Universal single-qubit quantum controller-observers

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Abstract

A feedback loop consists of three dynamical systems: an observer, a controller, and the system to be manipulated. Because of back-action in quantum systems, quantum controllers can act as quantum observers and vice-versa. We demonstrate that under very general conditions, a single quantum bit can serve as both a full controller and full observer of a quantum system for a feedback loop. By similar techniques, we show that a qubit can be harnessed to serve as a quantum communication channel between two systems, and that multiple systems can be connected together to create an efficient universal quantum computer. We propose experimental realizations of our approach, and explore the implications for controllability, observability, and quantum information processing.

A practical problem in quantum control and quantum computation is understanding when a given complex behavior of a quantum system can be constructed out of simpler operations. Geometric control theory has been used to show the universality of simple quantum operations for performing coherent control [1, 2]. In particular, almost any pair of Hamiltonians that can be applied to a closed, finite-dimensional quantum system render it controllable [3]. Geometric control has also been used to show that almost any quantum logic gate is universal [3]. Dual to the problem of controlling a complex system is observing it: to perform quantum feedback control, both controllers and observers are necessary [2, 4]. Although less attention has been paid to the problem of observability, it is known that coherent controllability of a quantum system combined with the ability to perform simple measurements on it renders the system observable [5].

Quantum controllers and observers can be combined to perform tasks such as engineering open-systems dynamics [5], and quantum error-correction and noise suppression [6, 7]. All such tasks are fundamentally based on getting and processing information [8]. We present a simple quantum device—a universal quantum interface, or UQI—that is able to perform all these tasks simply and efficiently. The universal quantum interface consists of a single two-state quantum system, or quantum bit, that couples to a Hamiltonian system to be controlled or observed via a fixed Hamiltonian interaction. The primary purpose of this article is to show that by controlling and observing the quantum bit on its own, one can fully control and observe the system to which it is coupled.

Consider a d-dimensional quantum system S whose dynamics are described by a Hamiltonian H. Consider a two-level system Q coupled to S via a fixed Hamiltonian interaction $A \otimes \sigma_z$ where A is an Hermitian operator on S and σ_z is the z Pauli matrix with eigenvectors $|+1\rangle$ corresponding to eigenvalue $+1$ and $|-1\rangle$ corresponding to eigenvalue -1. Assume that we can both make measurements on Q in this basis, and apply Hamiltonians $\gamma\sigma$ to Q, where σ is an arbitrary Pauli matrix and γ is a real control parameter. That is, taken on its own, Q is controllable and observable (the ability to measure with respect to one basis combined with the ability to perform arbitrary rotations translates into the ability to measure with respect to any basis).

It can immediately be shown that in the absence of environmental interactions the system is generically coherently controllable. As long as H and A are not related by some symmetry, the algebra generated by $\{H + A \otimes \sigma_z, \gamma\sigma\}$ is the whole algebra of Hermitian matrices for S and Q taken together. As a result, by the usual constructions of geometric control theory [1], one can perform arbitrary Hamiltonian transformations of the system and qubit by turning on and off various σs. One such Hamiltonian transformation is an arbitrary Hamiltonian transformation on the system on its own, so the system is coherently controllable.

Now turn to observability. Since by controlling the qubit on its own we can engineer any desired Hamiltonian transformation of the system and qubit together, we can apply any evolution of the form $e^{-iG\otimes\sigma_x t}$, where G is an arbitrary Hermitian operator on S and σ_x is the x-Pauli matrix on Q. Prepare the interface in the state $|+1\rangle$ (e.g., by measuring the qubit and rotating it to $|+1\rangle$), apply this evolution, and measure Q in the $\{|+1\rangle, |-1\rangle\}$ basis. As a result of this preparation, evolution, and measurement, the system state evolves from $\rho_S(0)$ into either $\rho_S^+ = \cos(\gamma tG)\rho_S(0)\cos(\gamma tG)$ or $\rho_S^- = \sin(\gamma tG)\rho_S(0)\sin(\gamma tG)$, with probabilities $p_+ = \mathrm{tr}\cos^2(\gamma tG)\rho_S(0)$ and $p_- = \mathrm{tr}\sin^2(\gamma tG)\rho_S(0)$ respectively. In other words, this procedure effects the generalized "Yes-No" measurement on S having Hermitian Kraus operators $\cos(\gamma tG)$, $\sin(\gamma tG)$. This is the form of the most general minimally-disturbing two-outcome measurement on S [9]. In [5], it is shown how one can perform any desired generalized measurement corresponding to Kraus operators $\{A_k\}$ by making a series of such two-outcome measurements. So by the construction outlined above, where the results of the two-outcome measurements are copied to classical memory, Q can effect an arbitrary generalized measurement on S and is therefore a full semiclassical observer for S [2].

