j$
Measurement
Single-parameter estimation
$|S\rangle$
$|S\rangle$
$|S\rangle$
V
$U_\phi = e^{-ih_1\phi}$
$U_\phi = e^{-ih_2\phi}$
$U_\phi = e^{-ih_3\phi}$
W
M
M
M
M
ancilla

Heisenberg limit

S. L. Braunstein, C. M. Caves, and G. J. Milburn, Ann. Phys. **247**, 135 (1996).
V. Giovannetti, S. Lloyd, and L. Maccone, PRL **96**, 041401 (2006).



W, U_ϕ, and ancilla measurements can be interleaved.

$$U = e^{-ih\phi}\,, \qquad h = \sum_{j=1}^{N} h_j$$

$$\Delta\phi \ge \frac{1}{2\Delta h} \ge \frac{1}{N(\Lambda - \lambda)}$$

Generalized uncertainty principle (Cramér-Rao bound)

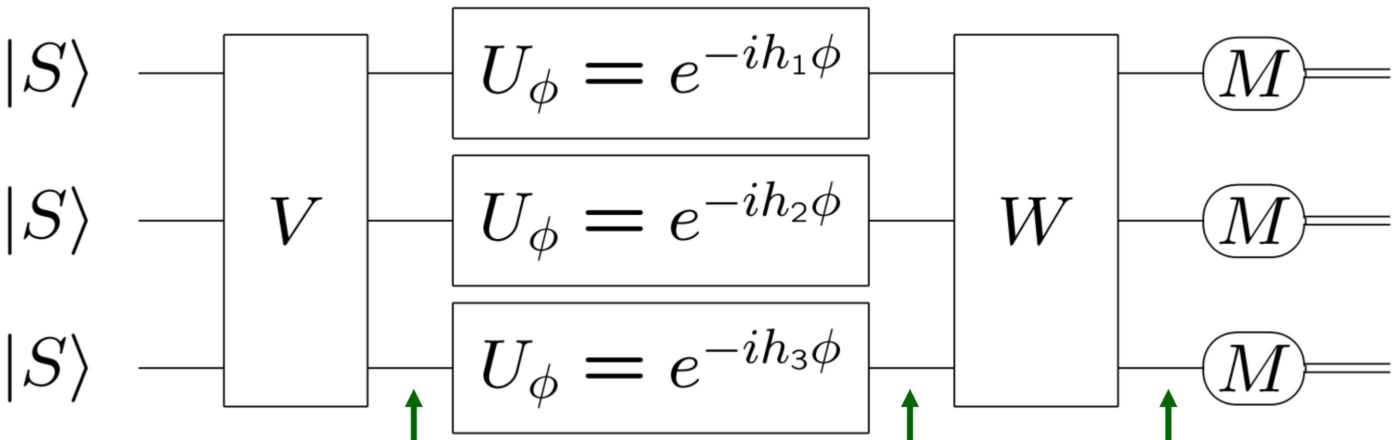
$$\Delta h \le \frac{1}{2}||h|| = \frac{1}{2} N(\Lambda - \lambda)$$

Separable inputs

$$\Delta h \le \frac{1}{2}\sqrt{N}(\Lambda - \lambda)$$

$$\Delta\phi \ge \frac{1}{\sqrt{N}(\Lambda - \lambda)}$$

Achieving the Heisenberg limit



cat state

$$\frac{1}{\sqrt{2}}(|\Lambda,\ldots,\Lambda\rangle + |\lambda,\ldots,\lambda\rangle)$$

$$\frac{1}{\sqrt{2}}\left(e^{-iN\Lambda\phi}|\Lambda,\ldots,\Lambda\rangle + e^{-iN\lambda\phi}|\lambda,\ldots,\lambda\rangle\right)$$

$$e^{-iN(\Lambda+\lambda)\phi/2}\Big(\cos[N(\Lambda-\lambda)\phi/2]|\Lambda,\ldots,\Lambda\rangle - i\sin[N(\Lambda-\lambda)\phi/2]|\lambda,\ldots,\lambda\rangle\Big)$$

Fringe pattern with period $2\pi/N(\Lambda-\lambda)$

$$\Delta\phi = \frac{1}{N(\Lambda-\lambda)}$$

II. Quantum metrology and resources

Oljeto Wash
Southern Utah

Making quantum limits relevant

Optimal sensitivity: $\Delta\omega \sim \frac{1}{TN}$

The serial resource, *T*, and the parallel resource, *N*, are equivalent and interchangeable, *mathematically.*

The serial resource, *T*, and the parallel resource, *N*, are not equivalent and not interchangeable, *physically.*

