Entanglement and Reality

As we've seen, Schrodinger regarded entanglement (rather than mere superposition and probability) as the fundamental difference between classical and quantum physics.

It is also well known that Einstein took issues with accepting QM as a fundamental theory of physics.

Despite being quoted as saying "God does not play dice", Einstein too was less concerned with superposition and probability, and more concerned with entanglement and its implications.

These concerns were put forth most directly in a classic 1935 paper with Podolsky and Rosen, titled (sic)

Can quantum-mechanical description of reality be considered complete?

Entanglement and Reality

For a modern audience, the essential point of EPR is clear from the original abstract:

In a complete theory there is an element corresponding to each element of reality.

A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system.

In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other.

Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality.

Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

Entanglement and Reality

The noncommuting variables used in the original EPR argument are position and momentum. Modern retellings tend to follow Bell 1964 and use qubits, with X and Z as the noncommuting observables.

Just like Schrodinger, EPR were bothered by the fact that particles which interacted at some point in the past may lose their independent existence ever after, being described only by a joint entangled state.

Suppose that after an interaction, two qubits are in the entangled state

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) = \frac{1}{\sqrt{2}}\left(|++\rangle + |--\rangle\right)$$

Now these two qubits are sent very to the keepers Alice and Bob who are very far apart. If Alice measures her qubit in the Z basis, then Bob can instantly measure his qubit in the Z basis to get the same result. The same is true in the X basis. This happens instantly, even when there is no time to exchange a light signal between A and B.

Entanglement and Reality

Suppose we have a red ball and blue ball. I give one to Alice and one to Bob, but they can't look at them. Then A and B go to far separated locations. If Alice measures her ball to be blue, then Bob's is "instantaneously" red.

Nothing interesting has happened here, because Bob's ball was red all along. This is object permanence.

This is what EPR meant by an "element of the theory for each element of reality." If the ball was red all along, then there is some variable recording this redness, even if it was hidden from Bob.

But in the EPR experiment, we get perfect correlations in both the Z and X basis. If Alice measures $Z = +1$ then Bob also should get $Z = +1$. Similarly for $X = +1$. But using QM, there is no way to assign an "element of the theory" to Bob's qubit which would determine these values. Thus EPR concluded that QM is incomplete.

Entanglement and Reality

To understand the mindset of EPR, it is worth pausing to distinguish multiple meanings of "determinism."

Perhaps the more popular meaning of "determinism" is causal determinism: the idea that the state of a physical system in the future is uniquely determined by the state in the past, together with the physical laws.

Causal determinism sometimes induces existential questions because it does not appear to allow for human free will. QM is causally deterministic because unitary evolution takes an initial state to a definite future state.

But another important notion of determinism is the question of whether quantities have definite values, independent of our measuring them. Is "my height" a well-defined quantity?

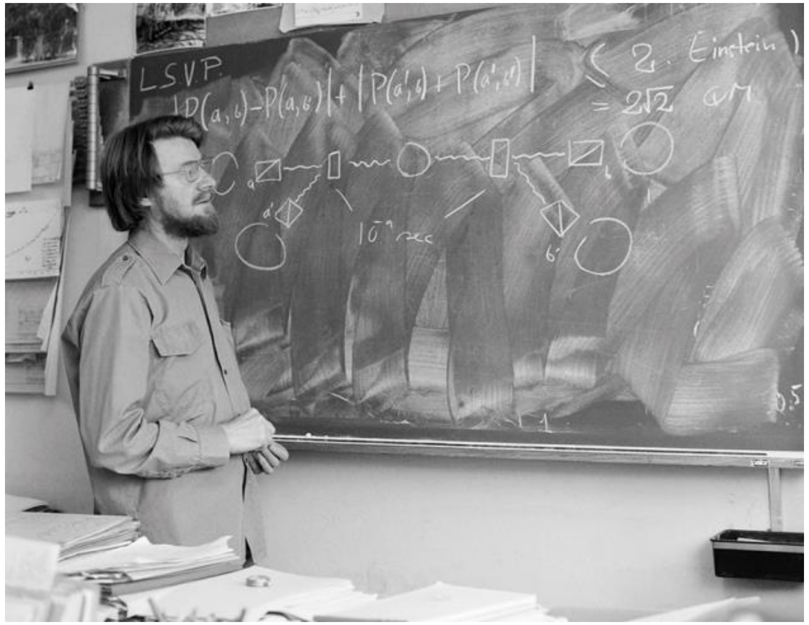
In this sense QM is not deterministic, because there are no definite underlying values of X and Z spin of Bob's qubit that explain the results of the previous experiment.

