Physics 581.0 : Quantum Optics II
Lecture 1: Review

In Physics 566, Quantum Optics I, we introduced some key concepts, which we review here.

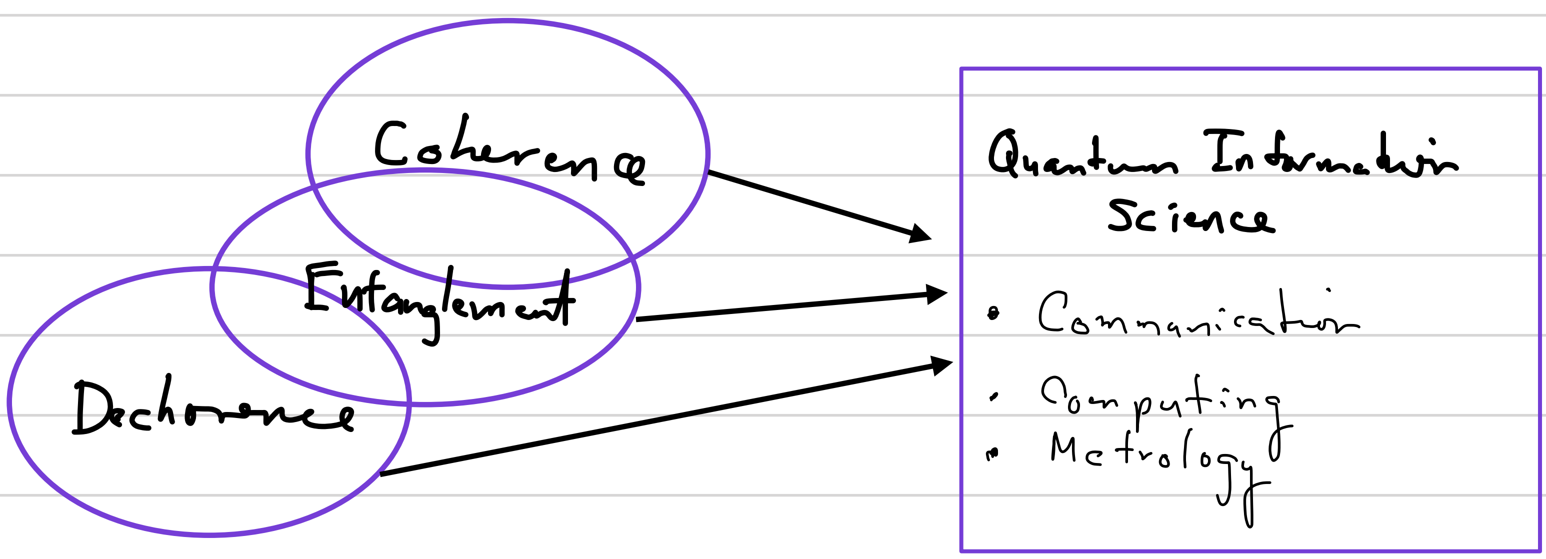
Overview: Coherence, Entanglement, Decoherence, and Quantum Information

The central subject of quantum optics is coherence in quantum system as induced by or intrinsic in optical (or more generally) electromagnetic fields. Coherence is the capacity of a system to exhibit interference. Interference is a "wave phenomenon" arising due to the principle superposition: the superposition of wave solutions to a linear differential equation is also a solution. In classical wave theory, this gives rise to phenomena such as Young's double slit and Michaelson interferometers. In quantum physics, "indistinguish processes" interfere. Each process is typically associated with "quanta" or "particles." Thus, quantum coherent phenomena are intimately connected with wave/particle duality. The quantum coherent phenomena of particles and waves will be a central theme this semester.