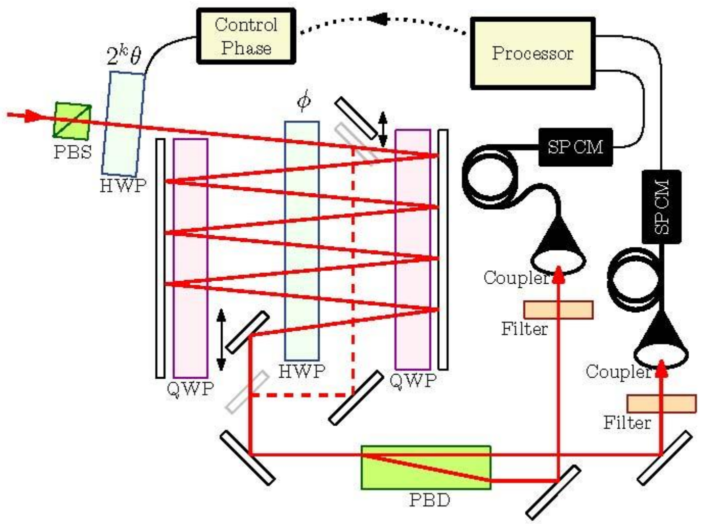
Information science perspective

Platform independence

Physics perspective

Distinctions between different physical systems

Working on T and N



B. L. Higgins, D. W. Berry, S. D. Bartlett, M. W. Mitchell, H. M. Wiseman, and G. J. Pryde, "Heisenberg-limited phase estimation without entanglement or adaptive measurements," arXiv:0809.3308 [quant-ph].

Laser Interferometer Gravitational Observatory (LIGO)

Advanced LIGO

$$\begin{pmatrix}\text{differential}\\ \text{strain}\\ \text{sensitivity}\end{pmatrix} \simeq 3\times 10^{-23}$$

from $10\,\text{Hz}$ to $10^3\,\text{Hz}$.

High-power, Fabry-Perot cavity (multipass), recycling, squeezed-state (?) interferometers

Livingston, Louisiana

Hanford, Washington

Making quantum limits relevant. One metrology story

Resources

- Overall measurement time τ or inverse bandwidth (the "*classical* serial resource")
- Coherent interaction time T for an individual probe (the "*quantum* serial resource")
- Rate R at which systems can be deployed ($R\tau = n$ is the "*classical* parallel resource")
- Entanglement within each probe consisting of N systems (N is the "*quantum* parallel resource")

Problem

Given τ and R, a decoherence rate Γ, and a marginal "cost" c for each nonclassical photon, what is the best strategy for estimating frequency $\omega = \phi/T$?

The answer has been worked out (in the case $c = 0$) for squeezed-state optical interferometry and for Ramsey interferometry with phase decoherence: The quantum resources—extended coherent evolution and entanglement—are useful only if $\Gamma\tau \lesssim 1$ and $R\tau \gg 1$. Other situations await analysis.

III. Beyond the Heisenberg limit

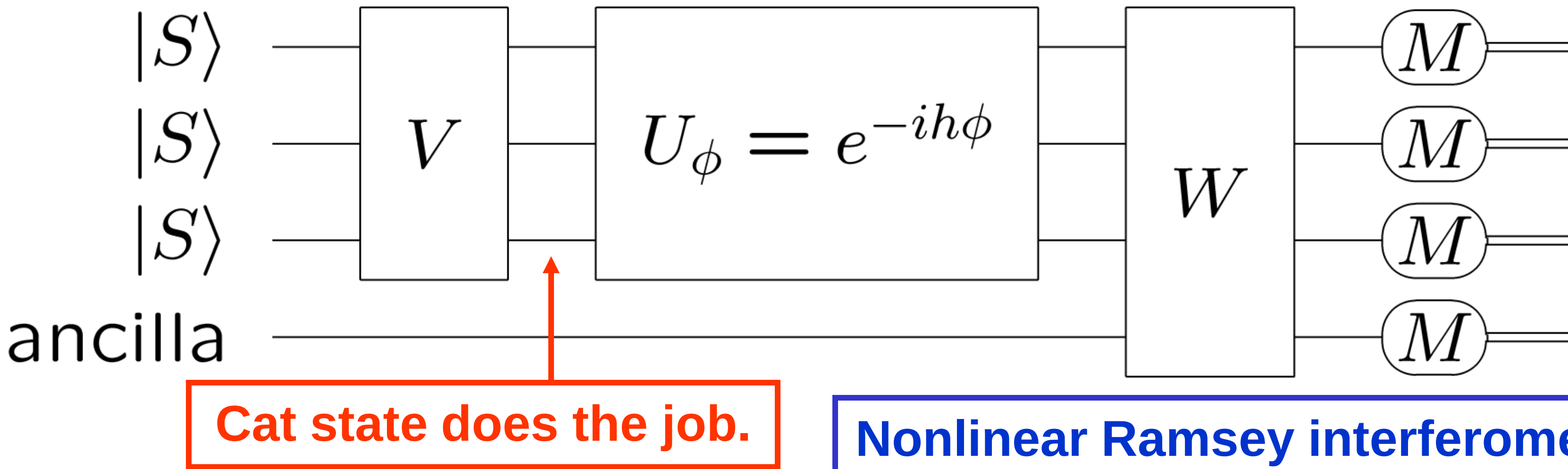
Truchas from East Pecos Baldy
Sangre de Cristo Range
Northern New Mexico

Beyond the Heisenberg limit

The purpose of theorems in physics is to lay out the assumptions clearly so one can discover which assumptions have to be violated.

