

Universal quantum walks and adiabatic algorithms by 1D Hamiltonians

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We construct a family of time-independent nearest-neighbor Hamiltonians coupling eight-state systems on a 1D ring that enables universal quantum computation. Hamiltonians in this family can achieve universality either by driving a continuous-time quantum walk or by terminating an adiabatic algorithm. In either case, the universality property can be understood as arising from an efficient simulation of a programmable quantum circuit. Using gadget perturbation theory, one can demonstrate the same kind of universality for related Hamiltonian families acting on qubits in 2D. Our results demonstrate that simulating 1D chains of spin-7/2 particles is BQP-hard, and indeed BQP-complete because the outputs of decision problems can be encoded in the outputs of such simulations.

I. INTRODUCTION

With quantum circuits [1, 2, 3], one can decompose even the most complex quantum computation into a sequence of simple operations called *gates* that act on simple parcels of information called *qubits*. Mathematically, the action of a T -gate quantum circuit on an n -qubit pure state can be expressed as $U_T \cdots U_1 |\psi\rangle$, where $|\psi\rangle$ represents the input state and each U_i represents a unitary transformation drawn from a quantum gate basis, such as the universal gate bases described in [1, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]. This mathematical description is usually interpreted as a set of instructions for applying the gates U_1, U_2, \dots, U_T in sequence to the qubits. Feynman noted in Ref. [15] that this description can instead be interpreted as a *blueprint* for a device in which the gate sequence is “printed” directly into hardware. Specifically, from the mathematical description of a quantum circuit, Feynman constructed the following time-independent Hamiltonian that acts collectively on both the input state $|\psi\rangle$ and an auxiliary system he called a “cursor” consisting of $(T + 1)$ qubits:

$$H_F = \frac{1}{\sqrt{T+1}} \sum_{t=1}^T U_t \sigma_t^+ \sigma_{t-1}^- + U_t^\dagger \sigma_{t-1}^+ \sigma_t^-, \quad (1)$$

where σ_k^+ and σ_k^- denote raising and lowering operators on the k th cursor qubit. Feynman proved that evolution for a time $T/2$ under H_F with the cursor initialized in the state $|1, 0, \dots, 0\rangle$ will efficiently approximate a sequenced implementation of the circuit. In the language of quantum information, one would say that Feynman proved that *continuous-time quantum walks* [16] are universal for quantum computation.

Feynman’s construction has received little attention as a possible quantum computing architecture. This is most likely because a direct implementation of H_F appears to

be far beyond the reach of any known quantum technology. Some of the difficulties are that H_F has *a*) four-body interactions (when two-qubit gates are used), *b*) spatially nonlocal interactions (because the cursor gets far away from the data as T and n grow), and is *c*) exponentially sensitive to decoherence via Anderson localization [17] (because the quantum walk generated by H_F is effectively on a uniform line). The central goal of this paper is to construct a Hamiltonian that achieves the same task as Feynman’s but which is simpler and hopefully closer to a technologically-reachable implementation.

There is a wealth of previous work on simplifying Feynman’s Hamiltonian in the context of exploring the computational universality of adiabatic algorithms and of demonstrating the QMA-completeness of finding Hamiltonian ground states [18, 19, 20, 21, 22, 23]. (Actually, these efforts are directed at simplifying a variant of Feynman’s Hamiltonian proposed by Kitaev in [18].) However, these simplifications are done with the goal of finding Hamiltonians whose *spectra* are close to that of H_F but which are not necessarily close to H_F *dynamically*. This latter kind of closeness is achieved when the *operator norm* of the difference between the Hamiltonians is close, because, as the following well-known inequality for Hermitian operators indicates [24], this implies that the corresponding dynamics generated by the Hamiltonians is close:

$$\|e^{-iH_F t} - e^{-iH'_F t}\| \leq \|H_F - H'_F\| t. \quad (2)$$

Examples of Hamiltonians whose spectra can efficiently be made close to that of H_F include a nearest-neighbor Hamiltonian on a two-dimensional grid of qubits [21], a translationally-invariant nearest-neighbor Hamiltonian on a line of 30-level systems [22], and nearest-neighbor Hamiltonians on a line of 9-level systems [23].

Although consideration of Feynman’s quantum computing architecture has been hampered by technological unreachability, it has not been overlooked entirely. For example, Margolus [25] presented an early generalization in which H_F is replaced by a spatially homogeneous finite-range 8-body Hamiltonian on a two-dimensional torus of qubits. Spatial homogeneity makes this model particularly simple—it might best be termed a

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continuous-time quantum cellular automaton [22, 26, 27, 28, 29]. However, the 8-body interactions and torus geometry keep it from being any more practical than Feynman’s original architecture. More significantly, Margolus’ model is only universal for classical, not quantum computation. Janzing and Wocjan [30] improved Margolus’ model so that it is programmable, universal for quantum computation, and homogeneous on the surface of a cylinder, but at the cost of requiring 10-body interactions among the qubits over a finite range. Janzing [31] has further modified this construction so that with mild spatial inhomogeneities in the interactions, it achieves programmable universality using only nearest-neighbor interactions among 3-level systems on a two-dimensional lattice. Recently, Nagaj and Wocjan removed these inhomogeneities in work concurrent to ours [32], and demonstrated that programmable universality can be achieved by a continuous-time cellular automaton in one dimension; their construction requires increasing the number of levels per system from three to ten to achieve this.

In this paper, we present a programmable, universal architecture for quantum computation that uses a time-independent nearest-neighbor Hamiltonian on a one-dimensional ring of 8-level systems. When combined with the results of Oliveira and Terhal about the operator norm closeness of Hamiltonians generated by a certain “gadget” perturbation theory technique [21], we show that we can modify our architecture to use only qubits at the expense of requiring a two-dimensional rather than a one-dimensional geometry. Additionally, we show how our model can also be used to reduce the state space per carrier from 9 to 8 in a recent proof of the universality of the adiabatic algorithm [23].

The remainder of our paper is organized as follows. In Sec. III, we motivate our architecture by defining a relevant programmably universal quantum circuit family \mathcal{F} . In Sec. IV, we define a family of one-dimensional 8-state Hamiltonians on a ring whose interactions mimic the gates used by circuits in \mathcal{F} . In Sec. IV, we show how these Hamiltonians can be used to drive continuous-time quantum walks that simulate circuits in \mathcal{F} efficiently. In Sec. V, we demonstrate the universality of our architecture under adiabatic evolution, following similar arguments to those of Aharonov *et al.* [20, 23]. In Sec. VI, we present a method for realizing this model using nearest-neighbor Hamiltonian interactions between qubits on a regular rectilinear lattice on the surface of a cylinder by leveraging results of Oliveira and Terhal [21]. Sec. VII concludes, followed by Appendix A which steps through how an example 3-gate simulation would be effected in our model.