Such phenomena take on a new dimension when they are associated with multiple quanta. In that case, the interference phenomena associated with the history of two or more particles has no classical wave interference analog. We have see one such example of great importance in quantum optics: the Hanbury-Brown & Twiss phenomenon. The generation to so-called "nonclassical light" will be another central topic of this semester. Of particular importance in understanding multi-quanta interference is the idea of entanglement, its relation to the foundations of quantum mechanics itself. Is quantum mechanics compatible with a "local realistist, objective description of nature? Quantum optics has been the physical platform in which these seemingly metaphysical questions have been put to the nuts-and-bolts test of laboratory experiment.

While quantum coherence can lead to unique a powerful phenomena, particularly in the realm of "many-body physics" and with entanglement, such coherence rarely survives into the macroscopic realm of our daily existence. The process by which a system goes from coherent – having the capacity to exhibit interference – to incoherent – without interference between said processes – is known as decoherence, and is an essential ingredient in understanding the quantum world, much as the thermodynamic equilibrium is essential to understanding the macroscopic classical world. Indeed, whereas thermodynamics arises in "open systems," where a small set of degrees of freedom are coupled to a reservoir, decoherence arises in "open quantum systems." Beyond its practical relevance, the study of decoherence and open quantum systems dynamics is an essential ingredient in "measurement theory." This too is an issue in the foundations of quantum mechanics – perhaps THE issue of the foundation. How are measurement outcomes, which are generically random, extracted from the microscopic deterministic predictions of the Schrödinger equation. In this semester we will study both the nuts-and-bolts on open quantum system dynamics and its relationship to understanding quantum measurement.

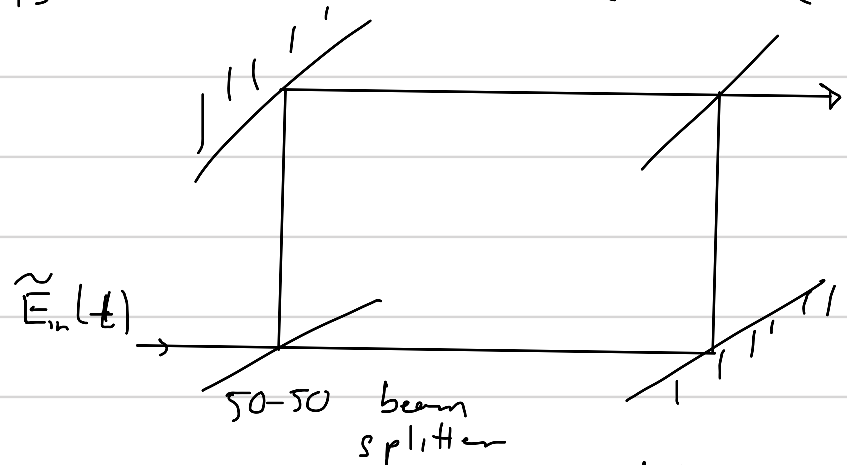
Historically, whereas the founders of quantum theory emphasized the ways in which quantum physics was "restricted" relative to classical physics, i.e., the uncertainty principle limiting the precision with which we could simultaneously predict the outcome of a measurement of position and momentum, in the 1990s, physicists, mathematicians, and computer scientists came to the understanding that exactly the opposite was true. Quantum physics, far from being a paler version of classical physics, gives nature more power than ever conceived. In particular, quantum physics opens the door to devices that can process information in ways that no classical information processor can. With its focus on the study of the foundations of quantum mechanics in the 1980's with the development of experiments to control measure individual quanta, and a theoretical framework to analyze and think out such systems, the subdiscipline of Quantum Optics became the natural forum in which to study the implementation of quantum information processing devices. Quantum opticians have populated the new discipline that has emerged, quantum information science (QIS), and these physicists have been some of its founders. We will study the applications of quantum optics to QIS as time permits.



