UNM Physics 262, Fall 2006
SAMPLE Midterm Exam 1: Optics

Name and/or CPS number: Dr. Landahl — Solution Key

Show all your work for full credit. Remember that quantities have units and vectors have components (or magnitude and direction). **ASK** if anything seems unclear.

CALCULATORS AND CELL PHONES ARE PROHIBITED.
USE OF THESE WILL RESULT IN A ZERO FOR THE EXAM.
Keep any factors of π, e, $\sqrt{2}$, etc. in your answers.

You may use a single 8.5" × 11" paper containing notes you have prepared ahead of time to assist you.

Apportion your time sensibly. Spend about 10–12 minutes per problem.

$\boxed{\text{Please put a box around your final answers.}}$

Useful constants:

$c = 3 \times 10^8$ m/s

This is for me to use in grading

Problem 1: 26/26

Problem 2: 24/24

Problem 3: 28/28

Problem 4: 22/22

100/100

1. Faraday's Law and Maxwell's equations [26 points]

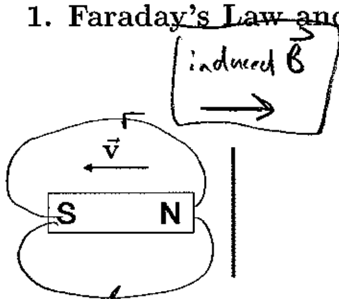


Fig. 1a: Side view

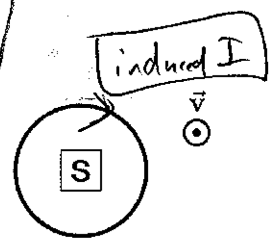


Fig. 1b: Front view

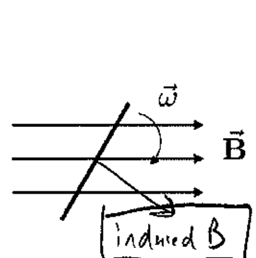


Fig. 2a: Side view

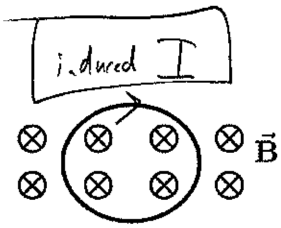


Fig. 2b: Front view

(*a*) A bar magnet is moving away from the wire loop in Fig. 1 with its North end closest to the loop.

[3] *i*) Draw and label the direction of the induced magnetic field in Fig. 1a. Explain your reasoning. The $\vec{B}$ field from the magnet goes through the loop to the right and is decreasing. Lenz' Law opposes this change, inducing a $\vec{B}$ field to the right to compensate for the loss.

[3] *ii*) Draw and label the direction of the current induced in the loop in Fig. 1b. Explain your reasoning.
In Fig 1b, the induced $\vec{B}$ field is into the page. By the right hand rule, the current circulates around this $\vec{B}$ field clockwise.

(*b*) A wire loop oriented perpendicular to the page is rotating clockwise in a magnetic field that points to the right. Fig. 2 depicts this situation with the wire loop drawn in cross-section.

[3] *i*) Draw and label the direction of the induced magnetic field in Fig. 2a. Explain your reasoning.
The $\vec{B}$ flux through the loop is decreasing as it rotates. Lenz' Law says a $\vec{B}$ field is created to oppose this loss, so the induced field is perpendicular to the loop to the right (and slightly down).

[3] *ii*) Draw and label the direction of the current induced in the loop in Fig. 2b. Explain your reasoning.
The current circulates via the right hand rule around the induced $\vec{B}$ field, which is into the page in this figure. Hence the current circulates clockwise.

(c) A circular wire loop of radius R lies flat in the xy plane, centered at the origin. It experiences a time-dependent magnetic field

$$\vec{\mathbf{B}}(x,y,z,t) = B_0\left(\frac{rt}{a}\hat{\mathbf{x}} + \frac{r^2t^2}{a^2}\hat{\mathbf{y}} + \frac{r^3t^3}{a^3}\hat{\mathbf{z}}\right),$$

where $r = \sqrt{x^2+y^2+z^2}$ is the distance from the origin and B_0 and a are parameters.

[8] *i)* Write down the integral for the flux of this field through the wire loop in terms of B_0, t, r, a, and R. Your integral should be solely in the integration variable r. Be sure to indicate the direction of this flux.

$$\Phi = \int \vec{B}\cdot\hat{n}\,da \qquad \hat{n} = \hat{z}, \quad da = 2\pi r\,dr$$

$$= \frac{B_0 t^3}{a^3}\int_{r=0}^{R} 2\pi r\,dr\; r^3\,\cancel{\hat{z}\cdot\hat{z}}^{\,1}$$

$(\hat{x}\cdot\hat{z} = \hat{y}\cdot\hat{z} = 0$, so only the $\hat{z}\cdot\hat{z}$ term survives$)$

$$\boxed{\Phi = \frac{2\pi B_0 t^3}{a^3}\int_{r=0}^{R} r^4\,dr}$$

[6] *ii)* Calculate the integral and use it with Faraday's Law to obtain an algebraic expression for the emf generated in the loop. Be sure to indicate the direction of this emf.