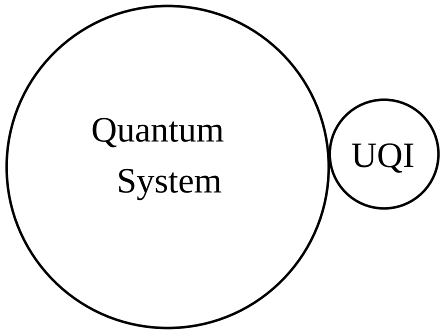

Figure 1: A universal quantum interface attaches itself to a system with Hamiltonian H via an interaction $A \otimes \sigma_z$. By measuring and manipulating the single qubit of the interface, one can control and observe the quantum system in any desired way.

Generalized measurements and generalized open-system transformations are closely related. By making a generalized measurement and ignoring the outcomes one effects the open-system transformation $\rho_S(0) \to \sum_k A_k \rho_S(0) A_k^\dagger$. So our universal quantum interface Q is not only a full semiclassical observer for S, but also a universal controller capable of performing any desired completely positive linear trace-preserving map on S [10] (see Fig. 1).

True to its name, the universal quantum interface can also act as a quantum communication channel between two quantum systems, S and S'. Let Q be coupled to S with a coupling $A \otimes \sigma_z$ and to S' with a coupling $A' \otimes \sigma_z$. As long as the algebras generated by $\{H, A\}$ and by $\{H', A'\}$ close only on the full algebras for the two systems on their own, then the algebra generated by $\{H + H' + A \otimes \sigma_z + A' \otimes \sigma_z,\ \gamma\sigma\}$ closes on the full algebra for the two systems together with Q. Consequently, Q can be used to shuttle quantum information from S to S' and *vice versa* (see Fig. 2).

The ability of quantum interfaces to perform communication tasks as well as coherent quantum information manipulation and measurement allows one to envisage a quantum control system, including sensors, controllers, and actuators, constructed of quantum systems linked via quantum interfaces, or even constructed entirely of quantum interfaces in series and parallel. Such quantum control systems could effect either coherent or incoherent quantum feedback [2, 4].

The universal quantum interface can control a quantum system, observe it, and shuttle quantum information between systems. How efficiently can it perform these tasks? Here we can use an argu-

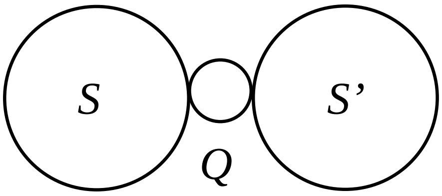

Figure 2: A universal quantum interface that interacts with two systems can serve as a quantum communication channel, mediating the flow of information between the two systems.

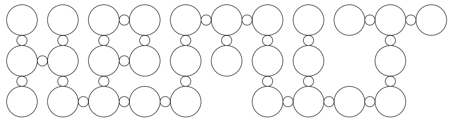

Figure 3: A set of quantum interfaces connecting low-dimensional systems makes up a quantum computer, capable of performing quantum logic operations and shuttling information between any two subsystems.

ment based on the Solovay-Kitaev theorem [11]. The transformations on the system and interface correspond to a time-dependent Hamiltonian $H + A \otimes \sigma_z + \gamma(t)\sigma(t)$. Arbitrary unitary transformations on system and interface can be built up this way. Let τ be the characteristic time that it takes to build up two unitary transformations U, U' that differ significantly from each other (i.e., $\mathrm{tr}\, U^\dagger U' \ll d$). Assuming that the unitary transformations that can be built up over times much greater than τ are distributed essentially uniformly over the space of all unitary transformations, one sees that in time t one can perform an arbitrary control or observation on a d-dimensional quantum system to an accuracy proportional to $e^{-t/\tau d^2}$. To obtain exponential accuracy requires time of $O(d^2)$.

Of course, some control tasks and observations can be performed in less time. In the case of quantum computing, we are interested in transformations of n qubits, so that the dimension of the Hilbert space is $d = 2^n$. A generic transformation can be built up out of $O(2^{2n})$ quantum logic gates. But some computations (Shor's algorithm [12], and quantum simulation [13], for example) can be performed in time polynomial in n, i.e., polylogarithmic in d.

A universal quantum interface can effect any desired transformation on the system to which it is connected, including quantum logic transformations. But if the system to which it is connected is high-dimensional, e.g., $d = 2^n$, the interface cannot necessarily effect those transformations efficiently. In particular, a desired quantum logic operation could take time $O(2^{2n})$ to effect. The general condition on H and A under which it is possible to perform quantum computation efficiently on a $d = 2^n$ dimensional system is an open question.