Improving the scaling with N

S. Boixo, S. T. Flammia, C. M. Caves, and JM Geremia, PRL **98**, 090401 (2007).



Nonlinear Ramsey interferometry

$$\Delta\phi \geq \frac{1}{2\Delta h} \geq \frac{1}{||h||} = \frac{1}{N^k(\Lambda^k - \lambda^k)}$$

Metrologically relevant *k*-body coupling

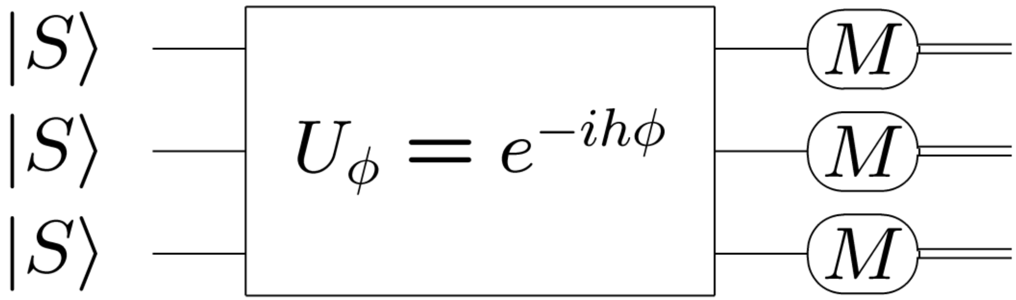
$$h = \left(\sum_{j=1}^{N} h_j\right)^k = \underbrace{\sum_{j_1,\ldots,j_k} h_{j_1} h_{j_2} \cdots h_{j_k}}_{N^k \text{ terms in sum}}$$