$$\Phi = \frac{2\pi B_0 t^3}{5a^3}R^5. \qquad \mathcal{E} = \frac{-d\Phi}{dt} \quad [\text{Faraday's Law}]$$

$$\boxed{\mathcal{E} = -\frac{6\pi B_0 t^2}{5a^3}R^5}$$

By the right hand rule, circulation around $\hat{n} = \hat{z}$ is counterclockwise when viewed from above, so the emf is in this direction. Because of the minus sign, positive emf is instead clockwise

2. E & M Optics [24 points]

A sinusoidal plane electromagnetic wave in free space has an electric field of

$$\vec{\mathbf{E}} = E_0 \cos\left(\frac{2\pi}{\lambda}(z + ct)\right)\hat{\mathbf{x}},$$

where $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$ forms a right-handed coordinate system, as usual.

[6] *a*) In what direction is the wave traveling? Explain your reasoning.

Left-moving/right moving solutions to the wave equation are of the form $f(z+ct)$ / $f(z-ct)$. Moving left along the z-direction is in the minus z-direction, so the wave travels $\boxed{\text{in the } -\hat{z} \text{ direction.}}$

(The fact that $\vec{E}$ is polarized in the $\hat{x}$-direction doesn't affect this answer.)

[10] *b*) Write down a **vector** expression for the magnetic field associated with this wave in terms of the quantities given in the electric field and any additional constants you may need. Explain your reasoning. You may find the following relationships useful.

$$\frac{\partial}{\partial z}E_x(z,t) = -\frac{\partial}{\partial t}B_y(z,t) \qquad \frac{\partial}{\partial z}B_y(z,t) = -\frac{1}{c^2}\frac{\partial}{\partial t}E_x(z,t).$$

Using either equation, we can extract B_y from E_x. Because $\vec{B} \perp \vec{E}$ for EM waves and because $\vec{B} \perp (-\hat{z})$ (the propagation direction), $\vec{B} = B_y\hat{y}$. Using the left formula yields

$$B_y = -\int dt \frac{\partial}{\partial z} E_0 \cos\left(\frac{2\pi}{\lambda}(z+ct)\right) = \frac{E_0 2\pi}{\lambda}\int dt \sin\left(\frac{2\pi}{\lambda}(z+ct)\right)$$

$$= -\frac{E_0 \cancel{2\pi}}{\cancel{\lambda}}\frac{\cancel{\lambda}}{\cancel{2\pi} c}\cos\left(\frac{2\pi}{\lambda}(z+ct)\right) + \cancel{\text{const}}$$

not part of wave.

$$\Rightarrow \boxed{\vec{B} = \frac{-E_0}{c}\cos\left(\frac{2\pi}{\lambda}(z+ct)\right)\hat{y}}$$

[8] *c*) What average radiation pressure does this wave exert at normal incidence on a totally light absorbing wall? Explain your reasoning.

$$P_{rad}^{emit} = \frac{I}{c} = P_{rad}^{absorb} \qquad \text{(definition of radiation pressure)}$$

from part (b)

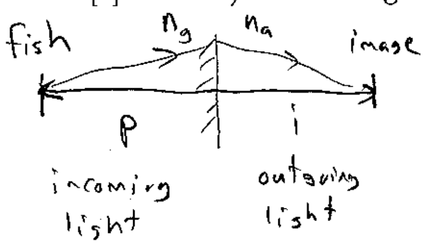
$$I = \langle|\vec{S}|\rangle = \frac{1}{2\mu_0}|\vec{E}_{max}\times\vec{B}_{max}| = \frac{E_0^2}{2\mu_0 c} \qquad \text{(definition of avg. intensity)}$$

$$\boxed{P_{rad}^{absorb} = \frac{E_0^2}{2\mu_0 c^2}}$$

3. Geometric Optics [28 points]

The Albuquerque Aquarium has glass portals on its water tanks where patrons can view the fish inside. At one portal, an 8 cm tall fish is 24 cm behind the glass and forms an image 6 cm on the other side of the glass. Ignore the glass on the wall and take the portal to be a refractive interface between water (refractive index 1.5) and air (refractive index 1.0).

[6] *a*) Is the image upright or upside-down? Explain your reasoning.



Sign conventions: p, incoming same side: $p > 0$

i, outgoing same side: $i > 0$

Magnification: $m = -\frac{n_g}{n_a}\frac{i}{p} < 0 \Rightarrow$ $\boxed{\text{upside-down image}}$

[6] *b*) What is the magnitude of the height of the image? Explain your reasoning.

$$h = |\text{image height}| = |m|\cdot 8\text{ cm} = \left|\frac{-1.5}{1.0}\,\frac{6}{24}\right|\cdot 8\text{ cm} = \frac{9}{24}\cdot 8\text{ cm} = \frac{9}{3}\text{ cm}$$

$$\boxed{h = 3\text{ cm}}$$

[8] *c*) What is the magnitude of the radius of curvature of the wall? Explain your reasoning.