II. A PROGRAMMABLY UNIVERSAL QUANTUM CIRCUIT FAMILY

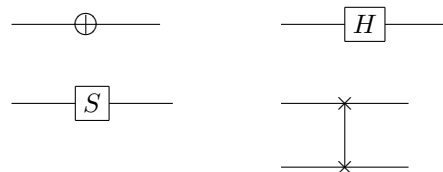
A. Notation and basic definitions

We begin by reviewing relevant notation and basic definitions, much of which can be found in standard textbooks on quantum computation, such as [18, 33]. A *qubit* is a quantum two-state system, the *computational basis* of which we define as $|0\rangle := \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $|1\rangle := \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. A *gate* is a unitary transformation on qubits and a *circuit* is a composition of gates. Gates discussed in this paper include the *NOT gate*, the *Hadamard gate*, the *phase gate*, and the *swap gate*, defined in the computational basis as

$$X := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad H := \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad (3)$$

$$S := \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}, \quad SWAP := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (4)$$

Each gate may be depicted as a *circuit element* in which qubits are represented by horizontal lines (*wires*) entering on the left and exiting on the right. The circuit elements depicting the gates above are

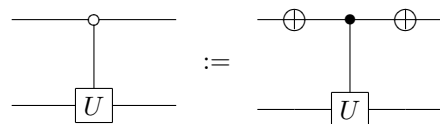


The *controlled- U gate*, denoted $\Lambda(U)$, applies the gate U to a *target* state conditioned on a *control* qubit being in the state $|1\rangle$. The matrix and circuit element representing $\Lambda(U)$ are

$$\Lambda(U) := \begin{pmatrix} I & 0 \\ 0 & U \end{pmatrix}, \quad \begin{array}{c} \bullet \\ | \\ \square U \end{array}$$

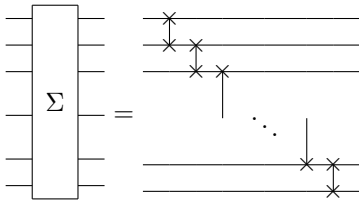
where I represents the identity matrix with $\dim I = \dim U$.

We use two non-standard shorthand notations for gates in this paper. The first is that we denote gates controlled by the state $|0\rangle$ rather than the state $|1\rangle$ as



The second is that we denote a cascaded array of nearest-

neighbor swap gates as



A *gate basis* is a set of gates; a circuit is expressed *over* a gate basis \mathcal{G} if all of its gates are elements of \mathcal{G} . A circuit family is *uniformly generated* if a description of its elements can be constructed by finite means, *e.g.*, by a Turing machine [34]. A gate basis \mathcal{G} is *universal* if for any unitary transformation U and for any desired precision ϵ , there is a uniformly generated circuit family over \mathcal{G} that can approximate U to within ϵ . An example of a universal gate basis is the *Kitaev gate basis* $\{H, \Lambda(S)\}$ [14]. A *programmable* circuit is one that can be conceptually divided into “program” and “data” regions, where the state of the program region controls which gates are to be applied to the state of the data region. For example, Fig. 1 depicts a programmable circuit over the gate basis $\{\Lambda(H), \Lambda^2(S)\}$ in which the program controls which gates from the Kitaev basis are to be applied to the data. A gate basis \mathcal{G} is *programmably universal* if for any desired precision ϵ and any number of qubits n , there is a uniformly generated programmable circuit V over \mathcal{G} such that for any n -qubit unitary transformation U , there is a uniformly generated program state that enables V to approximate U to within ϵ on the data portion of V .

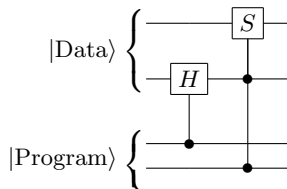


FIG. 1: A simple programmable circuit over the Kitaev gate basis.

B. Programmably simulating circuits over the Kitaev gate basis exactly

Consider the programmable circuit V depicted in Fig. 2. Its program region is decomposed into three parts labeled *cursor*, *swap*, and *Hadamard*. Given a description of a quantum circuit U on n qubits containing T gates that is expressed over the Kitaev gate basis, the program region of V can be initialized so that $\mathcal{O}(nT)$ iterations of V will simulate U exactly. To see how, we first trace through one pass of the circuit V to see how it responds to various initializations of the program regions. We then describe how to initialize these regions to carry out the simulation of U .

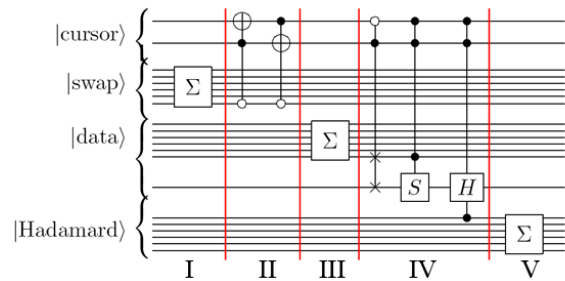


FIG. 2: The quantum circuit V , whose action is described in the text.

Phase I. The Σ gate moves the upper-most *swap* qubit state to the lower-most *swap* qubit and moves every other *swap* qubit state up one qubit to accommodate.

Phase II. If the lower-most *swap* qubit is in the state $|0\rangle$, then the two-qubit *cursor* state permutes from $|00\rangle \rightarrow |00\rangle$ and from $|01\rangle \rightarrow |10\rangle \rightarrow |11\rangle \rightarrow |01\rangle$. We depict this permutation in glyphs as $\ominus \rightarrow \ominus$ and $\odot \rightarrow \odot \rightarrow \omin� \rightarrow \odot$. The reason for this choice of glyphs will become apparent later.

Phase III. The Σ gate moves the upper-most *data* qubit state to the penultimate *data* qubit and moves the state of every *data* qubit in between up one to accommodate.

Phase IV. If the two-qubit *cursor* is in the state \odot , then the penultimate *data* qubit state is swapped with the final *data* qubit state. If the *cursor* is in the state $\omin�$, then first $\Lambda(S)$ is applied between the last two *data* qubits, followed by an application of H to the last *data* qubit, conditioned on the upper-most *Hadamard* qubit being in the state $|1\rangle$. If the *cursor* is in any other state, no action is performed on the *data* qubits.

Phase V. The Σ gate moves the upper-most *Hadamard* qubit state to the lower-most *Hadamard* qubit and moves every other *Hadamard* qubit state up one to accommodate.

Given this action of one iteration of V , we program V so that it follows a sequence of “behaviors” that depend primarily on the state of the two-qubit *cursor*:

Behavior 1. The two-qubit *cursor* is initialized in the state \odot and the first $n - k$ *swap* qubits are initialized in the state $|1\rangle$. The effect of applying V a total of $n - k$ times given this initialization is to cyclically permute the states of qubits in each program region and in the data region upwards $n - k$ times. Importantly, the state of the k th *data* qubit from the top before this behavior is stored in the lower-most *data* qubit after this behavior. The glyph \odot is intended to remind that when the *cursor* is in this state, the *data* qubit states are all cyclically permuted.

Behavior 2. The *cursor* is initialized in the state \odot , the upper-most *swap* qubit state is initialized in the state $|0\rangle$ and the next $n - j - 2$ *swap* qubits are initialized in

the state $|1\rangle$. The effect of applying V once given this initialization is to cyclically permute the *swap* qubit's states upwards once, cycle the *cursor* state from \ominus to \odot , cyclically permute all but the state of the last *data* qubit states upwards once, and cyclically permute all *Hadamard* qubit states upwards once. The effect of applying V a total of $n - j - 2$ more times given this initialization is to repeat this behavior $n - j - 2$ times, except it will not cycle the state of the *cursor* qubits. Importantly, the state of the j th *data* qubit from the top before this behavior becomes stored in the third-to-last *data* qubit after this behavior. The glyph \odot is intended to remind that when the cursor is in this state, the effect is to cyclically permute all but the last *data* qubit state.