$$||h|| = N^k(\Lambda^k - \lambda^k)$$

Improving the scaling with N without entanglement

S. Boixo, A. Datta, S. T. Flammia, A. Shaji, E. Bagan, and C. M. Caves, PRA **77**, 012317 (2008).

Product input

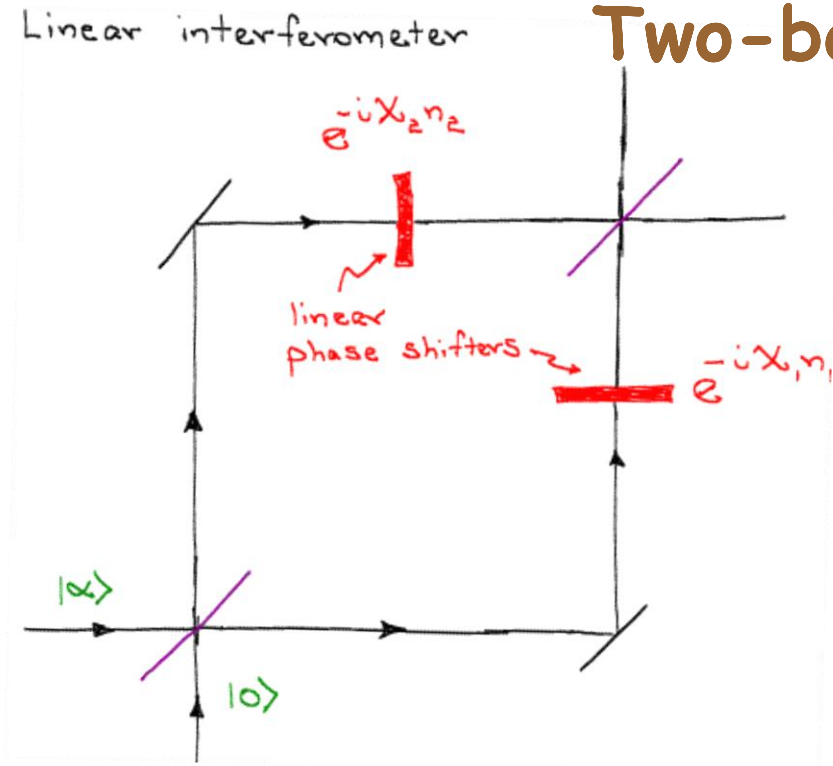


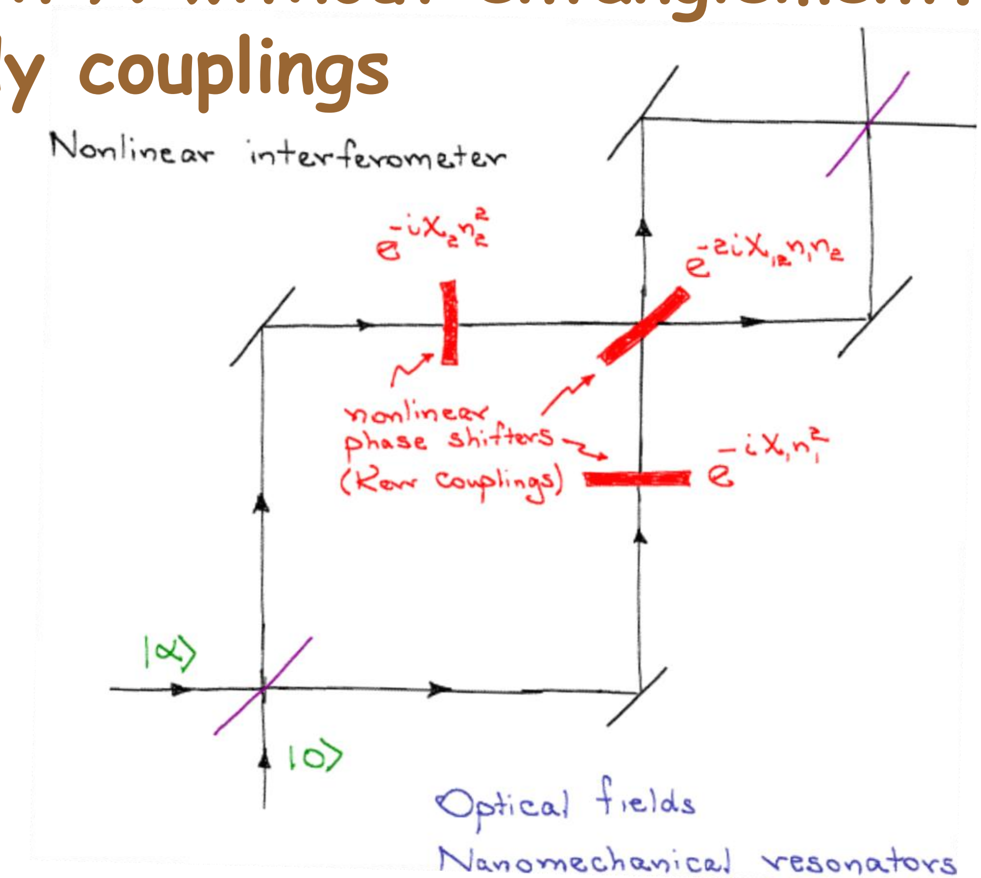
Product measurement

$$h = \left(\sum_{j=1}^{N} Z_j/2 \right)^k = J_z^k$$

$$\Delta\phi \sim \frac{1}{N^{k-1/2}}$$

Improving the scaling with N without entanglement. Two-body couplings





$$\chi_1 n_1 + \chi_2 n_2 = \tfrac{1}{2}(\chi_1+\chi_2)N + \underbrace{(\chi_1-\chi_2)}_{\equiv\,\phi} J_z$$

