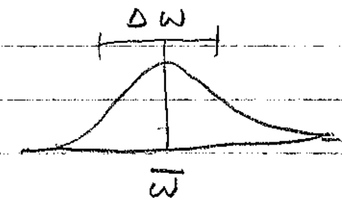
Ph262 Lecture 13

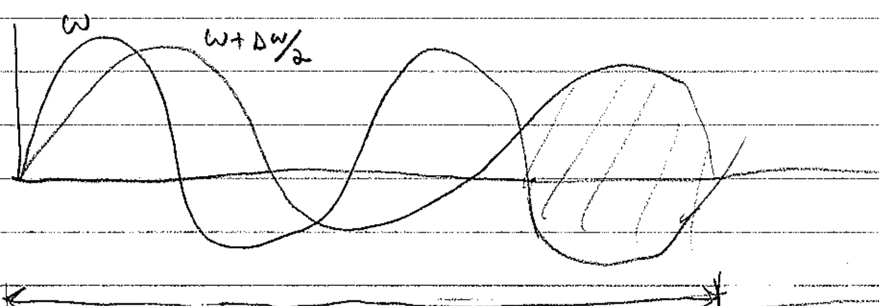
Coherence: (not in book)

$$\sum_i E_i \cos(\underbrace{\omega_i}_{\text{unknown}} t + \phi)$$



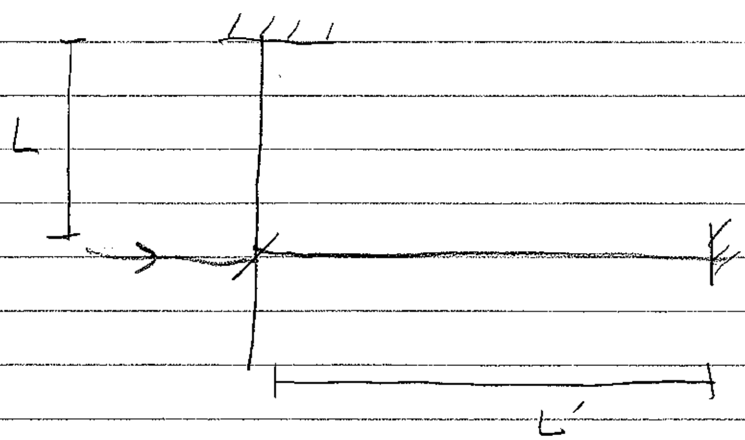
$\frac{\Delta\omega}{2} t = \pi + 2\pi m$ destructive interference

$$\boxed{\tau_c = \frac{2\pi}{\Delta\omega} = \frac{1}{\Delta\nu}}$$ [Coherence time]



$$\boxed{l_c = c\tau_c = \text{longitudinal coherence length}}$$

Michelson Interferometer measures temporal coherence.



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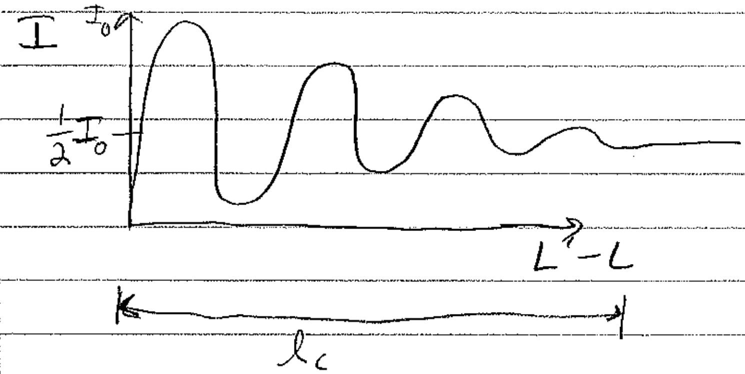
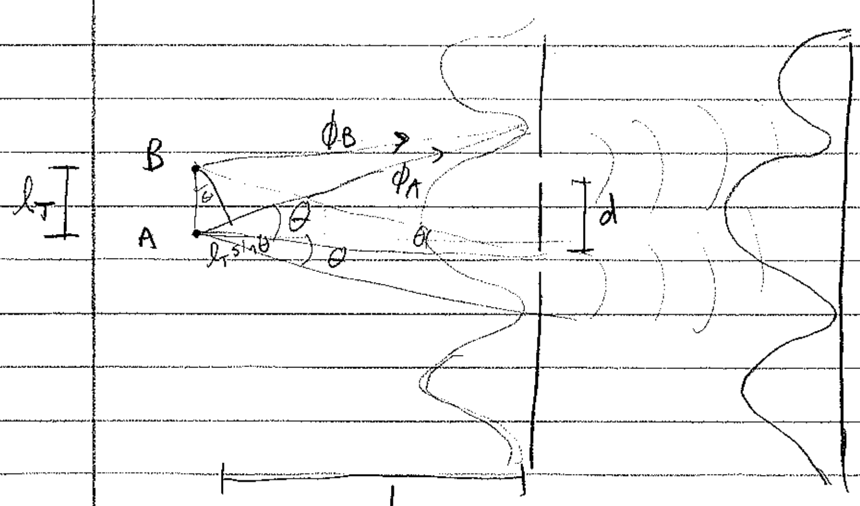


Improve temporal coherence by adding a temporal frequency filter $\Delta\omega \to$ small

$\Rightarrow \tau_c \to$ long

Spatial Coherence

Young's Double Slit measures spatial coherence.