This is the philosophical hang up, the belief in determinism of physical quantities and idea that fundamental physics theories should capture this, that led EPR to conclude that QM is incomplete.

Entanglement and Reality

This perceived incompleteness was addressed by J.S. Bell in 1964, but rather than patching up QM with a complete description of the kind Einstein expected, he showed that no such description is possible.

More specifically, Bell showed that no theory based on local classical hidden variables can recover the predictions of QM. Therefore either Einstein was wrong, or else QM is not a correct theory.



It was not just Einstein who believed in these "elements of reality" and held out hope for a deeper underlying classical theory to explain the results of QM.   Without Bell's impossibility proof, it's possible that misguided intuition would have derailed modern physics and turned it into an endless fruitless search for a deeper theory.

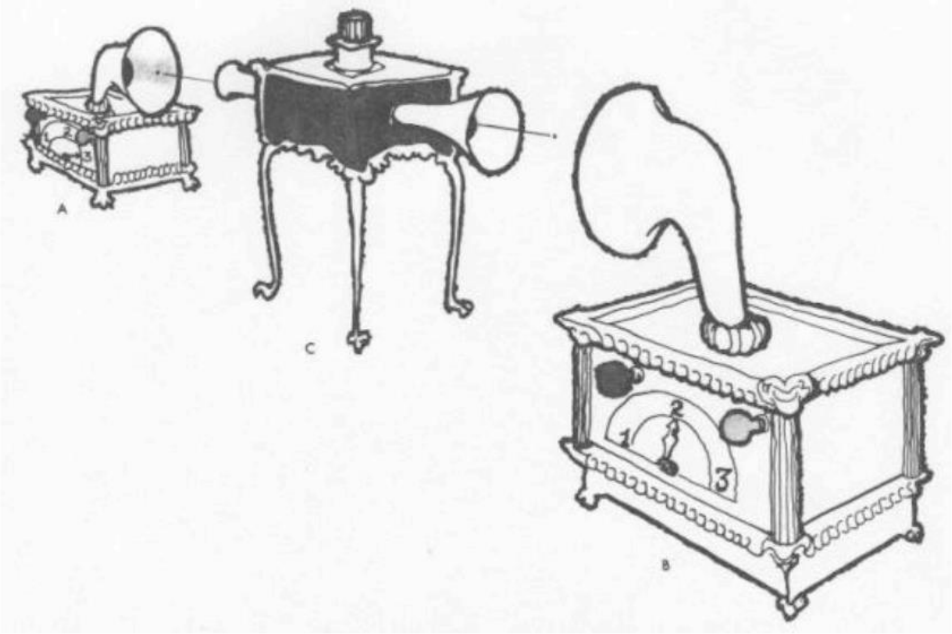
In this sense Bell's result is similar to Godel's incompleteness theorem: it tells us that our naïve hopes for completeness will never be fulfilled, and reality is both more difficult to understand, but also more rich, than we had imagined.

Entanglement and Reality

The general idea of Bell's theorem and its extension is that quantum correlations between measurements in multiple bases can be incompatible with any underlying classical explanation ("hidden variable theory").

Before moving to formal inequalities that these correlations must obey, we will give a fully intuitive formulation of Bell's theorem that is originally due to Mermin, in a 1985 Physics Today popular article titled

Is the moon there when nobody looks? Reality and the quantum theory



Mermin's "EPR machine." Each detector has three settings, and flashes a bulb that is either red or green.