Behavior 3. The *cursor* is initialized in the state \odot and the upper-most *swap* qubit is initialized in the state $|0\rangle$. The effect of applying V once given this initialization is to cyclically permute the *swap* qubit states upwards once, cycle the *cursor* state from \odot to \ominus , cyclically permute all but the last *data* qubit state upwards once, apply $\Lambda(S)$ between the last two *data* qubits, apply H to the last *data* qubit if the upper-most *Hadamard* qubit is in the state $|1\rangle$, and cyclically permute the *Hadamard* qubit states upwards once. Importantly, the states of qubits k and j manipulated in behaviors 1 and 2 have the gate $\Lambda(S)$ applied between them and the state of qubit k has the gate H applied to it conditioned on the state of the upper-most *Hadamard* qubit before this behavior. The glyph \ominus is intended to remind that when the cursor is in this state, the effect is to apply gates from the Kitaev gate basis to the *data* qubits.

Behavior 4. The *cursor* is initialized in the state \ominus and the upper-most *swap* qubit is initialized to the state $|0\rangle$. The effect of applying V once given this initialization is to cyclically permute the states of the *swap* qubits upwards once, cycle the *cursor* state from \ominus to \odot , and cyclically permute the *data* and *Hadamard* qubits upwards once. Importantly, after this behavior, the *cursor* is in the same state it started in for behavior 1.

Behaviors 1–3 describe how, given a suitable initialization of program qubits, the gate $(I \otimes H)\Lambda(S)$ or $\Lambda(S)$ can be applied to any two desired data qubits. Moreover, behavior 4 shows that by lengthening the program region, any sequence of such gates can be implemented. However, the Kitaev gate basis is not $\{\Lambda(S), (I \otimes H)\Lambda(S)\}$, but $\{H, \Lambda(S)\}$. In order to apply H to any desired qubit, we use a trick: we program V to apply $(I \otimes H)\Lambda(S) \cdot \Lambda(S)^3$. Because $\Lambda(S)^4 = I$, this is the same as applying H .

If a circuit U over the Kitaev gate basis contains $T_{\Lambda(S)}$ controlled- S gates and T_H Hadamard gates on n data qubits, then the number of program region qubits (and gates) needed by V is at most $4n(T_{\Lambda(S)} + 2T_H)$, so the simulation by V is *linear* in the size of U .

It is worth noting that V is defined over a gate basis that contains gates that are not spatially local when the qubits are arranged in a line. Spatial locality in one

dimension could have been achieved much more easily (and without the need for programmability) with, say, the gate basis $\{H, \Lambda(S), SWAP\}$. Nevertheless, all the nonlocality in V is between the cursor program qubits and the other data qubits and enables V to be repeated in a simple fashion to simulate arbitrary circuits U over the Kitaev gate basis exactly. By replacing each of the non-cursor qubits with eight-state systems (to simulate one data qubit plus two cursor qubits), it is possible to derive a related programmable circuit that requires only spatially local gates in one dimension. Instead of going through this exercise, we move on to how to construct a Hamiltonian that would simulate such a circuit via a continuous-time quantum walk.

III. A FAMILY OF NEAREST-NEIGHBOR HAMILTONIANS COUPLING 8-LEVEL SYSTEMS ON A RING

In this section, we construct a Hamiltonian H_8 acting on nearest-neighbor eight-state systems on a ring. The interactions in H_8 are closely related to the gates used by the circuit V of Fig. 2 of the last section. We defer a discussion of how H_8 is used to achieve universal quantum computation to subsequent sections.

A. State space and geometry

Let V be the circuit depicted in Fig. 2, and let $V^{(K)}$, $K = \mathcal{O}(nT)$, be a circuit with suitably initialized inputs that simulates a T -gate, n -qubit circuit over the Kitaev gate basis. Let \mathcal{H}_8 denote the Hilbert space of a quantum system having eight possible orthogonal states and let $\mathcal{H}_8^{\otimes K}$ denote the Hilbert space of a collection of K of these systems arranged in a one-dimensional ring. For convenience, we will think of the eight-state system as arising from the combined state space of quantum systems having two and four states respectively. In other words, we will use the conceptual decomposition of the Hilbert space of each of these systems as $\mathcal{H}_8 = \mathcal{H}_2 \otimes \mathcal{H}_4$, where \mathcal{H}_d indicates a Hilbert space of dimension d .

As illustrated in Figure 3, this ring can be divided into two parallel “lines” and three regions. This layout mimics that of the circuit V rotated on its side, but with periodic boundary conditions. However instead of having a cursor program region, each qubit on the lower line has its own adjacent 4-state cursor system on the upper line that can be in one of the “active” states \odot , \ominus , or $\omin�$, or in $\omin�$, a new “inactive” state not used by V . Just as the program region qubits in V are always in classical states $|0\rangle$ or $|1\rangle$ for any simulation of a quantum circuit, so too will the qubits in the program regions of this ring always be in classical states, which we label schematically by the glyphs \odot and $\omin�$. However, qubits in the data region may be in any superposition of $|0\rangle$ and $|1\rangle$, and indeed even entangled with the state of all other data

qubits, so we represent an arbitrary state of a data qubit by the glyph $\textcircled{?}$. Table 4 is a legend for the glyphs we use to denote the cursor states, qubit states, and region boundaries in this geometry.

B. Hamiltonian construction

Like Feynman’s Hamiltonian (1), we construct H_8 on \mathcal{H}_8 as a sum of terms with g_i representing forward computation and g_i^\dagger representing backwards computation:

$$H := \sum_{i=1}^K g_i + g_i^\dagger. \quad (5)$$

Each operator g_i is an interaction involving only the 8-state systems at sites i and $(i+1) \bmod K$ in the ring. A given g_i is generically responsible for two things. Firstly, it examines the current cursor state at site i , replaces it with \ominus and appropriately sets the cursor at site $i+1$. This effectively passes the cursor down the ring and ensures that only a single site is “active” (non- \ominus). Secondly, a g_i is charged with performing any necessary gates on the qubits at sites i and $i+1$, which depend on both the location i and the current state of the cursor.

At most of the ring sites, g_i merely swaps the state of the cursor. Namely,

$$g_i := \left| \begin{array}{c} \ominus \textcircled{?} \\ \textcircled{0} \textcircled{?} \end{array} \right\rangle \left\langle \begin{array}{c} \textcircled{?} \ominus \\ \textcircled{?} \textcircled{0} \end{array} \right| + \left| \begin{array}{c} \ominus \textcircled{?} \\ \textcircled{1} \textcircled{?} \end{array} \right\rangle \left\langle \begin{array}{c} \textcircled{?} \ominus \\ \textcircled{?} \textcircled{1} \end{array} \right|, \quad (6)$$

where $\textcircled{?}$ represents an unspecified $\textcircled{Q}, \textcircled{C}$ or $\textcircled{!}$ state that is preserved during the transition and $\textcircled{?}$ represents either a $\textcircled{0}$ or $\textcircled{1}$ that is preserved during the transition. In other words, Eq. (6) is actually a sum of terms, one for each possible value of $\textcircled{?}$ and $\textcircled{?}$.