A + B act as single point source at slits if both slits fall in first maximum.

$$\phi_A - \phi_B = \frac{2\pi}{\lambda} l_T \sin\theta = \pi + 2\pi m \quad \text{(destructive)}$$

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$$\sin \theta_m = \frac{\lambda}{l_T}\left(m + \frac{1}{2}\right)$$

$$d \leq L(\sin\theta_1 - \sin\theta_0)$$

$$\leq L\frac{\lambda}{l_T}$$

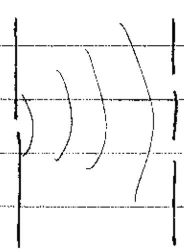
$$l_T \leq \lambda\left(\frac{L}{d}\right)$$

$$\tan\theta = \frac{d}{L} \sim \theta \qquad (\text{small } \theta)$$

$$\Rightarrow l_T \leq \frac{\lambda}{\theta}$$

$$\boxed{l_T = \lambda/\theta} \qquad [\text{Transverse coherence length}]$$

Improve spatial coherence by adding a spatial frequency filter.



Resolvability (roughly)

$$l_{object} \geq \lambda/\theta_{Detector} \Rightarrow \text{resolvable}$$

$$= \lambda L / D_{detector}$$

$$\boxed{\lambda/D_{detector} \leq \frac{l_{object}}{L} = \theta_{object}}$$

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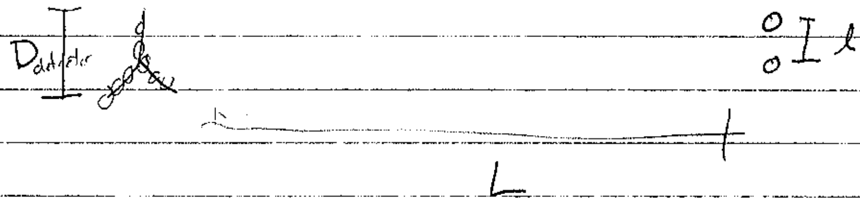
Example: VLA detecting binary stars. How close is resolvable?



$D_{VLA} \sim 36\ \text{km} = 36 \times 10^3\ \text{m}$

$L \sim 1\ \text{pc} \sim 3\ \text{ly} \sim \underline{3\pi \times 10^7\ \cancel{s} \cdot 3 \times 10^8\ \frac{m}{\cancel{s}}} \sim 27 \times 10^{15}\ \text{m}$

$F \sim 1000\ \text{MHz} \sim 10^9\ s^{-1}$

$c = f\lambda \Rightarrow \lambda = \frac{3 \times 10^8\ m/s}{10^9\ s^{-1}} \sim 3 \times 10^{-1}\ \text{m}$

$\frac{\lambda}{D} \sim \frac{l}{L}$

$l \sim \frac{L\lambda}{D} = \frac{(27 \times 10^{15}\ m)(\cancel{3} \times 10^{-1}\ m)}{\underset{12}{\cancel{36}} \times 10^3\ m} \sim 2.25 \times 10^{11}\ \text{m}$

$\sim \cancel{2} \times 10^{11}\ m \left| \frac{s}{\cancel{3} \times 10^8\ m} \right| \frac{AU}{5 \times 10^2\ s} \sim 0.2 \times 10^1\ AU$

$\sim \boxed{2\ AU}$

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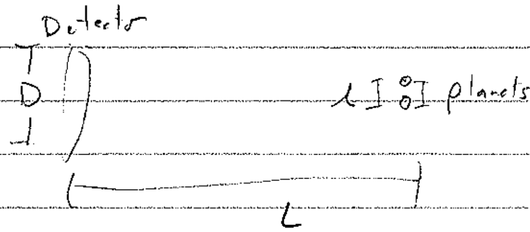
Example: Planet Imaging (Prob 36.72)

$D_{detector} \sim 6000\ km$



$\lambda_{IR} \sim 10\ \mu m$

$l_{object} \sim 200\ km$

How far can 200 km features be seen from?

$\frac{\lambda}{D} \sim \theta \sim \frac{l}{L}$

$L \sim \frac{Dl}{\lambda} = \frac{(6\times10^{6}\ m)(2\times10^{5}\ m)}{1\times10^{-5}\ m}$

$= 12\times10^{16}\ m \left| \frac{s}{3\times10^{8}\ m} \right| \frac{y}{\pi\times10^{7}\ s} = \frac{4}{\pi}\times10^{1}\ ly$

$\boxed{L \sim 13\ ly}$