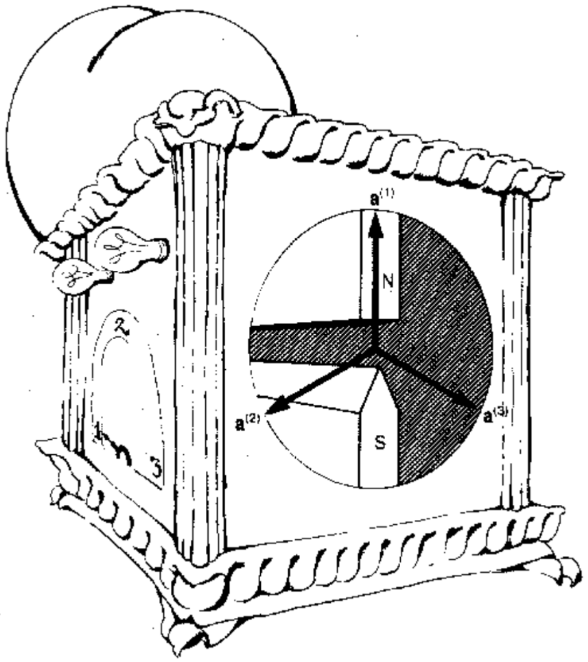
Entanglement and Reality

We will assume the emitter in the center outputs pairs of qubits in the state:

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) = \frac{1}{\sqrt{2}}(|+-\rangle - |-+\rangle) = \frac{1}{\sqrt{2}}(|i,-i\rangle - |-i,i\rangle)$$

A nice feature of this singlet state is that it is a zero eigenstate of $\sigma^2_{\text{tot}} = \|(\vec{\sigma}_1 + \vec{\sigma}_2)\|^2_2$, and it has the same form no matter which axis we choose as the computational basis.

In Mermin's thought experiment, the three different settings on the detector correspond to measuring this state along three separate axes, which are separated from each other by $2\pi/3$ rads.



Entanglement and Reality

On each run, the results of the experiment are denoted by two numbers (the detector settings) and two colors (the results indicated by the flashing bulb). E.g.

32RG 11RR 12GG 13RG 33GG 31RG 22RR

Observation 1: if the two detectors have the same setting, then their results are perfectly (anti)correlated.

Observation 2: if we average over the results of all the runs, then the flashes coincide half the time.

Observation 1 follows from the singlet state having the same form no matter which axis we choose to be the computational basis. Observation 2 follows because

$$\sum_{ij} \langle\psi|(\mathbf{a}^{(i)} \cdot \vec{\sigma}_1) \otimes (\mathbf{a}^{(j)} \cdot \vec{\sigma}_2)|\psi\rangle = 0$$

Entanglement and Reality

Observation 1: if the two detectors have the same setting, then their results are perfectly (anti)correlated.

Observation 2: if we average over the results of all the runs, then the flashes coincide half the time.

Now we come to the interesting part: showing that no classical hidden variable theory can explain O1 and O2.

We assume that each particle has some definite classical property that determines its behavior. There are three detector settings, and for each one the particle must decide on Red or Green.

For example, the theory may contain some type of particle with the behavior RGG, which yields Red if the detector is set to 1, Green if it is set to 2, and Green if it is set to 3. There can be 8 such species of particles.

Entanglement and Reality

Observation 1: if the two detectors have the same setting, then their results are perfectly (anti)correlated.

Observation 2: if we average over the results of all the runs, then the flashes coincide half the time.

Suppose the emitter sends out two particles, each with the "strategy" RRG.

This will satisfy O1: if the detectors have the same setting they see the same flash.

But count the number of coinciding flashes: 11, 22, 33, 12,21. This is $5/9 \neq 1/2$, which violates O2.

Because 9 is odd, there is no deterministic strategy that will yield coinciding flashes exactly ½ the time.

Therefore no local "elements of reality" can explain the results of this thought experiment!

Entanglement and Reality

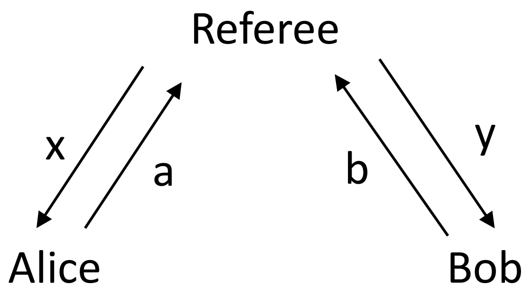
Bell's inequality is an upper bound on the correlation between various random variables representing measurements in the preceding scenario, if one assumes that the measurements are explained by an underlying classical hidden variable theory.

Bell's theorem is the statement that the predictions of QM violate the inequality described above, and therefore cannot be so explained. Therefore "violating Bell's inequality" is a strong test for QM to pass.