The only places where g_i deviates from this behavior is near the boundaries between regions. This is because the circuit $V^{(K)}$ consists of *SWAP* gates everywhere except near these boundaries. We define how g_i acts near these boundaries below:

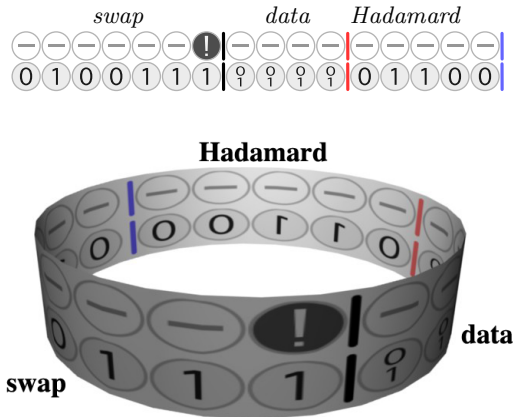


FIG. 3: Architecture Layout (Color online)

Cursor Line	Program & Data Line
\ominus : Inactive/Non-cursor site	$\textcircled{0}$: “Classical” zero bit
$\textcircled{!}$: Gate phase cursor	$\textcircled{1}$: “Classical” one bit
\textcircled{Q} : Cycle phase cursor	$\textcircled{?}$: Arbitrary qubit state
\textcircled{C} : Hold-cycle phase cursor	
$\textcircled{?}$: Any one of $\textcircled{Q}, \textcircled{C}, \textcircled{!}$	
Boundary Markers	
	Between <i>swap</i> and <i>data</i>
	Between <i>data</i> and <i>Hadamard</i>
	Between <i>Hadamard</i> and <i>swap</i>

FIG. 4: Glyphs for cursor states, qubits states, and region boundaries. (Color online.)

• Swap Region Near the Swap-Data Boundary

In order to simulate the action of $V^{(K)}$, which has a definite beginning and end, by H_8 , which has periodic boundary conditions, we replace Eq. (6) for the last two states in the swap program region with projections onto *start* and *stop* states:

$$g_i := \frac{1}{2} \left| \begin{array}{c} \ominus \textcircled{!} \\ \textcircled{1} \textcircled{1} \end{array} \right\rangle \left\langle \begin{array}{c} \ominus \textcircled{!} \\ \textcircled{1} \textcircled{1} \end{array} \right| + \frac{1}{2} \left| \begin{array}{c} \textcircled{!} \ominus \\ \textcircled{1} \textcircled{0} \end{array} \right\rangle \left\langle \begin{array}{c} \textcircled{!} \ominus \\ \textcircled{1} \textcircled{0} \end{array} \right|, \quad (7)$$

where the factors of $1/2$ are to accommodate the fact that both g_i and g_i^\dagger are included in H_8 .

The second term in (7) is a projection onto a stop state that is never reached by a valid programming of $V^{(K)}$. Specifically, “Behavior 4” in Sec. II prevents this from happening. (A cursor in the state $\textcircled{!}$ never sees a swap program qubit in the state $\textcircled{1}$.) By augmenting the swap program with one extra qubit in the state $\textcircled{1}$, the general swapping interaction (6) will evolve a state with a $\textcircled{!}$ atop $\textcircled{1}$ to the last two swap program sites discussed here and stop propagating further. Of course, because a g_i^\dagger term is in H_8 for every g_i term in H_8 , it is possible for evolution to “undo” some of the computation and compute in reverse from the stop state. However, no additional forward computation will accrue after this stop state is reached.

The first term in (7) is also one that is never reached by a valid programming of $V^{(K)}$. Moreover, it is never reached by the extended “stop state” implementation by H_8 just discussed. Indeed, it is precisely the stop state projection term that prevents the start state from ever being reached. Hence no computation can be “undone” by g_i^\dagger terms once this start state is reached (in reverse).

The net effect of incorporating stop and start state interactions is that evolution by H_8 on a ring can be “unfurled” into an equivalent interaction by a re-

lated Hamiltonian on a line. This will be described in more detail in Sec. IV.

To simulate the action of $V^{(K)}$ on these two qubits for a *valid* programming of the swap program register, we need to add to g_i of Eq. (7) the following terms:

$$g_i := \left| \begin{array}{c} \ominus \textcircled{?} \\ \textcircled{?} \textcircled{0} \end{array} \right\rangle \left\langle \begin{array}{c} \textcircled{?} \textcircled{-} \\ \textcircled{?} \textcircled{0} \end{array} \right| + \left| \begin{array}{c} \ominus \textcircled{?} \\ \textcircled{1} \textcircled{?} \end{array} \right\rangle \left\langle \begin{array}{c} \textcircled{?} \textcircled{-} \\ \textcircled{?} \textcircled{1} \end{array} \right| \quad (8)$$

$$+ \left| \begin{array}{c} \ominus \textcircled{?} \\ \textcircled{0} \textcircled{?} \end{array} \right\rangle \left\langle \begin{array}{c} \textcircled{?} \textcircled{-} \\ \textcircled{1} \textcircled{0} \end{array} \right| + \left| \begin{array}{c} \ominus \textcircled{?} \\ \textcircled{1} \textcircled{?} \end{array} \right\rangle \left\langle \begin{array}{c} \textcircled{?} \textcircled{-} \\ \textcircled{?} \textcircled{1} \end{array} \right|$$

$$+ \left| \begin{array}{c} \ominus \textcircled{!} \\ \textcircled{?} \textcircled{0} \end{array} \right\rangle \left\langle \begin{array}{c} \textcircled{!} \textcircled{-} \\ \textcircled{0} \textcircled{?} \end{array} \right|.$$

Finally, although the following state only arises when V simulates one of the rather banal one-gate circuits $U = (I \otimes H)\Lambda(S)$ or $U = \Lambda(S)$, we add to g_i a projection onto a second stop state for completeness' sake:

$$g_i := \frac{1}{2} \left| \begin{array}{c} \textcircled{!} \textcircled{-} \\ \textcircled{1} \textcircled{1} \end{array} \right\rangle \left\langle \begin{array}{c} \textcircled{!} \textcircled{-} \\ \textcircled{1} \textcircled{1} \end{array} \right| \quad (9)$$

- *Swap-Data Boundary*

To simulate the permutation of cursor states $\textcircled{?} \rightarrow \textcircled{?} \rightarrow \textcircled{!} \rightarrow \textcircled{?}$ enacted by $V^{(K)}$ when the state of the lower-most qubit in the swap program is a 0, we replace (6) at the swap-data boundary with

$$g_i := \left| \begin{array}{c} \ominus \textcircled{?} \\ \textcircled{0} \textcircled{?} \end{array} \right\rangle \left\langle \begin{array}{c} \textcircled{?} \textcircled{-} \\ \textcircled{0} \textcircled{?} \end{array} \right| + \left| \begin{array}{c} \ominus \textcircled{!} \\ \textcircled{0} \textcircled{?} \end{array} \right\rangle \left\langle \begin{array}{c} \textcircled{?} \textcircled{-} \\ \textcircled{0} \textcircled{?} \end{array} \right| \quad (10)$$

$$+ \left| \begin{array}{c} \ominus \textcircled{?} \\ \textcircled{0} \textcircled{?} \end{array} \right\rangle \left\langle \begin{array}{c} \textcircled{!} \textcircled{-} \\ \textcircled{0} \textcircled{?} \end{array} \right|.$$

To simulate the movement of the cursor from the swap region to the data region when the last qubit's state of the swap program is a 1 (and no cycling of the cursor's state is performed), we add to g_i at this location one more term:

$$g_i := \left| \begin{array}{c} \ominus \textcircled{?} \\ \textcircled{1} \textcircled{?} \end{array} \right\rangle \left\langle \begin{array}{c} \textcircled{?} \textcircled{-} \\ \textcircled{1} \textcircled{?} \end{array} \right|. \quad (11)$$