Coherence:

Standard paradigm: Two path interferometer (e.g. Mach-Zender)

Classical: Wave interference



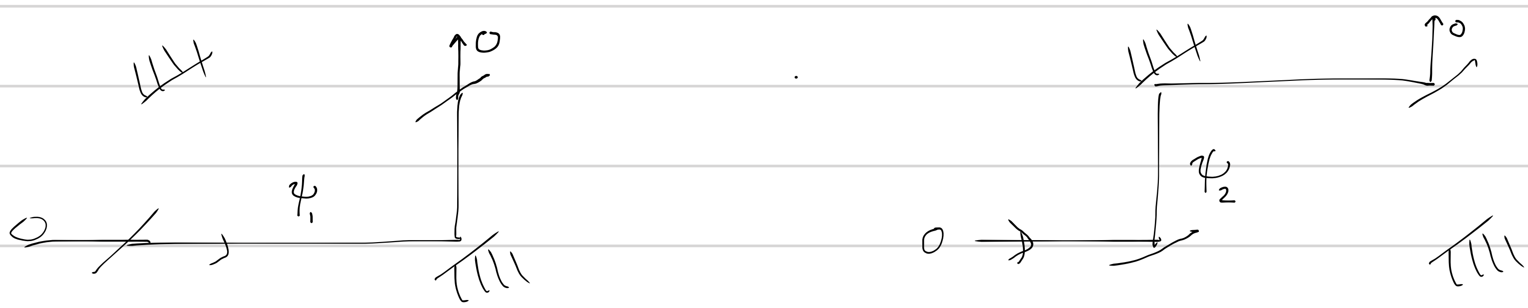
$$\tilde{E}_{out}(t) = \frac{\tilde{E}_{in}(t - \frac{l_1}{c}) + \tilde{E}_{in}(t - \frac{l_2}{c})}{2}$$

$$I_{out} = \overline{\tilde{E}^*_{out}(t)\,\tilde{E}_{out}(t)}$$ ← ensemble average over probability distribution ≡ time ave (ergodic)

$$= \frac{1}{2}\Big(I_{in} + \mathrm{Re}\big(\underbrace{\overline{\tilde{E}^*(t-\tau)\,\tilde{E}(t)}}_{\text{Field-field (auto) correlation } G^{(1)}(\tau)}\big)\Big) \qquad \tau = \frac{l_2 - l_1}{c}$$

$$G^{(1)}(\tau) = \int d\{\alpha_k\} P(\{\alpha_k\})\ \tilde{E}^*(t-\tau)\ \tilde{E}(t) \quad , \quad \tilde{E}(t) = \sum_k E_{0k}\,\alpha_k\, e^{-i\omega_k t}$$

Quantum: Alternative paths



In first order coherence, each photon "interfere only with itself"

First order interference involves "single particle physics" ⇒ linear optical phenomenona.

Density matrix and quantum coherence

$\hat{\rho} \equiv$ State of a quantum system

$\langle a|\hat{\rho}|a\rangle$ = Probability of finding system in state $|a\rangle$ (Population)

$\langle a|\hat{\rho}|b\rangle$ = The ability to see interference between alternatives $|a\rangle$ and $|b\rangle$ (coherence)

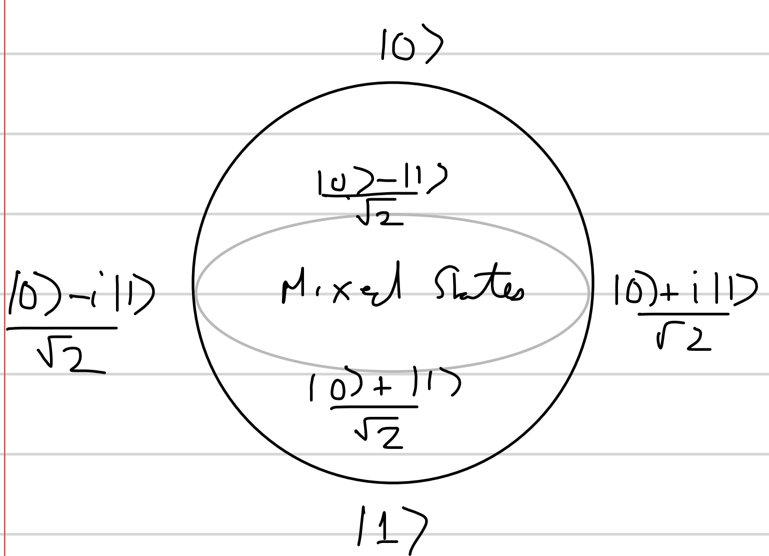
Qubit: Quantum system in a 2D Hilbert space
"Computation basis" $\{|0\rangle, |1\rangle\}$