$$N = n_1 + n_2\,, \qquad J_z = \tfrac{1}{2}(n_1 - n_2)$$

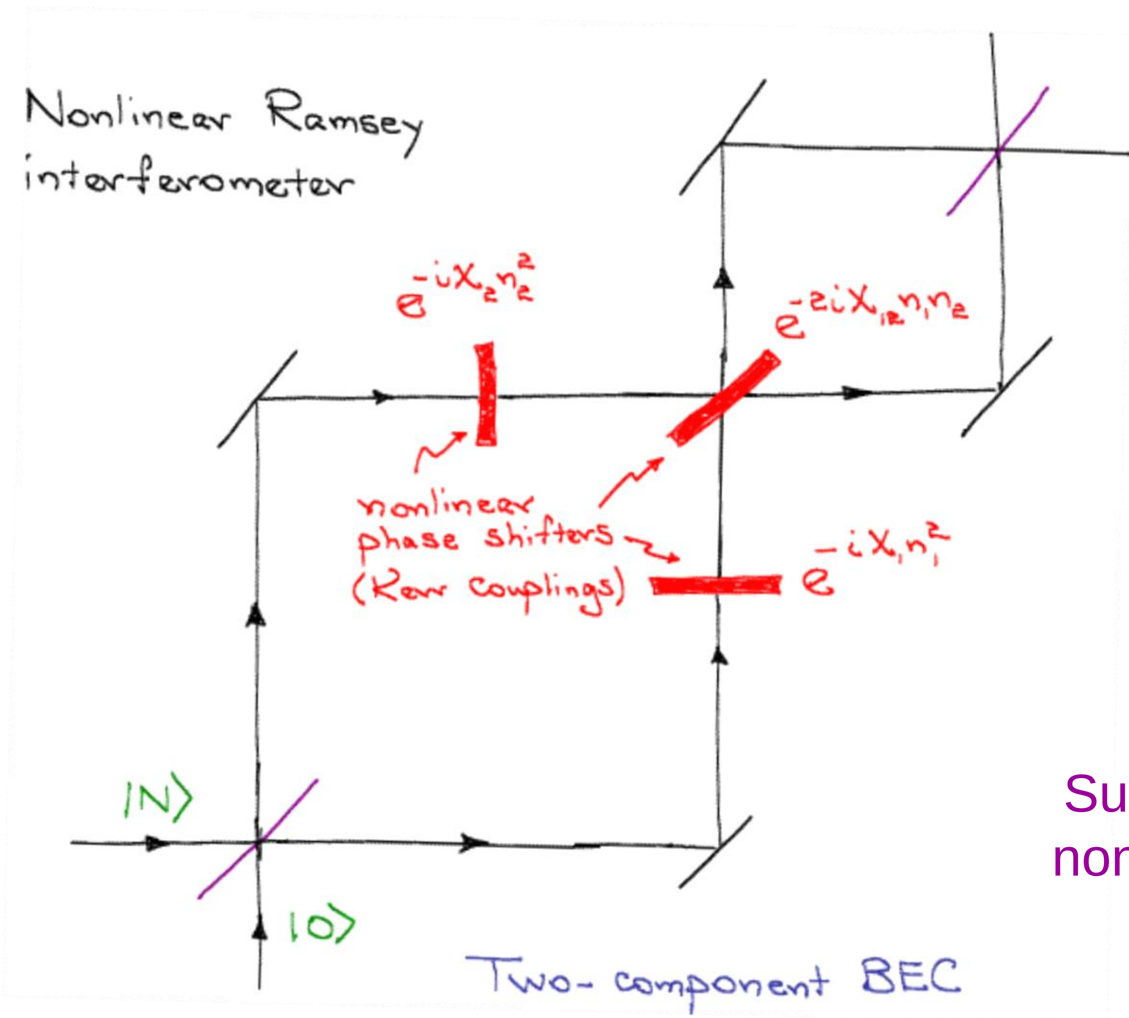
$$\Delta\phi = 1/\sqrt{N}$$

$$\chi_1 n_1^2 + \chi_2 n_2^2 + 2\chi_{12} n_1 n_2 = \tfrac{1}{4}(\chi_1+\chi_2+2\chi_{12})N^2 + (\chi_1-\chi_2)NJ_z + (\chi_1+\chi_2-2\chi_{12})J_z^2$$

$$\Delta\phi = 1/N^{3/2}$$

S. Boixo, A. Datta, S. T. Flammia, A. Shaji, E. Bagan, and C. M. Caves, PRA **77**, 012317 (2008); M. J. Woolley, G. J. Milburn, and C. M. Caves, NJP **10**, 125018 (2008);

Improving the scaling with N without entanglement. Two-body couplings



$$\chi_1 n_1^2 + \chi_2 n_2^2 + 2\chi_{12} n_1 n_2 = \frac{1}{4}(\chi_1 + \chi_2 + 2\chi_{12})N^2 + \underbrace{(\chi_1 - \chi_2)}_{\equiv \phi} N J_z + \underbrace{(\chi_1 + \chi_2 - 2\chi_{12})}_{=0} J_z^2$$

$$\Delta\phi = 1/N^{3/2}$$

Super-Heisenberg scaling from nonlinear dynamics, without any particle entanglement

Scaling robust against decoherence

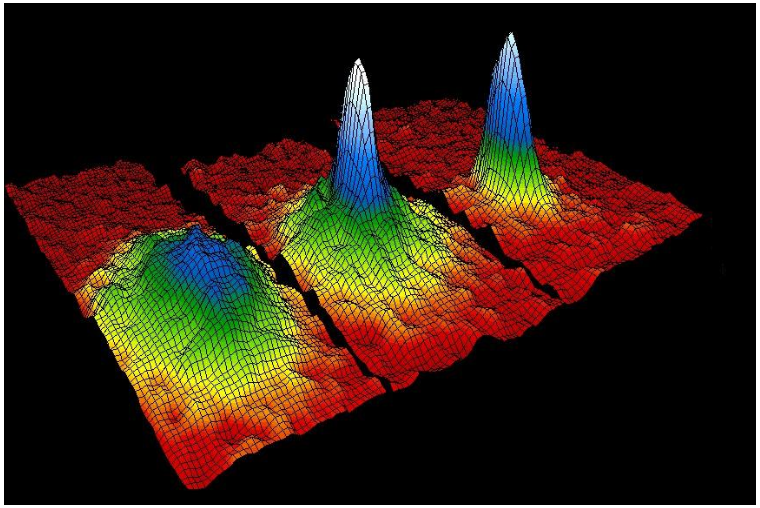
S. Boixo, A. Datta, M. J. Davis, S. T. Flammia, A. Shaji, and C. M. Caves, PRL **101**, 040403 (2008); S. Boixo, A. Datta, M. J. Davis, A. Shaji, A. B. Tacla, and C. M. Caves,PRA **80**, 032103 (2009).

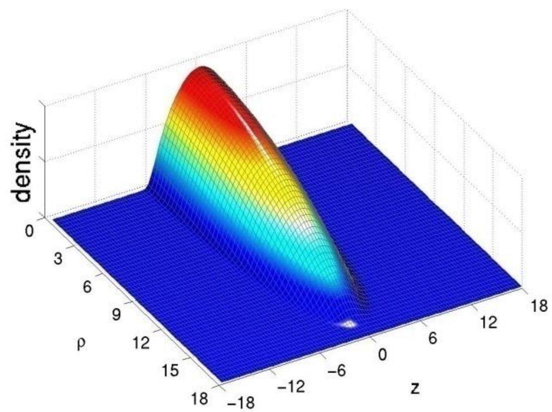
IV. Two-component BECs for quantum metrology

Czarny Staw Gąsienicowy
Tatras
Poland

Two-component BECs

S. Boixo, A. Datta, M. J. Davis, S. T. Flammia, A. Shaji, and C. M. Caves, PRL 101, 040403 (2008); S. Boixo, A. Datta, M. J. Davis, A. Shaji, A. B. Tacla, and C. M. Caves, PRA **80**, 032103 (2009).





Nonlinear BEC Ramsey interferometer

^{87}Rb atoms cooled to spatial ground state in hyperfine level $|F=1;M_F=-1\rangle$. Other relevant hyperfine level is $|F=2;M_F=1\rangle$, which sees the same trapping potential.