- *Data Region Near the Data-Hadamard Boundary*

Because $V^{(K)}$ swaps the last two data qubits only when the cursor is in the state $\textcircled{?}$, and because $V^{(K)}$ only applies a $\Lambda(S)$ between these qubits when the cursor is in the state $\textcircled{!}$, we replace (6)

at the last two data sites with

$$g_i := \left| \begin{array}{c} \ominus \textcircled{?} \\ \textcircled{?} \textcircled{0} \end{array} \right\rangle \left\langle \begin{array}{c} \textcircled{?} \textcircled{-} \\ \textcircled{?} \textcircled{0} \end{array} \right| + \left| \begin{array}{c} \ominus \textcircled{?} \\ \textcircled{?} \textcircled{1} \end{array} \right\rangle \left\langle \begin{array}{c} \textcircled{?} \textcircled{-} \\ \textcircled{?} \textcircled{1} \end{array} \right| \quad (12)$$

$$+ \left| \begin{array}{c} \ominus \textcircled{?} \\ \textcircled{0} \textcircled{?} \end{array} \right\rangle \left\langle \begin{array}{c} \textcircled{?} \textcircled{-} \\ \textcircled{?} \textcircled{0} \end{array} \right| + \left| \begin{array}{c} \ominus \textcircled{?} \\ \textcircled{1} \textcircled{?} \end{array} \right\rangle \left\langle \begin{array}{c} \textcircled{?} \textcircled{-} \\ \textcircled{?} \textcircled{1} \end{array} \right|$$

$$+ \left| \begin{array}{c} \ominus \textcircled{!} \\ \textcircled{?} \textcircled{0} \end{array} \right\rangle \left\langle \begin{array}{c} \textcircled{!} \textcircled{-} \\ \textcircled{?} \textcircled{0} \end{array} \right| + \left| \begin{array}{c} \ominus \textcircled{!} \\ \textcircled{?} \textcircled{1} \end{array} \right\rangle \left\langle \begin{array}{c} \textcircled{!} \textcircled{-} \\ \textcircled{?} \textcircled{1} \end{array} \right|,$$

where the bar over the qubits in the last two terms indicates that a $\Lambda(S)$ gate has been applied between them.

- *Data-Hadamard Boundary*

Because no swapping occurs across the data-Hadamard boundary and because additionally a Hadamard gate is applied to the state of the last data qubit if the state of the upper-most Hadamard program qubit is a 1 and the cursor is in the state $\textcircled{!}$, we replace Eq. (6) at the data-Hadamard boundary with

$$g_i := \left| \begin{array}{c} \ominus \textcircled{?} \\ \textcircled{?} \textcircled{0} \end{array} \right\rangle \left\langle \begin{array}{c} \textcircled{?} \textcircled{-} \\ \textcircled{?} \textcircled{0} \end{array} \right| + \left| \begin{array}{c} \ominus \textcircled{?} \\ \textcircled{?} \textcircled{1} \end{array} \right\rangle \left\langle \begin{array}{c} \textcircled{?} \textcircled{-} \\ \textcircled{?} \textcircled{1} \end{array} \right| \quad (13)$$

$$+ \left| \begin{array}{c} \ominus \textcircled{?} \\ \textcircled{?} \textcircled{0} \end{array} \right\rangle \left\langle \begin{array}{c} \textcircled{?} \textcircled{-} \\ \textcircled{?} \textcircled{0} \end{array} \right| + \left| \begin{array}{c} \ominus \textcircled{?} \\ \textcircled{?} \textcircled{1} \end{array} \right\rangle \left\langle \begin{array}{c} \textcircled{?} \textcircled{-} \\ \textcircled{?} \textcircled{1} \end{array} \right|$$

$$+ \left| \begin{array}{c} \ominus \textcircled{!} \\ \textcircled{?} \textcircled{0} \end{array} \right\rangle \left\langle \begin{array}{c} \textcircled{!} \textcircled{-} \\ \textcircled{?} \textcircled{0} \end{array} \right| + \left| \begin{array}{c} \ominus \textcircled{!} \\ \textcircled{?} \textcircled{1} \end{array} \right\rangle \left\langle \begin{array}{c} \textcircled{!} \textcircled{-} \\ \textcircled{?} \textcircled{1} \end{array} \right|,$$

where the bar over the qubit in the last term indicates that a Hadamard gate has been applied to it.

- *Hadamard-Swap Boundary*

The only subtlety about the Hadamard-swap boundary is that qubit states are not swapped across them in $V^{(K)}$. Hence Eq. (6) is replaced at this boundary by

$$g_i := \left| \begin{array}{c} \ominus \textcircled{?} \\ \textcircled{1} \textcircled{?} \end{array} \right\rangle \left\langle \begin{array}{c} \textcircled{?} \textcircled{-} \\ \textcircled{1} \textcircled{?} \end{array} \right| + \left| \begin{array}{c} \ominus \textcircled{?} \\ \textcircled{0} \textcircled{?} \end{array} \right\rangle \left\langle \begin{array}{c} \textcircled{?} \textcircled{-} \\ \textcircled{0} \textcircled{?} \end{array} \right|. \quad (14)$$

Given this definition for our Hamiltonian H_8 , our next task is to demonstrate how it may be programmed to simulate quantum circuits. In the next section, we show how this can be done when H_8 drives a continuous-time quantum walk. In the subsequent section, we show how this can be done when (a slightly modified version of) H_8 is the final Hamiltonian of an adiabatic algorithm.

IV. UNIVERSAL QUANTUM COMPUTATION VIA A CONTINUOUS-TIME QUANTUM WALK DRIVEN BY H_8

The Hamiltonian H_8 defined in the previous section enables universal quantum computation by driving a continuous-time quantum walk in the following way. First choose a precision ϵ and a quantum circuit W to simulate to within precision ϵ . Then, construct a circuit U over the Kitaev gate basis that approximates W to within ϵ . This can be done using standard techniques, *e.g.*, by the Solovay-Kitaev algorithm [35]. Next, determine how to program the circuit V defined in Sec. II so that it simulates U exactly. Initialize the *swap*, *data*, and *Hadamard* regions of $\mathcal{H}_4 \otimes \mathcal{H}_2$ to the same states that one would initialize the *swap*, *data*, and *Hadamard* regions for V . However, because the *swap* region on the 1D ring has one more state than in the V circuit, initialize the last *swap* qubit on the 1D ring to be in the state $|1\rangle$. (Note that a valid programming obtained from V will necessarily also have the penultimate *swap* qubit also initialized to the state $|1\rangle$.) Finally, initialize the cursor line on the 1D ring to be \ominus everywhere, except over the last *swap* qubit, where it is in the state $\omin�$. We will denote the state of the 1D ring so initialized by $|\psi_0\rangle$.

Only two g_i terms in H_8 act nontrivially on $|\psi_0\rangle$. The first is the *start* state projector (7) and the second is the transition term across the *swap-data* boundary (11). Thus evolution by H_8 can either keep the ring in the state $|\psi_0\rangle$ or advance it with some amplitude to a unique successor state $|\psi_1\rangle$. If the ring is in the state $|\psi_1\rangle$, the Hamiltonian H_8 only couples it back to $|\psi_0\rangle$ or to a unique successor state $|\psi_2\rangle$ given by the general swap rule (6). This line of reasoning continues, with a unique successor state and predecessor state existing for $|\psi_t\rangle$ in the ring until the final $t = \bar{T} := \mathcal{O}(nT)$. In this configuration, only the *stop* state projector and the predecessor state match. The Hamiltonian H_8 can therefore be restricted to a subspace of size $\bar{T} + 1$, on which it looks like

$$H_8^{(\text{eff})} := \sum_{i=1}^{\bar{T}} |\psi_i\rangle\langle\psi_{i-1}| + |\psi_{i-1}\rangle\langle\psi_i| \quad (15)$$

$$+ |\psi_0\rangle\langle\psi_0| + |\psi_{\bar{T}}\rangle\langle\psi_{\bar{T}}|. \quad (16)$$

At this point, one can argue as Feynman originally did [15] that evolution by H_8 is the same as by a quantum walk on a line. Straightforward analysis demonstrates that the time at which this walk has a maximum amplitude for moving from $|\psi_0\rangle$ to $|\psi_{\bar{T}}\rangle$ is time $T/2$, at which the amplitude is

$$\langle\psi_{\bar{T}}|e^{-iH_8 T/2}|\psi_0\rangle \approx \bar{T}^{-1/3}. \quad (17)$$

A detailed analysis in terms of Bessel functions of why this is the case can be found in numerous places, for example in [36].