Eg. Spin-$\frac{1}{2}$ $|0\rangle = |\uparrow_z\rangle$, $|1\rangle = |\downarrow_z\rangle$ $\Rightarrow$ Pauli algebra

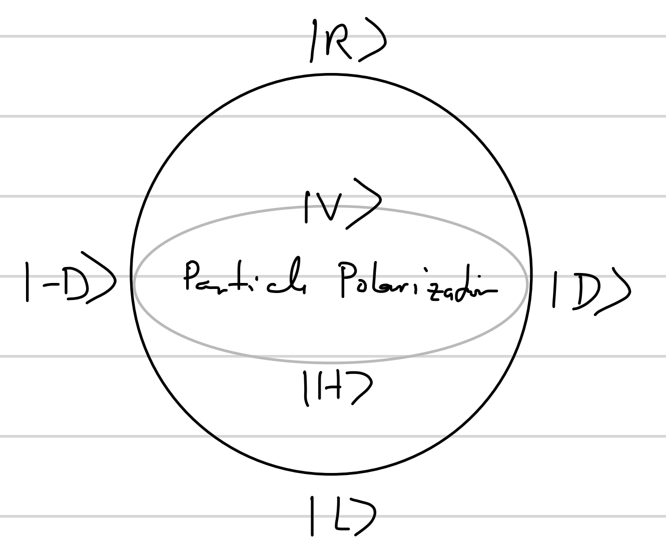
Photon: Polarization: $|0\rangle = |R\rangle$, $|0\rangle = |L\rangle$

Spatial mode $|0\rangle = |0_a, 1_b\rangle$, $|1\rangle = |1_a, 0_b\rangle$
"Dual rail"

Bloch sphere: Space of density matrices for a qubit



$\Longleftrightarrow$
isomorphism



Spin - Bloch sphere

Polarization Poincaré sphere

Quantum Field

The quantized electromagnetic field exhibits properties of particles and waves.

Free field, periodic boundary conditions

$$\hat{\vec{E}}(\vec{r},t) = \underbrace{\sum_{\vec{k},\mu} \sqrt{\frac{2\pi\hbar\omega_k}{V}}\, \vec{\epsilon}_{k,\mu}\, e^{i(\vec{k}\cdot\vec{r}-\omega_k t)}\, \hat{a}_{\vec{k},\mu}}_{\hat{\vec{E}}^{(+)}(\vec{r},t)} + \underbrace{h.c.}_{\hat{\vec{E}}^{(-)}(\vec{r},t)}$$

$\hat{a}^\dagger_{\vec{k},\mu}$ ($\hat{a}_{\vec{k},\mu}$) creates (annihilates) a photon - quantum of excitation in the plane wave mode $\vec{\epsilon}_{\vec{k}\mu}\, e^{i(\vec{k}\cdot\vec{r}-\omega_k t)}$

For given mode: Fock state $|n\rangle_{\vec{k}\mu} = \dfrac{(\hat{a}^\dagger_{\vec{k}\mu})^n}{\sqrt{n!}}|0\rangle$

"Discrete variables" $\quad n = 0, 1, 2, \ldots \quad$ "particles"