- $\pi/2$ transition.
- Atoms in $|1\rangle$ see nonlinear phase shift $\frac{1}{2}(g_{11}n_1^2+g_{12}n_1n_2)$, and atoms in $|2\rangle$ see nonlinear phase shift $\frac{1}{2}(g_{12}n_1n_2+g_{22}n_2^2)$, where $g_{jk}=4\pi\hbar^2 a_{jk}/m$.
- $\pi/2$ transition.
- Measure number of atoms in $|1\rangle$ and $|2\rangle$.

$$a_{11}=100.40a_0\ ,\quad a_{22}=95.00a_0\ ,\quad a_{12}=97.66a_0$$

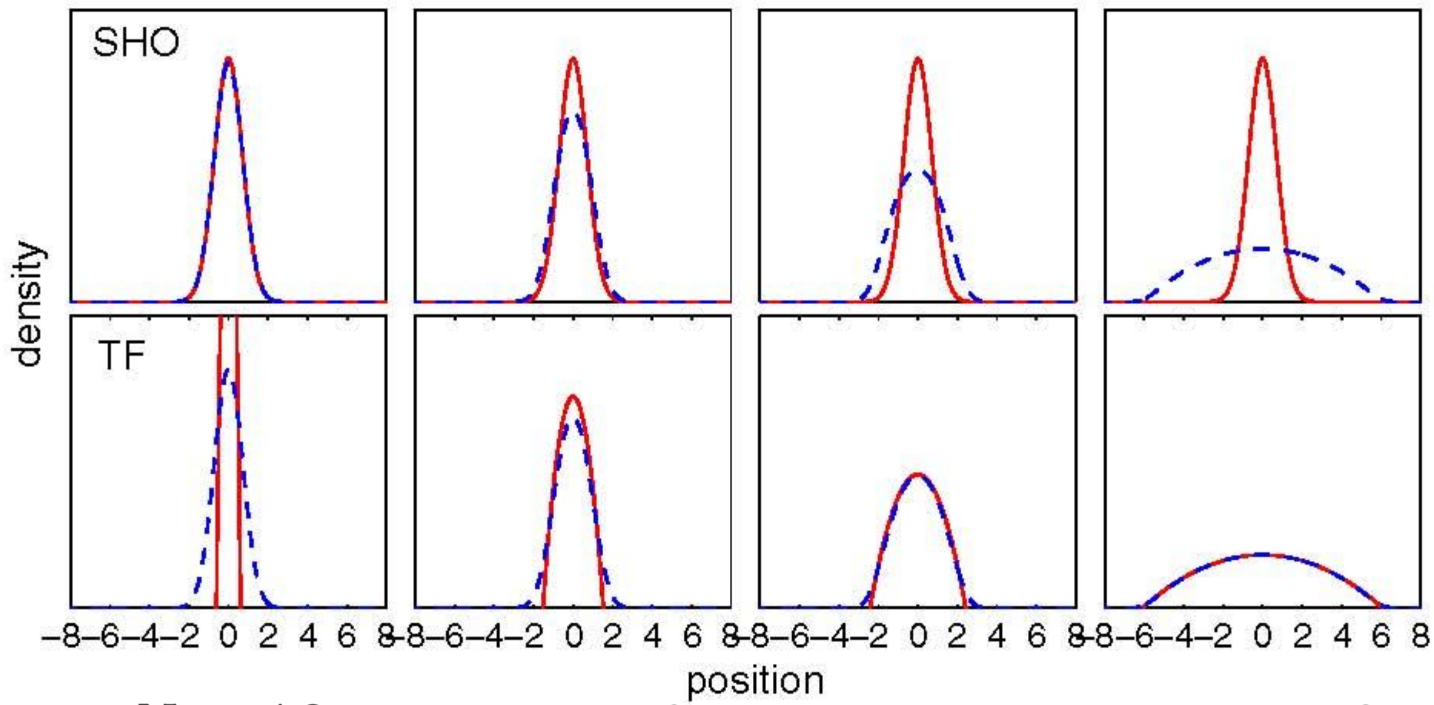
$$\frac{1}{2}(a_{11}-a_{22})=2.70a_0\ ,\quad \frac{1}{2}(a_{11}+a_{22})-a_{12}=0.04a_0$$

Nearly pure NJ_z coupling to measure $\gamma=\frac{1}{2}(g_{11}-g_{22})$

Two-component BECs

Isotropic, harmonic trap with bare ground-state width r_0

$$\begin{pmatrix}\text{critical atom}\\ \text{number}\end{pmatrix} = N_c \simeq 1 + \frac{r_0}{6a}$$



$N = 10$ $N = 10^3$ $N = 10^4$ $N = 10^6$

J. E. Williams, PhD dissertation, University of Colorado, 1999.

Two-component BECs

Isotropic, harmonic trap with bare ground-state width r_0

$$\begin{pmatrix}\text{critical atom}\\ \text{number}\end{pmatrix} = N_c \simeq 1 + \frac{r_0}{6a}$$

Renormalization of scattering strength

$$\frac{r_N}{r_0} \sim \left(\frac{N-1}{N_L-1}\right)^{1/5} \qquad \frac{g}{r_N^3} \sim \frac{g}{r_0^3}\left(\frac{N_L-1}{N-1}\right)^{3/5}$$

$$\Delta\gamma \sim 1/N^{9/10}$$

Let's start over.