Refractive imaging equation:

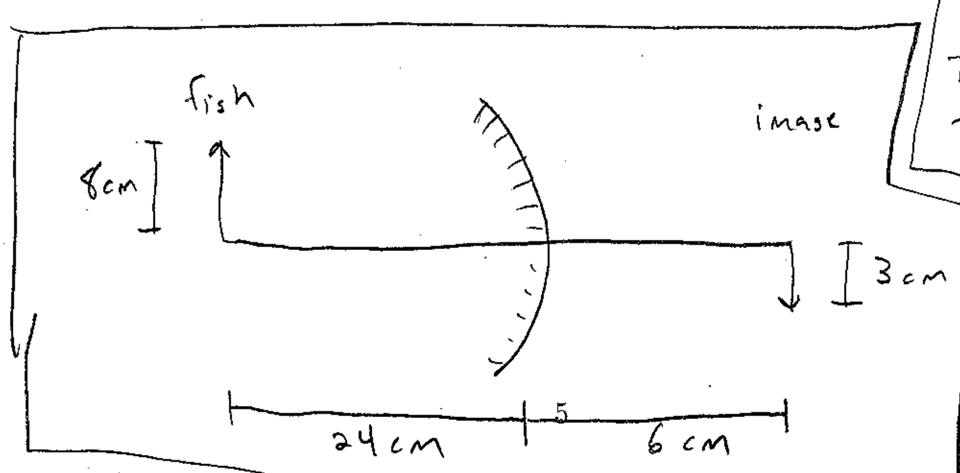
$$\frac{n_r}{p} + \frac{n_i}{i} = \frac{(n_i - n_r)}{r}$$

$p > 0, i > 0$ so we can use this form.

$$\frac{1.5}{24\text{cm}} + \frac{1.0}{6\text{cm}} = \frac{(1-1.5)}{r} \to \frac{1.5+4.0}{24\text{ cm}} = \frac{-1/2}{r} \to r = \frac{-1/2\cdot 24\text{ cm}}{5.5} = \frac{-24}{11}\text{ cm}$$

$$\boxed{|r| = 24/11\text{ cm}}$$

[8] *d*) Draw a picture of the situation, indicating where the object and image are located in relation to the portal, what their heights are, and which way the portal is curved. Explain your reasoning for the curvature you draw.

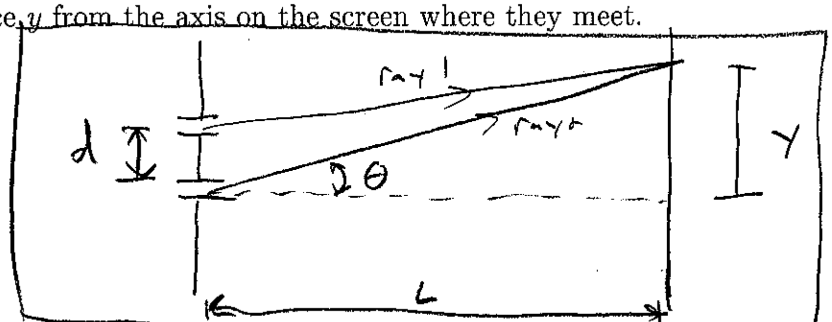


Since $r < 0$ from (c), The sign conventions say the radius of curvature is on the incoming side of light.

4. Wave Optics [22 points]

Coherent light of wavelength λ enters a double slit apparatus having slit separation d. The light creates an interference pattern on a screen placed a distance L far away from the slits.

[6] *a*) Sketch the situation, labeling d, L, and rays for the two waves that interfere on the screen. Also label the angle θ that these rays make with the the slit-screen axis and the distance y from the axis on the screen where they meet.



[8] *b*) Write down an expression for the phase difference between these two waves after they pass through the apparatus and recombine on the screen. Express your answer in terms of the variables in part (*a*) and λ. Explain your reasoning.

$$\Delta\phi = k\, d \sin\theta = \boxed{\frac{2\pi}{\lambda} d\sin\theta}$$

This is because ray 2 travels further than ray 1 by a distance $d\sin\theta$.

[8] *c*) For $\lambda = 500$ nm, the first off-axis minimum is produced on the screen a distance 10 mm from the axis. Where is the second minimum located on the screen? Explain your reasoning.

$\Delta\phi = \pi + 2\pi m$ at a minimum.

$$2\pi \frac{d\sin\theta}{\lambda} = \pi + 2\pi m \quad \rightarrow \quad \sin\theta = \lambda(m + 1/2)$$

The locations of the minima on the wall are

$$Y_m = L\sin\theta = \frac{L\lambda}{d}(m + 1/2)$$

So $Y_1 = \frac{L\lambda}{d}(3/2)$ and $Y_0 = \frac{L\lambda}{d}(1/2)$

Hence $Y_1 = 3Y_0 = 3(10\text{ mm}) = \boxed{30\text{ mm}}$