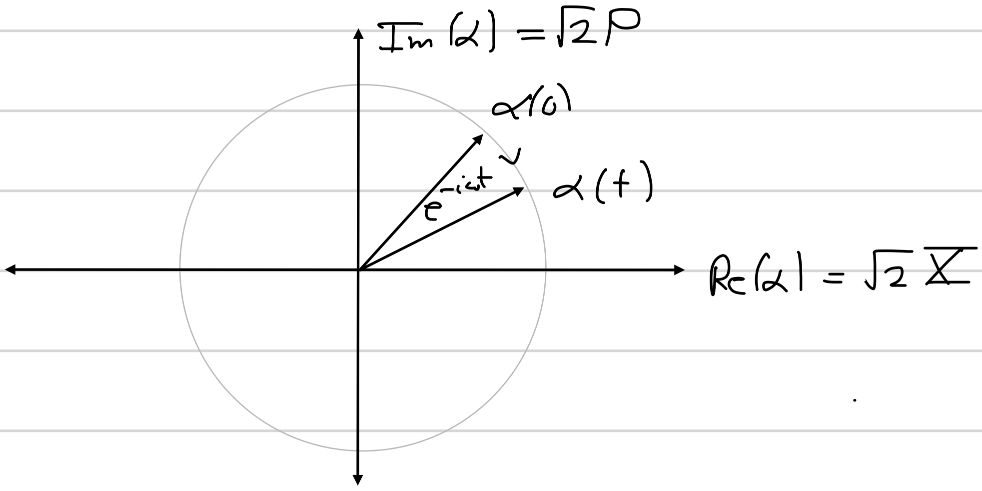
$\hat{a}$ is the quantized complex amplitude for a classical phasor, α.

$\alpha = Ae^{i\phi} = \dfrac{X+iP}{\sqrt{2}}$, $\quad$ Amplitude $A = |\alpha| = \sqrt{\alpha^*\alpha}$

$\phi = \mathrm{Arg}(\alpha) = \tan^{-1}\left(\dfrac{\mathrm{Im}\,\alpha}{\mathrm{Re}\,\alpha}\right)$

Quadratures: $X = \sqrt{2}\,\mathrm{Re}(\alpha) = \dfrac{\alpha+\alpha^*}{\sqrt{2}}$, $\quad P = \sqrt{2}\,\mathrm{Im}(\alpha) = \dfrac{\alpha-\alpha^*}{i\sqrt{2}}$

SHO $\alpha(t) = \alpha(0)e^{-i\omega t} \Rightarrow X(t) = \mathrm{Re}(\alpha(0)e^{-i\omega t}) = X(0)\cos\omega t + P(0)\sin\omega t$



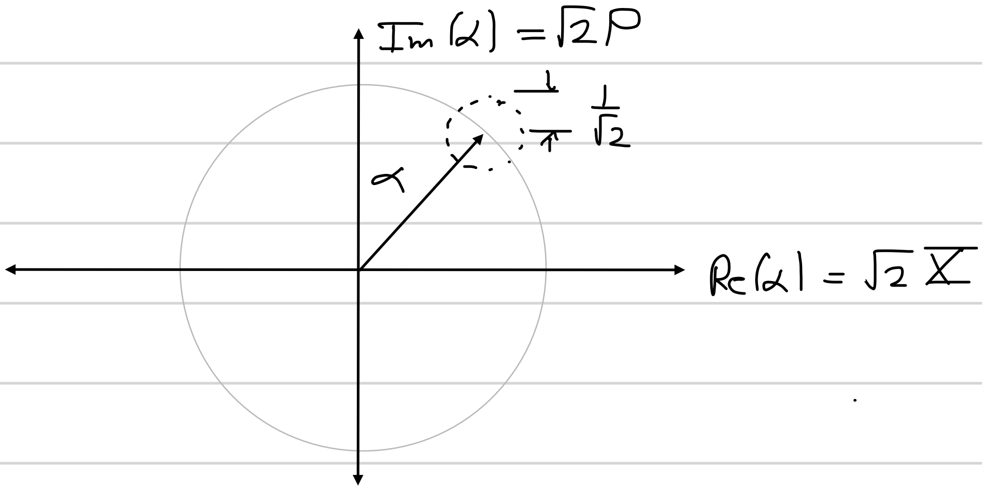
Quantum mode: $[\hat{X}, \hat{P}] = i$, $[\hat{a}, \hat{a}^\dagger] = 1$ (Bosons)