Two-component BECs

Anisotropic, nonharmonic trap: d dimensions loosely confined by a power-law potential $V = \frac{1}{2}kr^q$, with bare ground-state width $r_0 \simeq (\hbar^2/mk)^{1/(q+2)}$; $D = 3 - d$ dimensions tightly confined in a harmonic potential with bare ground-state width $\rho_0 \ll r_0$.

$$\begin{pmatrix} \text{critical atom} \\ \text{number} \end{pmatrix} = N_L \simeq 1 + \beta_d \frac{r_0}{a}\left(\frac{\rho_0}{r_0}\right)^D , \qquad \beta_d = \begin{cases} 1\,, & d = 1, \\ \sqrt{\pi}/4\,, & d = 2, \\ 1/6\,, & d = 3. \end{cases}$$

Renormalization of scattering strength

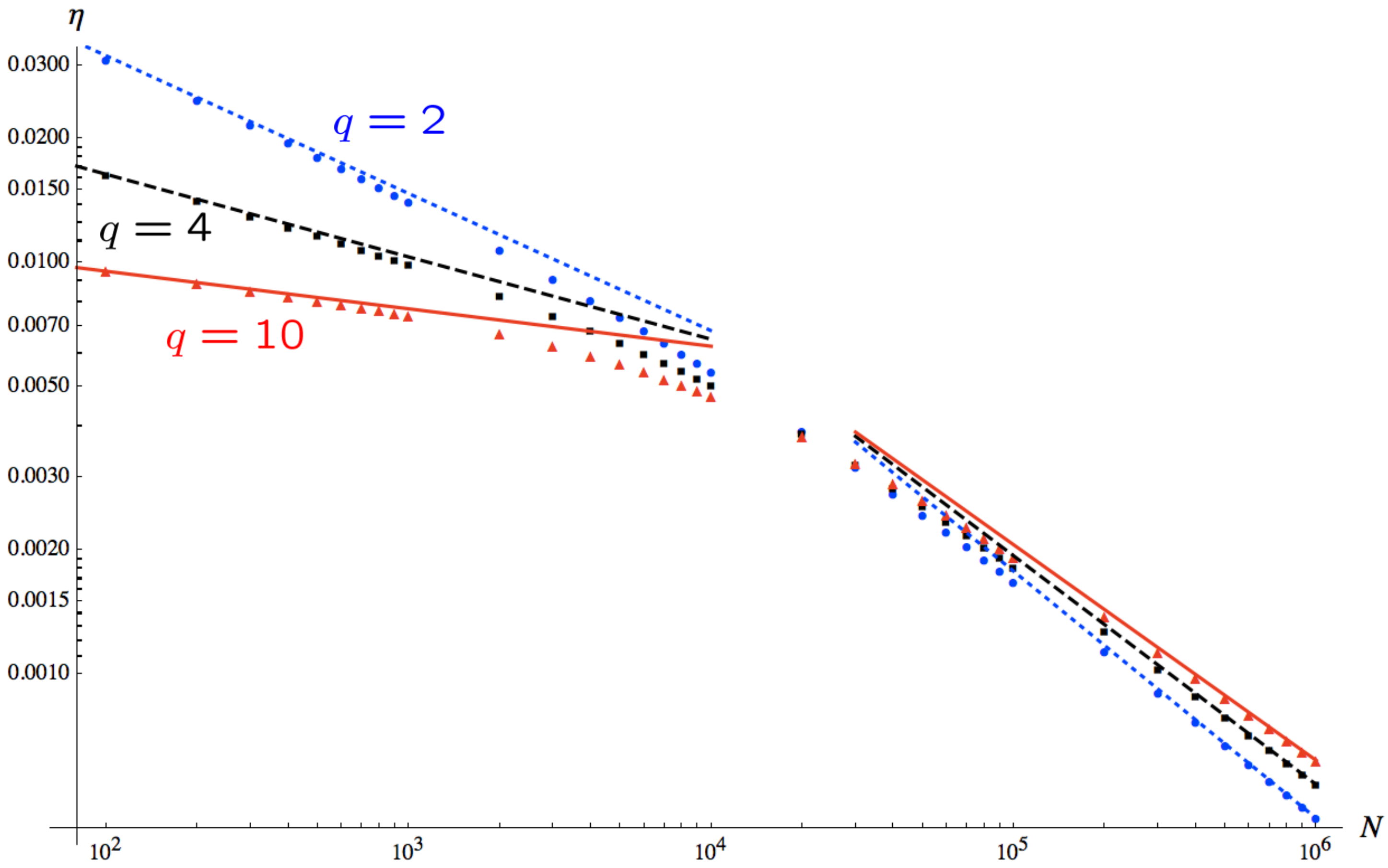
$$\frac{r_N}{r_0} \sim \left(\frac{N-1}{N_L-1}\right)^{1/(d+q)} \qquad \frac{g}{\rho_0^D r_N^d} \sim \frac{g}{\rho_0^D r_0^d}\left(\frac{N_L-1}{N-1}\right)^{d/(d+q)}$$

$$\Delta\gamma \sim 1/N^{(d+3q)/2(d+q)}$$

Two-component BECs: Renormalization of scattering strength

$$\eta = \int d\mathbf{r}\,|\psi(\mathbf{r})|^4$$

$d = 1$



A. B. Tacla, S. Boixo, A. Datta, A. Shaji, and C. M. Caves, "Nonlinear interferometry with Bose-Einstein condensates," in preparation.