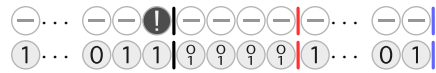


FIG. 5: Start State: \dots represents an arbitrary number of qubit/qudit vertical pairs

Given these insights, a continuous-time quantum walk driven by H_8 can be made to simulate V arbitrarily well in several different ways:

- One could simply repeat the preparation and evolution $\mathcal{O}(\bar{T}^{2/3})$ times, availing upon the Chernoff bound that it is exponentially likely in \bar{T} that one of the final measurements will find the system in the state $|\psi_{\bar{T}}\rangle$, representing the output of the computation.
- One could replace the projection onto the stop state in H_8 with $\mathcal{O}(\bar{T}^{2/3})$ general swap transitions defined by Eq. (6) on $\mathcal{O}(\bar{T}^{2/3})$ additional qubits in the swap program region. By use of the Chernoff bound once again, it becomes exponentially likely in \bar{T} that the cursor is in one of these new “dummy” locations. Note that simply adding $\mathcal{O}(\bar{T}^{2/3})$ identity gates to the end of the circuit V being simulated is not trivial as it was in similar refs. [15, 37] because V simulates the identity by applying $\Lambda(S)^4$.
- One could replace both the start state and stop state projections in H_8 by additional “runway” and “landing pad” qubit chains of size $\mathcal{O}(\bar{T}^{2/3})$ that are swapped by the general swap rule (6). As shown by Feynman [15] (and argued in greater detail in [38]) this will allow the computation to proceed ballistically with arbitrarily high probability.
- One could redefine H_8 so that the t th term had a coefficient $\sqrt{\bar{T}(\bar{T} - t)}$, enabling perfect fidelity state transfer to the final state of the computation [39, 40].

Running a quantum algorithm in this model is particularly simple. One first initializes the state $|\psi_0\rangle$, then waits a time $T/2$, and finally measures the state of the ring destructively using one of the methods described above. Because no dynamical controls are required during the course of the computation, “gate errors” appear as fabrication errors in H_8 . Rather than having to deal with such errors during runtime, which is computationally expensive and must be done in the circuit model, these errors may instead be dealt with during the fabrication of H_8 in a much more controlled environment. Decoherence and other environmentally-induced noise processes will still be present, but at least they will not be conflated with dynamical control errors.

V. UNIVERSAL QUANTUM COMPUTATION VIA ADIABATIC EVOLUTION TO H_δ

Given that comparable physical models have appeared in proofs for adiabatic universality [20, 23], we will demonstrate that one-dimensional Hamiltonians on 8-level quantum systems on a ring are universal under adiabatic evolution. Our derivation will closely mirror these previous works and follows straightforwardly from results of Aharonov *et al.* [20] that have been further refined by Deift *et al.* [41]. We begin by quoting the adiabatic theorem:

The Adiabatic Theorem (adapted from [41], quoted from [20]) Let H_{init} and H_{final} be two Hamiltonians acting on a quantum system and consider the time-dependent Hamiltonian $H(s) = (1-s)H_{\text{init}} + sH_{\text{final}}$. Assume that for all s , $H(s)$ has a unique ground state. Then for any fixed $\delta > 0$, if

$$T \geq \Omega \left(\frac{\|H_{\text{final}} - H_{\text{init}}\|^{1+\delta}}{\epsilon^\delta \min_{s \in [0,1]} \{\Delta^{2+\delta}(H(s))\}} \right), \quad (18)$$

then the final state of an adiabatic evolution according to H for time T (with an appropriate setting of global phase) is ϵ -close in l_2 -norm to the ground state of H_{final} . The matrix norm is the spectral norm $\|H\| = \max_w \|Hw\|/\|w\|$.

Loosely speaking, the adiabatic theorem states that if the variation between the Hamiltonians is slow enough, then the quantum state will be in the ground state of H_{final} at the end of evolution if it starts in the ground state of H_{initial} . Consequently, to demonstrate that our model is universal under adiabatic evolution, we must pick an H_{init} and H_{final} whose ground states are appropriately related to the initial and final states of computation.

First consider H_{init} . We have previously detailed the configuration of a valid initial state, $|\psi_0\rangle$, which encodes both the *program of execution* and the *input data state*. We define H_{init} to be a sum over local projectors for which $|\psi_0\rangle$ is the zero-eigenvalue ground state. This differs from previous adiabatic proofs involving non-programmable architectures, which only needed to encode the input data state. We will now construct H_{init} piece by piece.

In a shorthand notation analogous to the one used in Sec. IV, the first term is

$$I - \left| \begin{array}{c} \ominus \ominus \\ \ominus \ominus \\ \ominus \ominus \\ \ominus \ominus \end{array} \right\rangle \left\langle \begin{array}{c} \ominus \ominus \\ \ominus \ominus \\ \ominus \ominus \\ \ominus \ominus \end{array} \right| \quad (19)$$

where the projector is for the two locations to the left of the swap program/data region boundary. While this will ensure that the ground state is an initial state, it does not ensure that it is valid or that it executes the desired program (i.e. that it is $|\psi_0\rangle$). However, we can pick out our desired initial state by adding nearest-neighbor projectors to this Hamiltonian in a chained fashion. The

first such projector will be between the leftmost qubit in the above projector and its left neighbor. Without loss of generality, suppose that the program qubit for our desired program is a \ominus in this location; we would then add terms

$$\sum \left| \begin{array}{c} \ominus \ominus \\ \ominus \ominus \\ \ominus \ominus \\ \ominus \ominus \end{array} \right\rangle \left\langle \begin{array}{c} \ominus \ominus \\ \ominus \ominus \\ \ominus \ominus \\ \ominus \ominus \end{array} \right|_{-1} \quad (20)$$

$$+ \sum_{\ominus \neq \ominus} \left| \begin{array}{c} \ominus \ominus \\ \ominus \ominus \\ \ominus \ominus \\ \ominus \ominus \end{array} \right\rangle \left\langle \begin{array}{c} \ominus \ominus \\ \ominus \ominus \\ \ominus \ominus \\ \ominus \ominus \end{array} \right|_{-1} \quad (21)$$

where the first sum is over all qudit symbols, ensuring that anything over a \ominus has a higher energy. Similarly, the second sum ensures that all configurations except the desired \ominus over a \ominus have a higher energy. Thus, we sum over all non-desired configurations, of which there are seven. This process is continued by sliding over one location and again summing over projectors where the right qudit-qubit pair is fixed to the desired initial configuration and the left qudit-qubit pair runs over all seven illegal or non-desired configurations. Moving around the entire ring step by step, we add similar projectors which tack on all undesired nearest-neighbor pairs. This ensures that the ground state is precisely $|\psi_0\rangle$; it is both valid and encodes only the program we want to execute.