"Continuous variables": Quantum wave amplitude

Coherent state $|\alpha\rangle$: $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$

$$\langle\alpha|\hat{X}|\alpha\rangle = X, \quad \langle\alpha|\hat{P}|\alpha\rangle = P$$

$$\Delta X = \Delta P = \frac{1}{\sqrt{2}}, \quad \Delta X \Delta P = \frac{1}{2} \quad \left(\begin{matrix}\text{minimum}\\ \text{uncertainty}\end{matrix}\right)$$



Continuous variable vs. discrete variable: Number-Phase uncertainty

$$\hat{a} = \widehat{e^{i\phi}}\sqrt{\hat{n}}, \quad \hat{a}^\dagger = \sqrt{n}\,\widehat{e^{i\phi}}^\dagger$$

$$\sqrt{\hat{n}} = \sum_{n=0}^{\infty}\sqrt{n}|n\rangle\langle n| \qquad \widehat{e^{i\phi}} = \sum_{n=0}^{\infty}|n\rangle\langle n+1| \quad \left(\begin{matrix}\text{Partial}\\ \text{isometry}\end{matrix}\right)$$

$$\Delta n \Delta\phi \gtrsim 1$$

Definite $n \Rightarrow$ indefinite phase, Definite phase $\Rightarrow$ indefinite n

Coherent state: $|\alpha\rangle = \sum_{n=0}^{\infty} c_n |n\rangle$, $\quad c_n = \frac{\alpha^n e^{-|\alpha|^2/2}}{\sqrt{n!}}$

"Photon statistics": Probability of n-photons in the mode

$$P_n = |c_n| = \frac{\langle n\rangle^n e^{-\langle n\rangle}}{n!}, \quad \langle n\rangle = |\alpha|^2, \quad \Delta n^2 = \langle n\rangle, \quad \underline{\text{Poisson}}$$

The Poisson statistics represent the minimum fluctuation in photon number associated with a perfectly stable classical wave of definite intensity. Mixed states, associated with classical statistical fluctuations in the intensity add additional number fluctuations.

"Natural light" (thermal state): Bose-Einstein Statistics

$$\hat{\rho} = \sum_n P_n |n\rangle\langle n| \quad , \quad P_n = \frac{1}{\langle n\rangle + 1}\left[\frac{\langle n\rangle}{\langle n\rangle+1}\right]^n$$

$$\left(\text{In thermal equilibrium } \langle n\rangle = \frac{1}{e^{\hbar\omega/k_B T} - 1}\right)$$

Continuous variable representation: Statistical mixture of coherent states

$$\hat{\rho} = \int d^2\alpha \; P(\alpha) \; |\alpha\rangle\langle\alpha|$$

Glauber-Sudarshan P-representation

See homework

$$\begin{cases} P(\alpha) = \dfrac{1}{\pi \bar{n}} e^{-\frac{|\alpha|^2}{\bar{n}}} : \text{Gaussian amplitude fluctuations} \\ \left[\Delta|\alpha|^2\right]^2 = \langle n\rangle^2 \end{cases}$$

$$\Delta n^2 = \langle \hat{n}^2\rangle - \langle \hat{n}\rangle^2 = \langle \hat{a}^\dagger \hat{a} \hat{a}^\dagger \hat{a}\rangle - \langle \hat{a}^\dagger \hat{a}\rangle^2$$

$$= \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a}\rangle + \underbrace{\langle \hat{a}^\dagger \hat{a}\rangle}_{\text{term arising from the comutator (quantum fluct.)}} - \langle \hat{a}^\dagger \hat{a}\rangle^2$$

$$= \mathrm{Tr}\left[\hat{\rho}\,(\hat{a}^{\dagger 2} \hat{a}^2)\right] - \left(\mathrm{Tr}(\hat{\rho}\, \hat{a}^\dagger \hat{a})\right)^2 + \langle \hat{n}\rangle$$

$$= \int d\alpha \, P(\alpha) \, |\alpha|^4 - \left(\int d\alpha \, P(\alpha) |\alpha|^2\right)^2 + \langle \hat{n}\rangle$$

$$= \underbrace{\left[\Delta|\alpha|^2\right]^2}_{\text{Wave noise}} + \underbrace{\langle \hat{n}\rangle}_{\text{Particle noise}}$$