As this discussion also makes clear, one can derive a variety of Bell inequalities covering a range of measurement scenarios. Bell's original scenario was experimentally inconvenient, and this led to a scenario called the CHSH game becoming the standard example of a Bell inequality.

Entanglement and Reality

The CHSH (Clauser, Horne, Shimony, and Holt) game is an example of a nonlocal game. Two players, Alice and Bob, communicate with a referee but cannot communicate with each other.



In the CHSH game the referee distributes bits x and y to the players, and then the players respond with bits a and b to maximize their probability of meeting some win condition (to be described momentarily).

The point of a nonlocal game is that the players cannot communicate, but they can share some correlated randomness and/or entanglement. Nonlocal games provide a setting to contrast quantum and classical, with some games (like CHSH) having entangled strategies out perform classical strategies.

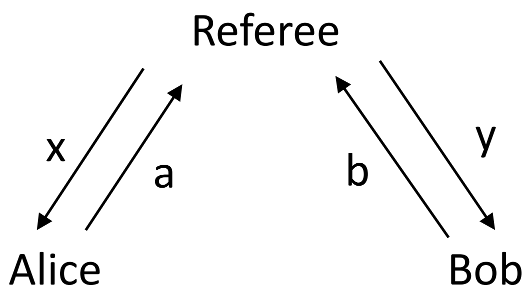
Entanglement and Reality

CHSH game: the referee chooses two bits, x and y, randomly and sends them to A and B.

The players send back two bits, a and b, and they win if

$$x \wedge y = a \oplus b$$

"*x* and *y* equals *a* exclusive or *b*"



x	y	$x \wedge y$
1	1	1
1	0	0
0	1	0
0	0	0

a	b	$a \oplus b$
1	1	0
1	0	1
0	1	1
0	0	0

Entanglement and Reality

Clearly Alice and Bob have a difficult task: if neither of them sees both x and y, they can never really know the truth value of x AND y.

But they do know that x and y are selected uniformly at random, so with probability ¾ , the truth value of x AND y is 0. Alice and Bob could guess this to always be the case, and win ¾ of the time.

Can they do better? We need a way to characterize all of their possible strategies.

In general any strategy for responding to the questions is a conditional distribution

$$p(a,b|x,y)$$

And we want to understand the implications for this distribution that follow from (1) Alice and Bob being far separated, and (2) the fact that their strategy depends on a hidden classical parameter.

Entanglement and Reality

Let $V(x,y,a,b) = 1$ if the win condition is satisfied, and 0 otherwise.

Then the expected winning probability is:

$$\sum_{x,y,a,b} V(x,y,a,b)p(a,b|x,y)p(x,y) = \frac{1}{4}\sum_{x,y,a,b} V(x,y,a,b)p(a,b|x,y)$$

Next we allow for an additional parameter λ , which represents either a quantum or classical strategy.

$$p(a,b|x,y) = \int_\lambda p(a,b|\lambda,x,y)p(\lambda|x,y)$$

So far this is completely general: the strategy is a random variable, which may depend on both questions x and y, and which together with the questions may influence both a and b.

Entanglement and Reality

If we assume the strategy is classical, then it is independent of the questions:

$$p(\lambda|x,y) = p(\lambda)$$

And further, each response can only depend on the strategy and the question that player received:

$$p(a,b|\lambda,x,y) = p(a|\lambda,x)p(b|\lambda,y)p(\lambda)$$

Therefore any classical strategy can be decomposed as

$$p(a,b|x,y) = \int_\lambda p(a|\lambda,x)p(b|\lambda,y)p(\lambda)$$

Entanglement and Reality

By introducing additional variables n and m, we can make the replacement

$$p(a,b|x,y) = \int_\lambda p(a|\lambda,x)p(b|\lambda,y)p(\lambda) = \int_\lambda f(a|\lambda,x,n)g(b|\lambda,y,m)p(\lambda)p(n)p(m)$$

Where f and g are now binary valued functions. We can always do this using a suitable distribution for n,m.