Now consider H_{final} . As discussed in [20, 23, 41, 42], we seek a H_{final} whose ground state is the sum-over-histories state

$$\frac{1}{\sqrt{\bar{T}+1}} \sum_{j=0}^{\bar{T}} |\psi_j\rangle. \quad (22)$$

This can be accomplished simply by adding penalty terms to Eq. 15 which “prefer” the transition elements. That is, we define

$$H_{\text{final}} := \sum_{i=1}^{\bar{T}-1} |\psi_i\rangle\langle\psi_i| \quad (23)$$

$$- \frac{1}{2} \sum_{i=0}^{\bar{T}} |\psi_i\rangle\langle\psi_{i-1}| + |\psi_{i-1}\rangle\langle\psi_i| \quad (24)$$

$$+ \frac{1}{2} |\psi_0\rangle\langle\psi_0| + \frac{1}{2} |\psi_{\bar{T}}\rangle\langle\psi_{\bar{T}}|. \quad (25)$$

Expressed in the $|\psi_i\rangle$ basis, H_{init} is simply

$$H_{\text{init}} = \begin{pmatrix} 0 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix} \quad (26)$$

as $|\psi_0\rangle$ is the unique groundstate. Similarly, H_{final} is

$$H_{\text{final}} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & 0 & \cdots & 0 \\ -\frac{1}{2} & 1 & -\frac{1}{2} & 0 & \ddots & \vdots \\ 0 & -\frac{1}{2} & 1 & -\frac{1}{2} & 0 & \ddots & \vdots \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & 0 & -\frac{1}{2} & 1 & -\frac{1}{2} & 0 \\ & & & 0 & -\frac{1}{2} & 1 & -\frac{1}{2} \\ 0 & \cdots & & 0 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}. \quad (27)$$

As proved in Sec 3.1.2 of [20] and simplified in [41], the spectral gap of these Hamiltonians is at least $1/[2(\bar{T} + 1)^2]$, which is an inverse polynomial in T , the number of gates in the initial quantum circuit (recall \bar{T} is polynomial in T). Thus, the evolution can be considered efficient, proving that Hamiltonians of 8-level quantum systems on a one-dimensional ring are universal for adiabatic quantum computation.

VI. A TWO-DIMENSIONAL CYLINDER QUBIT HAMILTONIAN H_2

Given the conceptual decomposition of the 8-level quantum system into a 4-level quantum system and a qubit, it is easy to imagine fully decomposing each 8-level quantum system into 3 qubits. Such an architecture would still be effectively one-dimensional, as the width of the ring is invariant. However, each nearest neighbor interaction in the 8-level system would become a 6-body interaction, which is a greater than Feynman's 4-body interactions.

Alternatively, one could use gadget Hamiltonian theory, as presented in [19], to reduce the degree of interactions between qubits. This procedure augments the original system \mathcal{S} with additional mediator qubits. Given an initial Hamiltonian H acting on \mathcal{S} , one constructs a new Hamiltonian H' which acts on the augmented system and whose action restricted to \mathcal{S} is bounded as $\|H' - H\| < \epsilon$ for a desired ϵ . For our purposes, we would seek to replace the 6-body terms required for the transition rules with lower body terms. While in theory it is straightforward to apply a 6-body gadget, each term in our original Hamiltonian will require 6 mediators. Generally, each 6-body term will then require more than 6 mediators, resulting in an expanding geometry. We have been unable to devise a scheme which maintains both spatially local interactions and an effective one-dimensional width under this replacement scheme.

An alternative and more promising approach, presented by Oliveira and Terhal [21], details an "efficient" gadget reduction procedure to a 2-local Hamiltonian which is planar, but two dimensional. We defer to the paper for the details of such a reduction, but note that there is a prescription for applying gadgets to reduce a

spatially-sparse k -local Hamiltonian to a 2-local Hamiltonian on a regular lattice. This reduction maintains the spatial setup of the initial Hamiltonian, so that for our ring geometry, the reduced setup would be a regular lattice on the surface of a cylinder. Moreover, the effective interaction is not only close to the initial Hamiltonian with respect to its eigenvalues, but the operator norm is close within a restricted subspace. Thus the dynamics are similarly close. In other words, given a desired ϵ , a "reduced" 2-local version of the architecture can be designed with dynamics which are ϵ -close to that of the original architecture. While the actual mapping requires fine-tuning perturbation coupling parameters to ensure only polynomial overhead and growth, the Oliveira-Terhal approach does provide a means for reducing the interaction degree of our architecture, though the resulting system is on the surface of a cylinder whose height is no longer fixed.

VII. CONCLUSION

We presented a family of time-independent Hamiltonians that can enable universal quantum computation either by driving a continuous-time quantum walk or by terminating an adiabatic algorithm. When used to drive a continuous-time quantum walk, quantum computation consists of 1) preparing an input that describes a quantum circuit to be executed and the quantum data onto which it is to be applied, 2) waiting the appropriate amount of time, and 3) measuring the output. The simplicity of the operation of such a machine is appealing, but Feynman's original proposal for realizing it [15] still remains far out of technological reach. Our work demonstrates that it suffices for such a machine to use only nearest-neighbor interactions between 8-level systems on a 1D ring, which may be more technologically feasible. It also demonstrates that simulating the dynamics of 1D time-independent Hamiltonians on 8-level systems is a BQP-complete problem, even though it is known how to simulate 1D spin systems in a way that scales polynomially with the number of spins [43].

When used to terminate an adiabatic algorithm, our Hamiltonian achieves universality via nearest-neighbor interactions between 8-level systems on a 1D ring, one level fewer than in a recent proof of universality of the adiabatic algorithm by 9-level nearest-neighbor 1D Hamiltonians [23].

Finally, using gadget perturbation theory [21], our Hamiltonian can be made spatially local using only qubits rather than 8-level systems, but at the expense of requiring the qubits to lie on the two-dimensional geometry of a cylinder rather than a one-dimensional geometry of a ring.

A remaining challenge is to address error correction and fault tolerance for adiabatic and quantum walk models driven by the Hamiltonian we constructed [44]. In particular, because the quantum walk describing quan-

tum computation is effectively a continuous-time quantum walk on a line, an imperfect implementation may be subject to Anderson localization which could exponentially suppress propagation along this line. One promising feature of the quantum walk model is that there are no dynamical controls during the operation of the quantum computer, so that control errors are all fabrication. This allows such errors to be handled at “compile time” rather than at “run time,” which could be much easier.

Another remaining challenge is to explore whether finding the ground state of our Hamiltonian is a QMA complete problem. We conjecture that it is, as several similar Hamiltonians used to prove adiabatic quantum computing universality are [20, 23]. However, the “clock” and its update rule as implemented by our Hamiltonian are not localized to a particular point in space, as computation winds around the ring many times during the course of a computation. Hence, previous proofs do not translate directly and creative ideas are required. Since our architecture is also programmable, the need to encode the program in the initial state further complicates the QMA question.

Addendum: As we were finishing this paper, we became aware of related work by Nagaj and Wocjan that demonstrates that translationally invariant quantum walks in one dimension (“continuous-time quantum cellular automata”) are also universal for quantum computation, albeit using ten-dimensional rather than eight-dimensional systems [32]. A final remaining challenge we state is to explore whether our model can be made translationally invariant or whether the dimension of the Nagaj-Wocjan model can be reduced to eight as ours is.

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APPENDIX A: EXAMPLE COMPUTATION

The following illustrates computing the circuit in Figure 6.