Two-component BECs

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Renormalization of scattering strength

$$\frac{r_N}{r_0} \sim \left(\frac{N-1}{N_L-1}\right)^{1/(d+q)} \qquad \frac{g}{\rho_0^D r_N^d} \sim \frac{g}{\rho_0^D r_0^d}\left(\frac{N_L-1}{N-1}\right)^{d/(d+q)}$$

$$\Delta\gamma \sim 1/N^{(d+3q)/2(d+q)}$$

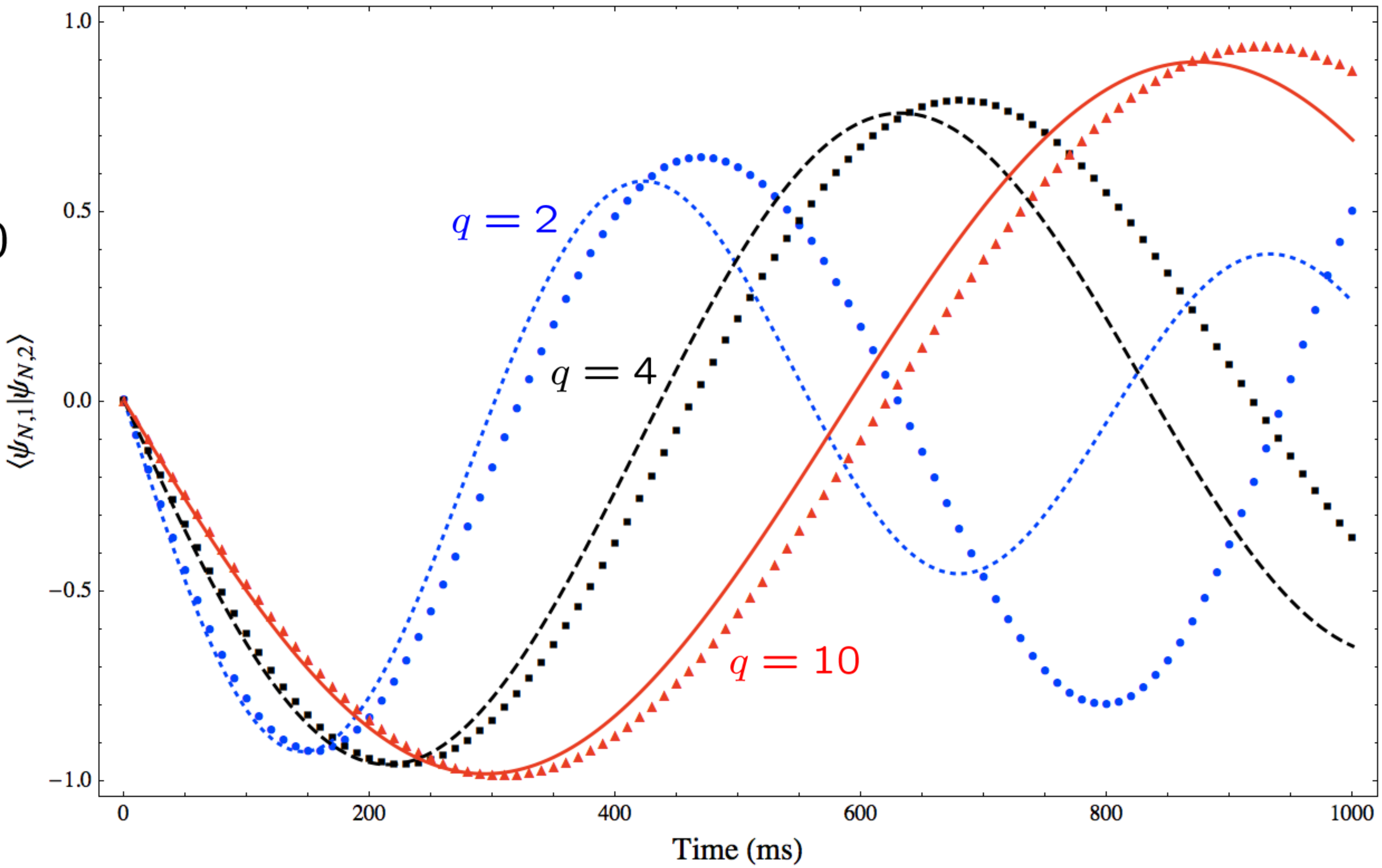
Integrated vs. position-dependent phase

$$\frac{\tau_{\text{pd}}}{\tau_{\text{int}}} = \sqrt{\frac{2(d+3q)}{d}}$$

Two-component BECs: Integrated vs. position-dependent phase

$d = 1$

$N = 1\,000$



A. B. Tacla, S. Boixo, A. Datta, A. Shaji, and C. M. Caves, “Nonlinear interferometry with Bose-Einstein condensates,” in preparation.

Two-component BECs for quantum metrology

? Perhaps ?
With hard, low-dimensional trap or ring

Losses ?
Counting errors ?

Measuring a metrologically relevant parameter ?

S. Boixo, A. Datta, M. J. Davis, A. Shaji, A. B. Tacla, and C. M. Caves, PRA **80**, 032103 (2009); A. B. Tacla, S. Boixo, A. Datta, A. Shaji, and C. M. Caves, "Nonlinear interferometry with Bose-Einstein condensates," in preparation.