If one uses quantum interfaces to control and connect a number of quantum systems, however, one can in general perform efficient universal quantum computation. A specific architecture in which universal quantum interfaces can be used to perform universal quantum computation is one in which n small-dimensional systems are coupled together via quantum interfaces as described above (see Fig. 3). Any set of pairwise couplings between systems that forms a connected graph now allows efficient universal quantum computation as follows.

First, consider the problem of performing coherent quantum logic operations on the coupled systems. Prepare the interfaces in the state $|+1\rangle$ by measuring them. Each interface is now in an eigenstate of the Hamiltonians $H_j + A_j \otimes \sigma_z$ that couples it to its connecting systems. As a result, the systems are all effectively uncoupled and evolve by renormalized versions of their respective Hamiltonians: the jth system evolves via the Hamiltonian $H_j + A_j$. By coherently controlling the interface between the jth and kth system, one can effect an arbitrary coherent transformation of these two systems together, returning the interface to the state $|+1\rangle$. That is, one can perform any desired quantum logic transformation on any two systems that are connected by an interface. While this quantum logic transformation takes place, the other systems evolve in an uncoupled fashion via known Hamiltonians.

Since the graph that describes the interfaces is fully connected, quantum information can be moved at will throughout the set of coupled systems by sequential pairwise couplings intermediated by the interfaces. The maximum number of pairwise operations required to bring any two qubits into adjacent systems is $O(n)$. Arbitrary quantum logic transformations can be performed on systems in a pairwise fashion. As a result, any desired quantum logic circuit of N logic gates can be built up using no more than $O(d^2 nN)$ pairwise operations, where d is the typical dimension of a subsystem. If the systems are qubits then the quantum logic circuit can be built up in $O(nN)$ operations. For example, the coupled systems could themselves be quantum interfaces, so that an entire quantum computer could be constructed from interfaces alone.

State preparation and measurement can be accomplished in a similar fashion. By manipulating and measuring a given interface, while keeping the other interfaces 'turned off' via the decoupling procedure given above, one can perform any desired generalized measurement on the systems to which that interface is coupled. This procedure allows one both to prepare and to measure the state of those systems. Since state preparation, coherent quantum logic operations, and measurement can all be accomplished efficiently, the set of systems coupled by universal interfaces can perform universal quantum computation.

Universal quantum interfaces are simple systems that can be used to perform arbitrary quantum operations—control, observation, and computation—on quantum systems. Note that the derivations above depend on the fact that the systems to be controlled or observed are closed apart from the interactions with their interfaces. If the systems to be controlled or observed are open to the environment, as all systems are to a greater or lesser degree ('no quantum system is an island entire unto itself'), then only those operations which can be performed efficiently within the system's decoherence time can actually be effected. An interesting open question for further research is the degree to which quantum interfaces can be used to protect quantum systems and effectively decouple them from their environment via the use of symmetries [14], bang-bang techniques [15], or analogs of quantum error correcting codes [6].

The straightforward requirements for universality allow many candidates for quantum interfaces. Many systems that are frequently used to couple to quantum systems are universal quantum interfaces. For example, a mode of the electromagnetic field that couples to an optical cavity can be used to control and observe the contents of the cavity, as in quantum computing using cavity quantum electrodynamics [16]. In an ion trap, the internal and vibrational states of the ions could be controlled and observed using just one ion in the trap (for example, an ion of a different species from the other ions in the trap [17]). In general, a single optically active site on a molecule, e.g., one held in optical tweezers to minimize coupling to the environment, could be used to control and observe the quantum states of the molecule. If the electronic and hyperfine states of the atoms in the molecule can be addressed either individually or in parallel, such a molecule addressed via an optical quantum interface is a good model for quantum computation. In liquid state NMR, it is possible to control and observe the state of the nuclear spins in a molecule by observing just one nuclear spin on the molecule while using coherent control to shuttle quantum information from the spins to be observed to the observed spin [18]. In coherent superconducting circuits, for example ones made up of several coupled charge or flux qubits, the state of the entire circuit can in general be coherently controlled and observed simply by controlling and observing a single qubit, which could be specially designed for this purpose [19].

Universal quantum interfaces are devices that can be used to control and observe a quantum system in any desired fashion. Because of their simple nature, universal quantum interfaces are considerably easier to exhibit experimentally than is a universal quantum computer. Indeed, existing interfaces with cavity QED, ion-trap, and NMR systems are already universal. Networks of quantum interfaces can be used to perform arbitrarily difficult quantum control tasks in principle, including full-blown quantum computation. In practice, complicated quantum information processing tasks involving many quantum interfaces are of the same order of difficulty to perform as quantum computation. Open questions include problems of efficiency, convergence [20], networkability, and interfaces with quantum systems that interact strongly with their environment.

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