But now we can incorporate n,m into the definition of λ, so we have

$$p(a,b|x,y) = \int_\lambda f(a|\lambda,x)g(b|\lambda,y)p(\lambda)$$

Entanglement and Reality

Therefore the win probability (also called the value of the game) for any classical strategy is

$$\frac{1}{4}\sum_{x,y,a,b} V(x,y,a,b)\left[\int_{\lambda} f(a|\lambda,x)g(b|\lambda,y)p(\lambda)\right] = \frac{1}{4}\int_{\lambda} p(\lambda)\sum_{x,y,a,b} V(x,y,a,b)f(a|\lambda,x)g(b|\lambda,y)$$

$$\leq \frac{1}{4}\sum_{x,y,a,b} V(x,y,a,b)f(a|\lambda^*,x)g(b|\lambda^*,y)$$

Where the inequality replaces the integral using the maximum value of the function, which occurs at the value λ^*. This step tells us that the optimal classical strategy is deterministic.

But deterministic strategies are very simple, because it means that Alice has a predetermined rule which decides which bit a she sends in response to the question x.

Entanglement and Reality

All deterministic strategies are described in the table on the right.

It is not possible to satisfy all 4 of these equations (check the column sums), but we can satisfy ¾. Therefore the optimal win prob for a classical strategy is ¾.

x	y	$x \wedge y$	$= a_x \oplus b_y$
0	0	0	$= a_0 \oplus b_0$
0	1	0	$= a_0 \oplus b_1$
1	0	0	$= a_1 \oplus b_0$
1	1	1	$= a_1 \oplus b_1$

This is Bell's inequality, the statement that the optimal classical value of this game is ¾.

Entanglement and Reality

To complete the proof of Bell's theorem, we need to exhibit a strategy in which Alice and Bob share an entangled state $|\phi_{AB}\rangle$ that will allow them to win with probability greater than ¾.

Alice and Bob will each measure in some basis local basis, $\sum_a \Pi_a^{(x)} = I, \sum_b \Pi_b^{(y)} = I$, so

$$p(a,b|x,y) = \langle\phi_{AB}|\Pi_x^{(a)} \otimes \Pi_b^{(y)}|\phi_{AB}\rangle$$

So that the winning probability achieved using the entangled state $|\phi_{AB}\rangle$ is

$$\frac{1}{4}\sum_{x,y,a,b} V(x,y,a,b)\langle\phi_{AB}|\Pi_x^{(a)} \otimes \Pi_b^{(y)}|\phi_{AB}\rangle$$

Entanglement and Reality

Alice and Bob will agree in advance on the following strategy. They share a maximally entangled state

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

If Alice receives x = 0, she measures her qubit in the Z basis and returns a = 0 for the state |0> , and a =1 for the state |1>. Similarly, if x = 1 she measures in the X basis and returns the outcome as a.

If Bob receives y = 0, he measures in the basis $(X+Z)/\sqrt{2}$ and returns the outcome as b, or if y = 1 he measures in $(X-Z)/\sqrt{2}$ and returns the outcome as b.

Entanglement and Reality

To describe this strategy we can use the basis vectors

$$|\nu_0(\theta)\rangle = \cos(\theta)|0\rangle + \sin(\theta)|1\rangle$$

$$|\nu_1(\theta)\rangle = \sin(\theta)|0\rangle - \cos(\theta)|1\rangle$$

If Alice receives question A0 (x = 0) she measures with angle θ_{A0}, and similarly for $\theta_{A1}, \theta_{B0}, \theta_{B1}$.

If (x,y) = (0,0) then winning requires the values of a,b to be equal.

$$\Pr(\text{win}|x=0, y=0) = |\langle \nu_0(\theta_{A0})\nu_0(\theta_{B0})|\Phi^+\rangle|^2 + |\langle \nu_1(\theta_{A0})\nu_1(\theta_{B0})|\Phi^+\rangle|^2$$

$$= \cos(\theta_{A0} - \theta_{B0})^2$$

Entanglement and Reality

Similarly, after the computer does its job one finds:

$$\Pr(\mathrm{win}|x=0,y=0)=\cos(\theta_{A0}-\theta_{B0})^2 \qquad \Pr(\mathrm{win}|x=1,y=0)=\cos(\theta_{A1}-\theta_{B0})^2$$

$$\Pr(\mathrm{win}|x=0,y=1)=\cos(\theta_{A0}-\theta_{B1})^2 \qquad \Pr(\mathrm{win}|x=1,y=1)=\sin(\theta_{A1}-\theta_{B1})^2$$