If $\Delta n^2 < \langle \hat{n}\rangle$ (Sub Poissonian)

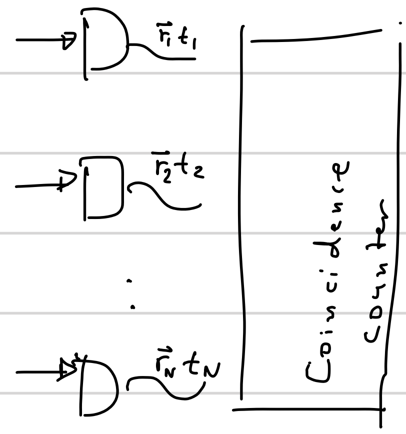
Nonclassical

Photon-Photon Correlations and higher-order interference

Glauber theory of photon coincidence:

Given N-photon counters, the probability of N-fold coincidence

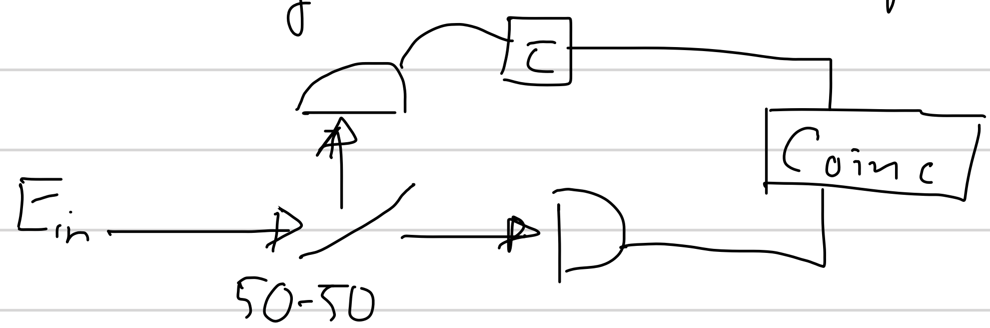
$\hat{I}(\vec{r}_1 t_1) = \hat{E}^{(-)}(\vec{r}_1 t_1)\, \hat{E}^{(+)}(\vec{r}_1 t_1)$



$$P_{coincidence} \propto \langle :\hat{I}(\vec{r}_1,t_2)\, \hat{I}(\vec{r}_2,t_2) \cdots \hat{I}(\vec{r}_N,t_N): \rangle$$ Normal ordering

$$= \langle \hat{E}^{(-)}(\vec{r}_1,t_1)\, \hat{E}^{(-)}(\vec{r}_2,t_2) \cdots \hat{E}^{(-)}(\vec{r}_N t_N)\, \hat{E}^{(+)}(\vec{r}_1 t_1)\, \hat{E}^{(+)}(\vec{r}_2 t_2) \cdots \hat{E}^{(+)}(\vec{r}_N t_N) \rangle$$

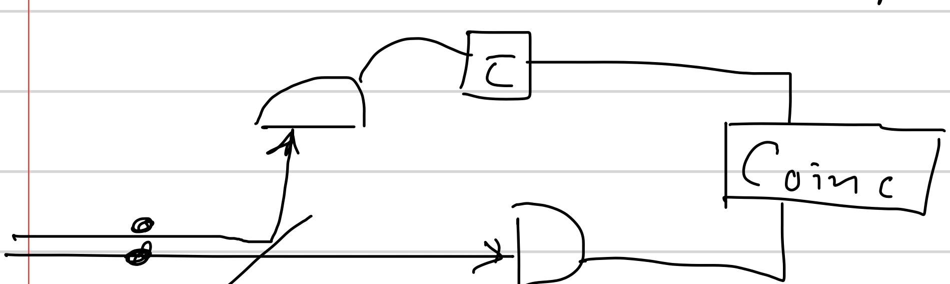
Example: Hanbury-Brown Twiss Temporal coherence