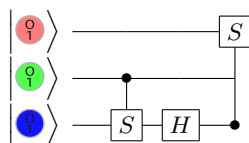
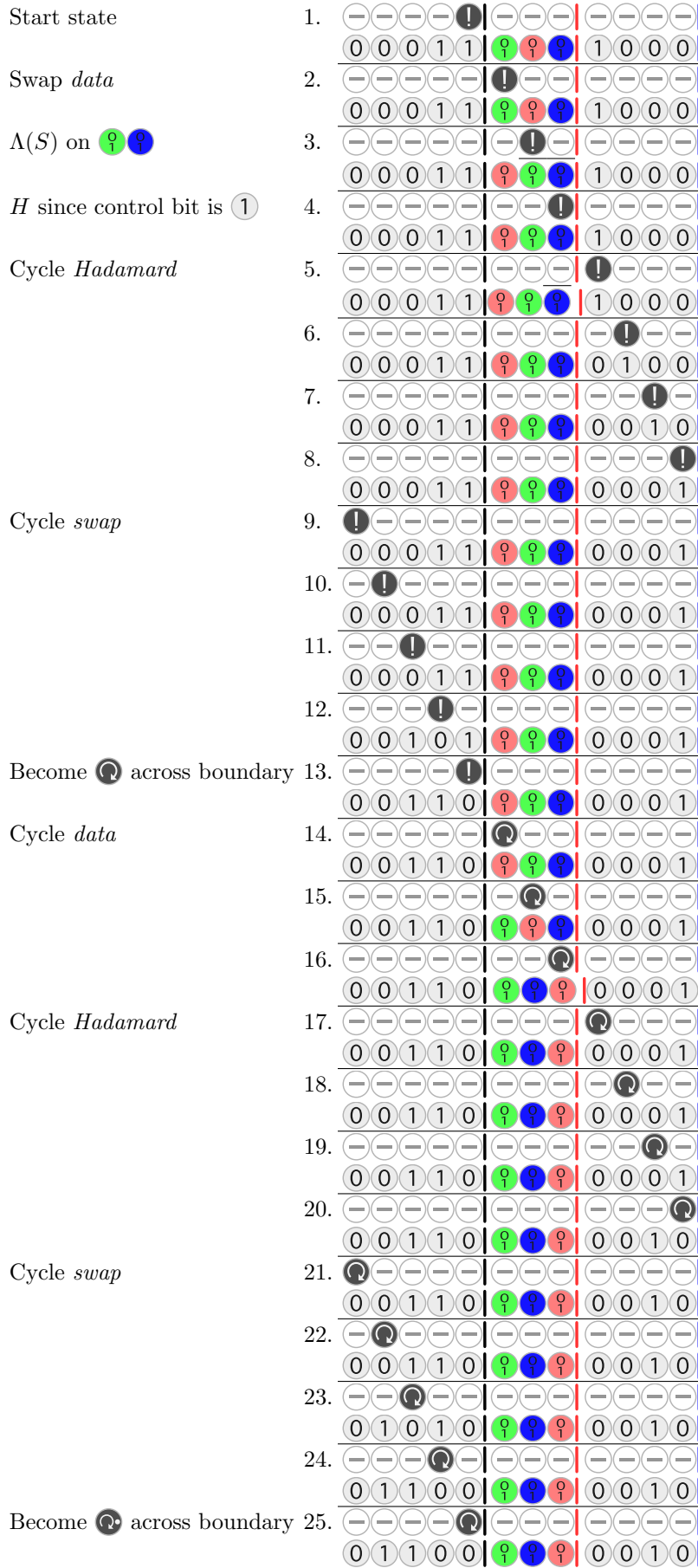
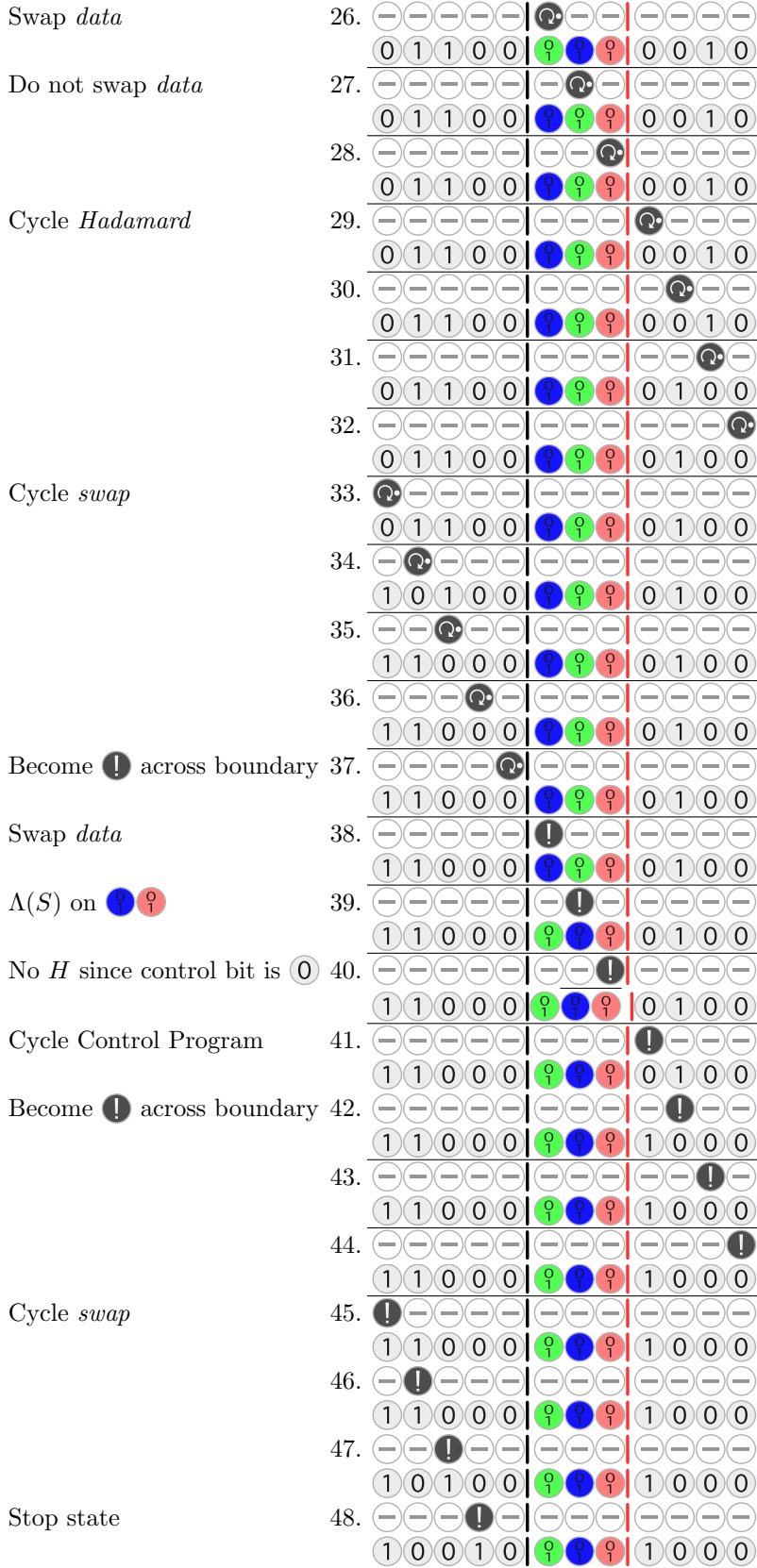


FIG. 6: Equivalent Circuit





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