And so the total win probability is

$$\frac{1}{4}\left(\cos(\theta_{A0}-\theta_{B0})^2+\cos(\theta_{A0}-\theta_{B1})^2+\cos(\theta_{A1}-\theta_{B0})^2-\sin(\theta_{A1}-\theta_{B1})^2\right)$$

The described strategy corresponds to: $\theta_{A0}=0,\ \theta_{A1}=\pi/4,\ \theta_{B0}=\pi/8,\ \theta_{B1}=-\pi/8$

Which yields a win probability of $\cos(\pi/8)^2=\frac{1}{2}+\frac{1}{2\sqrt{2}}=0.8535\ldots$

Entanglement and Reality

Therefore the quantum winning probability is ~85%, significantly higher than the best classical strategy that wins with probability 75%.

Bell's theorem: Any classical strategy for the CHSH game can win at most 3/4ths of the time, while there exists a quantum strategy for the CHSH game that wins at least a $\cos(\pi/8)^2$ fraction of the time.

To put the limitation on classical strategies in the form of an inequality, let a,b,c,d be random variables with $\nu(a) \in \{1,-1\}$ and similarly for b,c,d. Then

$$\nu(a)\nu(b) + \nu(a)\nu(d) + \nu(c)\nu(b) - \nu(c)\nu(d) = \pm 2$$

And therefore we have the **CHSH inequality**:

$$|\langle ab\rangle + \langle ad\rangle + \langle cb\rangle - \langle cd\rangle| \leq 2$$

Which is violated if a,b,c,d correspond to the outcomes of the directional measurements applied to the entangled state that we saw previously. This violation relates directly to the enhanced win probability.

Entanglement and Reality

Even in the quantum setting, one can derive an upper bound on these correlations:

$$|\langle ab\rangle + \langle ad\rangle + \langle cb\rangle - \langle cd\rangle| \leq 2\sqrt{2}$$

Which is called Tsirelson's bound. See Wilde's discussion of CHSH for a relatively short (two page) proof of Tsirelson's inequality. It shows that our win probability of $\cos(\pi/8)^2$ is optimal within QM.

But upper bounding the win probability of quantum strategies is a secondary point. The main result is that quantum strategies out perform classical strategies.

Put another way, the results of the EPR thought experiment cannot be explained by any underlying local classical hidden variables, the "elements of reality" that Einstein sought.

An experimental violation of Bell's inequality would force us to give up either locality (relativistic causality) , or else give up on the determinism of unobserved values.

Entanglement and Reality

The first experimental tests of Bell's inequality were proposed in 1969, using entangled photons.

These first experiments were performed in 1972. Because the method for producing entangled photons was a recent invention, the rate of particle pair production was extremely low. This meant one had to keep a highly sensitive experiment running for a week or more.

Because of these limitations, the first published tests found contradictory results: both support for Bell's inequality, and for its violation. The first conclusive violation was found in 1976.

But a major limitation of these early experimental results was that, unlike the ideal thought experiments of EPR and Bell, the two detectors were not sufficiently separated to rule out causal interaction.

Entanglement and Reality

The first experimental violations of Bell's inequality that prevent this "causality loophole" were performed by Alain Aspect in the early 1980s. These results were accepted as a definitive demonstration.

Nevertheless, various loopholes" remained where one could imagine pathological scenarios which allow us to be fooled by the results of the experiment.

For example, the "detection loophole" holds that we don't necessarily detect every pair of entangled photons. What if the full distribution obey's Bell's inequality, but we are accidentally selecting a subset of the particles that violates it? (fair sampling)

What about memory within the detectors? What about the initial randomness used to set detectors? The first tests closing all of these standard loopholes were performed in 2015.

Entanglement and Reality

Bell's theorem forces us to give up on "local realism." We could give up locality and keep realism, or give up realism and keep locality. The working version of QM appears to do the latter.

But don't underestimate the tendency to cling to realism in the face of quantum mechanics.

One solution to retaining a classical worldview is t'Hooft's superdeterminism. This theory holds that every event is predetermined since the beginning of time, so as to make it appear that QM is correct.

In particular, our choices of detector settings are predetermined to show us violations of Bell's inequality.

Superdeterminism is unfalsifiable.

Entanglement and Reality

Another theory that upholds elements of reality while sacrificing locality is called the pilot-wave theory.