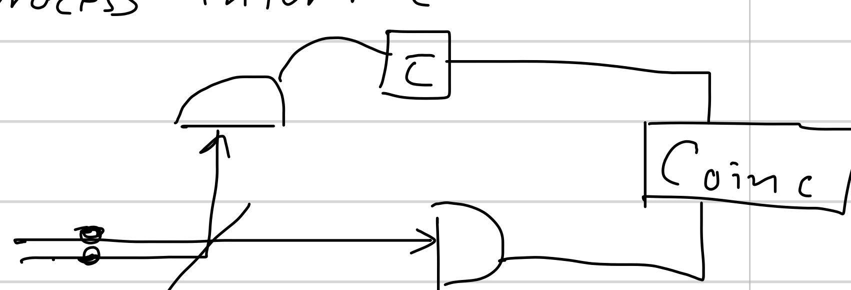


Photon counters, symmetrically placed on opposited output ports of a 50-50 beam splitter

$$P_{coincidence} \propto \langle :\hat{I}(\tau)\, \hat{I}(0): \rangle, \quad P_{\substack{statistical\\independent}} = \langle :\hat{I}: \rangle^2$$

Indistinguishable two-photon process interfere





Photon-bunching and antibunching:

Define: $g^{(2)}(\tau) \equiv \dfrac{\langle :\hat{I}(\tau)\,\hat{I}(0): \rangle}{\langle \hat{I} \rangle^2}$: Tendency of photons to arrive separated of τ / Statistical independent arrival

Photon bunching $g^{(2)}(0) > g^{(2)}(\tau)$ (Photons arrive in "clumps")
Photon antibunching $g^{(2)}(0) < g^{(2)}(\tau)$ (Photon arrival spread out)

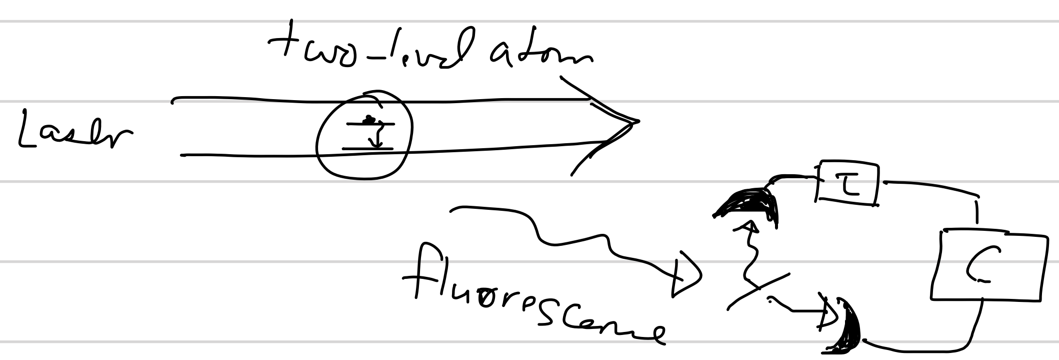
It follows from Cauchy-Schwartz:

If $\hat{\rho} = \int d^2\{\alpha_k\}\; P(\{\alpha_k\})\; |\{\alpha_k\}\rangle\langle\{\alpha_k\}|$, $P(\{\alpha_k\}) \geq 0$

Then $g^{(2)}(0) \geq g^{(2)}(\tau)$:
(Statistically independent if coherent, bunched if noisy intensity)

If $g^{(2)}(0) < g^{(2)}(\tau)$: Photon-antibunching
$\Rightarrow$ Nonclassical Light

Example: Resonance fluorescence from a single atomic emitter



If one photon counter clicks, we immediately measured that the atom jumped to the ground state. It take time for the atom to be reëxcited — the atom never emits two photons of this color at one time $\Rightarrow$ Photons are never arriving at the same time $\Rightarrow$ antibunching!