Pilot-wave theory has the status of an interpretation of quantum mechanics. It reproduces all the predictions of QM, because it is based on equivalent equations.

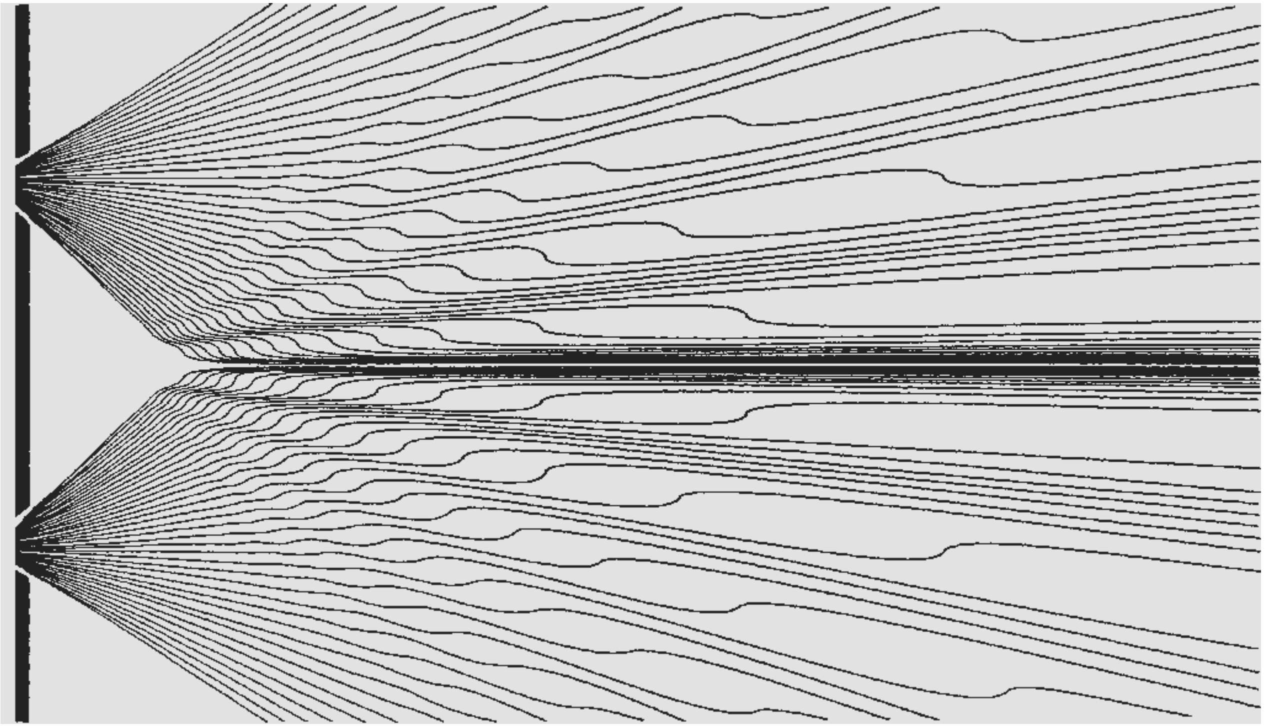
The pilot-wave description of a particle moving in space is as follows. There is a particle which moves in a force field according to some nonlinear equations of motion.   The field creating the force is called the pilot-wave, and it evolves by a nonlinear PDE.

Mathematically, these nonlinear equations for the particle and the wave are derived from the magnitude and phase of the wave function in the Schrodinger equation.

The field and the particle evolve together so that the average over particle trajectories reproduces the predictions of QM. To make this work, the field needs to change in an instantaneous and nonlocal way. Further, we need to introduce additional fields with more particles; the number of pilot-wave fields increases with the dimension of the quantum system.

Entanglement and Reality

This image displays the kind of particle trajectories that contribute to the double-slit wave function in pilot-wave theory.



This version of QM was put forth by deBroglie and developed further by Bohm (it is sometimes called "Bohmian mechanics"). It was also J.S. Bell's preferred interpretation.

Mathematically, the nonlinearity makes Bohmian Mechanics useless for predictions. More importantly, it hasn't opened any deep new directions to explore, and it strikes me as somewhat unscientific in its approach ("I don't like what nature is telling me, so I'll resort to something that sounds crazy in order to preserve my preconceived dogmas").